Growth rates of dike-fed plutons: Are they compatible with observations in the middle and upper crust?

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ABSTRACT
Diapirism and dike-fed expansion are two popular models for the ascent and emplacement of large batches of granitoid magma into the middle and upper crust. Many structural and petrologic relationships in and around plutons can be cited as permissive evidence for either model, and so it has been difficult to differentiate between them on these grounds. Here we consider wall-rock strain rates resulting from dike-fed expansion at a constant-volume pluton-filling rate. Considering end-member pluton shapes and dike flow rates, and dikes 1 km in plan length, the dike-fed expansion model predicts wall-rock strain rates over the entire pluton-growth period that are up to six orders of magnitude faster than the fastest accepted regional tectonic strain rates ($10^{-13}$ s$^{-1}$). Strain rates in the very early stages of growth can reach values several orders of magnitude faster. Our results suggest that evidence for or against very fast strain rates in deformation aureoles around plutons may form a key criterion for identifying dike-fed expansion. On the other hand, our ability to extract strain-rate information from pluton aureoles is currently extremely limited.

Keywords: deformation, diapirs, dikes, plutons, rates, strain.

INTRODUCTION
Igneous intrusions commonly make up 25%–75% of exposed rocks in eroded arcs, yet there is no consensus regarding how large volumes of granitoid magma ascend through the crust and become emplaced as middle- to upper-crustal plutons. Several papers published in the past 10 years have emphasized the importance of granitoid magma ascent in dikes and concluded that many large granitoid plutons are fed by such dikes (e.g., Clemens and Mawer, 1992; Petford et al., 1993). These models have led to the more provocative suggestion that magmatic diapirism be discarded as an ascent mechanism (Clemens et al., 1997; Clemens, 1998). If magmatic diapirism is unimportant, then a profound implication is that many or most plutons exposed in eroded arcs may form a key criterion for identifying dike-fed expansion. On the other hand, our ability to extract strain-rate information from pluton aureoles is currently extremely limited.

Keywords: deformation, diapirs, dikes, plutons, rates, strain.

rates and example plutons
To evaluate pluton-filling rates and wall-rock strain rates, we must choose a horizontal dike length and a realistic range of dike widths and magma-flow rates. Following Clemens et al. (1997), we consider dikes 1 km in plan length, end-member critical dike widths of 3 and 16 m, and end-member flow rates of $3 \times 10^{-3}$ and 8.0 m·s$^{-1}$. It is instructive to evaluate two end-member pluton volumes (Fig. 1), and for these we choose a sphere of 5 km radius and 524 km$^3$ volume, and a spheroid of 5 km radius in plan and 0.915 km vertical radius. This spheroid has a volume of 99 km$^3$ and reflects the following power-law relationship between thickness ($T$) and plan length ($l$) suggested by McCaffrey and Petford (1997) for granitoid plutons:

$$T = 0.29 l^{0.80}. \quad (1)$$

We choose these two bodies because they have very different volumes but would show identical map-view patterns in central sections. Table 1 shows filling rates and total filling times in years for the various combinations of dike widths, flow rates, and pluton volumes under consideration. To understand...
Figure 1. Three-dimensional surface plots of (A) sphere and (B) spheroid. Both have horizontal radii of 5 km in central sections.

Figure 2. Graph showing nonlinear relationship between radius and volume increase for sphere and spheroid.

Figure 3. Graphs showing nonlinear increase in radius with time for (A) sphere and (B) spheroid, for four fill rates shown in Table 1. Total time $T = 1$ yr.

Figure 4. Graphs showing change in strain rate with pluton radius for (A) sphere and (B) spheroid, for four fill rates shown in Table 1. Total time $T = 1$ yr.

Table 1. Strain Rates Averaged Over Total Pluton-Filling Time for Horizontal Dike Length of 1 km

<table>
<thead>
<tr>
<th>Dike width (m)</th>
<th>3</th>
<th>16</th>
<th>3</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate ($m \cdot s^{-1}$)</td>
<td>$3 \times 10^{-3}$</td>
<td>$3 \times 10^{-3}$</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Fill rate ($km^2 \cdot yr^{-1}$)</td>
<td>0.28</td>
<td>1.51</td>
<td>756.86</td>
<td>4036.61</td>
</tr>
<tr>
<td>Total filling time (yr)—sphere</td>
<td>1846.21</td>
<td>346.17</td>
<td>0.69</td>
<td>0.13</td>
</tr>
<tr>
<td>Total filling time (yr)—spheroid</td>
<td>348.81</td>
<td>65.40</td>
<td>0.13</td>
<td>0.02</td>
</tr>
</tbody>
</table>

how such rapid filling rates affect the surrounding wall rocks, we need expressions for the increase in radius ($R$) of the two hypothetical plutons with increase in volume ($V$), as follows:

$$R_{\text{sphere}} = \frac{3V}{4\pi}, \quad (2)$$

$$R_{\text{spheroid}} = \frac{3V}{(0.18)(\pi)(4)}, \quad (3)$$

Figure 2 shows the nonlinear relationship between radius and volume increase, and also the dramatic difference in volume between these two plutons of equal final plan radius (5 km). We can represent the horizontal radii of the two plutons in relation to time by the following two equations:

$$R_{\text{sphere}} = \frac{3(\Delta t)(dV/dt)}{4\pi}, \quad (4)$$

$$R_{\text{spheroid}} = \frac{3(\Delta t)(dV/dt)}{(0.18)(\pi)(4)}, \quad (5)$$

where $\Delta t$ is time and $dV/dt$ values are the fill rates given in Table 1. The graphs in Figure 3 suggest that strain rates in the adjacent wall rocks should change markedly with radius and time, and to determine these rates we first need to assign a strain-aureole width around the final pluton. We select a horizontal width of 3 km around both plutons and assume for now that the aureole deforms homogeneously by pure shear; we show later that partitioning of strain in the aureole has only a second-order effect on wall-rock strain rates. We can define the finite stretch ($S$) as follows:

$$S = \frac{l_0 - R}{l_0} = 1 - \frac{R}{l_0}, \quad (6)$$

where $l_0 = 8$ km (final pluton radius plus deformation aureole), $R$ is the pluton radius ranging from 0 to 5 km, and $l_0 - R$ is the deformed length of the aureole at any stage of pluton growth. Because we want to determine how strain rate changes with time and pluton radius, we define the instantaneous strain rate ($\dot{e}$) as the rate of change of the stretch (Twiss and Moores, 1992) as follows:

$$\dot{e} = \dot{S} = \frac{d}{dt} \left[ 1 - \frac{R}{l_0} \right], \quad (7)$$

which is equivalent to the “conventional” strain rate of Pffnfer and Ramsay (1982). Our results could also be expressed as the “natural” strain rate, which is simply the conventional strain rate divided by the stretch. Because the stretch in this instance becomes smaller with time, the natural strain rate would reach values of approximately one order of magnitude greater than the conventional strain rate in the final stages of pluton growth. Thus, the conventional strain rate is the more conservative of the two. Equation 7 can be solved to obtain the change in strain rate with radius for the sphere and spheroid (Fig. 4). More specifically, Figure 4 shows the change in shortening rate in the horizontal plane passing through the pluton centers. These graphs show that strain rates vary by up to six orders of magnitude during pluton filling, but the first three orders of magnitude occur before the pluton has a radius of 150 m. For simplicity, we have considered dikes that are 1 km in plan length from the smallest time step. A more realistic model would allow a dike’s plan length to grow with the pluton diameter until it reached 1 km, at which time the dike would
remain 1 km in length while the pluton continued to grow. Our preliminary results from such a model also indicate that strain rates will vary by up to six orders of magnitude, but the first three orders of magnitude occur before the pluton has a radius of 40 m. Thus, regardless of which model is considered, half of the strain-rate variation occurs very early in the inflation history, prior to the pluton radius reaching 3% of its final dimension.

**STRAIN PARTITIONING IN AUREOLES**

These calculations assume homogeneous deformation in the 3-km-wide aureole, but rocks exhibit both temperature- and strain-rate–dependent rheology, and so strain intensity in ductile aureoles is strongly concentrated near the pluton margins, falling off exponentially with distance (e.g., Paterson and Fowler, 1993; Weinberg and Podladchikov, 1995). We can model nonlinear variation of strain rate across the aureole by applying the exponential equation of the form $\dot{\varepsilon} = \dot{\varepsilon}_0 e^{-c x}$, where $\dot{\varepsilon}$ is strain rate ($s^{-1}$), $\dot{\varepsilon}_0$ is the initial strain rate at distance $= 0$ m and time $t$, $x$ is the distance from the contact between pluton and host rock, and $c$ is a constant that determines the total variation in strain rate through the aureole at any particular time. This constant was chosen by assuming 95% and 10% shortening in the inner and outer parts, respectively, of the 3-km-wide aureole. These are considered to be reasonable upper and lower limits of measurable strain, and so are reasonable for defining the deformation aureole. We calculated inner- and outer-aureole strain rates around both plutons for each of the four filling times. We then compared inner- and outer-aureole strain rates at specific times, and used the differences in these rates as the total variation across the aureole at any one time. The constant $c$ was set by iteration to a value of 0.000 90 m$^{-1}$ to reproduce these results.

We combine the pluton-radius and distance dependence of strain rate and establish the following relationship:

$$\dot{\varepsilon} = \frac{d}{dR} \left[ \frac{1 - R}{l_0} \right] e^{-c x}.$$  

Calculations were performed for those combinations of flow rate and feeder-dike dimension corresponding to the lowest and highest filling rates for the plutons with spherical and spheroidal geometry, respectively. Figure 5 emphasizes that the spatial variation in strain rate throughout the aureole (as defined) at any one time is less than one order of magnitude, which is consistent with the maximum difference in finite strain of less than one order of magnitude. Thus, a simple model like the one developed here that considers homogeneous strain is adequate for assessing what order-of-magnitude strain rates can be expected in the more highly strained rocks in an aureole, given particular rates of pluton inflation.

**MATERIAL TRANSFER PROCESSES IN DEFORMATION AUREOLES**

These calculations assume that all deformation in the aureole occurs by homogeneous, concentric shortening perpendicular to the growing pluton margins. However, quantitative strain studies around plutons (e.g., studies summarized in Paterson and Fowler, 1993; Johnson et al., 1999) indicate that generally only 25%–40% of a pluton’s volume can be accounted for by bulk wall-rock shortening, and so other processes must operate (e.g., stopping, assimilation, volume loss, rigid rotation, and sill formation followed by vertical pluton growth). The filling rates and wall-rock strain rates determined here suggest that there would be very little time for wall-rock volume loss or assimilation, and so the principal combination of processes would most likely include the brittle process of stopping driven by thermal and/or mechanical stress, the brittle processes of sill emplacement or fracture propagation, the ductile process of lateral wall-rock shortening, and rigid rotation of units, presumably by a flexural-slip mechanism.

**TESTING THE DIKE-FED MODEL**

Our analysis suggests that evidence for fast strain rates in pluton aureoles might be one of the best tests of the dike-fed expansion model, but what constitutes evidence for fast strain rates? Pluton aureoles are not characterized by steady-state thermal and dynamics conditions, and so quantifying strain rates from microstructures in these settings is currently tenuous. In light of this limitation, we offer the following as possible indicators of fast strain rates.

1. If a rapidly inflating pluton grows via an outward-migrating “shatter zone” or intrusive brecciation, then it seems reasonable to assume that examples of such zones may be preserved around some plutons. One example of such a shatter zone may be preserved around the margins of the Cadillac Mountain intrusive complex (e.g., Gilman and Chapman, 1988; Wiebe, 1996) in the coastal Maine magmatic province (Hogan and Sinha, 1989). We are investigating this possibility.

2. Other brittle features that could be caused by fast strain rates include: (a) radiating fractures sets oriented perpendicular to the pluton margin and the direction of minimum compressive stress; (b) local or extensive zones of brecciation; and (c) fracturing of minerals that were present prior to emplacement, although later dynamic recrystallization may mask these features as the thermal front passes outward from the pluton. These features could result from other factors (e.g., high fluid pressures), but in general we might expect their occurrence to increase with increasing strain rate.

3. If a rapidly-inflating pluton grows by brittle wall-rock deformation, a large volume of stoped wall rock may have been incorporated into the pluton at least in the early phases of emplacement. Recent work on the processes involved in stopping (e.g., stress fracturing, partial melting, mechanical disaggregation) suggests that stoped fragments may be reduced to a mixture of partial melts and individual mineral grains if residence times in the magma are long enough (e.g., Clarke et al., 1998; Paterson and Okaya, 1999). Detailed microstructural, isotopic, and rare-earth-element studies of ferromagnesian minerals in plutons that show evidence for stopping may provide important clues as to what percentage of these grains may have originated from the wall rocks.

4. Contact metamorphic minerals should grow relatively late in the emplacement-related deformation, owing to conductive heat transfer lagging behind chamber growth. An interesting implication is that “static” metamorphic aureoles around plutons may in some instances have been quite dynamic environments during pluton emplacement.

5. Wall rocks are commonly intruded by dikes from the main pluton during its emplacement. Rapidly inflating plutons may cause such dikes to become isoclinally folded before cooling below their solidus. Dikes folded in the magmatic state should show magnetic, rather than solid-state, axial surface foliations. This evidence would perhaps be most useful around middle- to upper-crustal plutons where ambient wall-rock temperatures are...
well below the solidus temperatures of the dikes.

EVIDENCE FOR HIGH WALL-ROCK STRAIN RATES?

Tectonic strain rates are generally thought to be in the range of $10^{-13}$ to $10^{-15}$ s$^{-1}$ (e.g., Piffner and Ramsay, 1982; Paterson and Tobisch, 1992; Dunlap et al., 1997; foster and Gray, 1999; Muller et al., 2000). In contrast, theoretical analysis of dike and diapir ascent rates suggests that faster-than-tectonic nearfield strain rates should be expected during pluton construction (e.g., Clemens and Mawer, 1992; Weinberg and Podladchikov, 1995; pavlis, 1996; petford, 1996; this study). Has any evidence for such fast rates been described?

Field studies of individual plutons have led some workers to suggest faster-than-tectonic strain rates during pluton emplacement. For example, rates on the order of $10^{-11}$ to $10^{-12}$ s$^{-1}$ have been postulated in the aureoles of the sheeЛike East Piute and Old Woman plutons in southern California (karlstrom et al., 1993; mccaffrey et al., 1999), accompanied by an increase in minimum fault-displacement rates to greater than tens of centimeters per year. Fernandez and Castro (1999) suggested that strain rates of $10^{-10}$ to $10^{-11}$ s$^{-1}$ formed in local “kink-like” openings during magma emplacement. John and Blundry (1993) examined expansion of the Adamello massif, Italy, and estimated strain rates of $10^{-13}$ s$^{-1}$ for ductile deformation in the aureole. Miller and Paterson (1994) described isoclinally folded tonalite to granodiorite dikes around the Mt. Stuart batholith with magmatic axial surface foliations. Finite difference thermal modeling suggests that these $0.5$–$1.5$-m-thick dikes crystallized in no more than a few hundred thousand years and typically record >$80\%$ shortening. Thus, a strain rate between $5 \times 10^{-13}$ and $1 \times 10^{-11}$ s$^{-1}$ is required to form these folds (Paterson and Tobisch, 1992).

All of these strain rates overlap the rates of $10^{-10}$ to $10^{-13}$ s$^{-1}$ suggested by Schmid (1989) for some high-strain mylonite zones. However, they do not approach the faster end of the range of strain rates shown in figure 4.

CONCLUSIONS

Because granitoid dikes must have minimum widths and flow rates to keep from freezing, dike-fed plutons can grow at extremely rapid rates. Such rapid growth should lead to rapid strain rates in the surrounding wall rocks. Under conditions of constant-volume filling, growth can easily outpace conductive heating of the wall rocks. Thus, theoretically, dike-fed plutons could be completely emplaced before the wall rocks undergo contact metamorphism. These two predictions of the dike-fed model provide possible structural and petrologic avenues for testing the model. In attempting to formulate tests, we have been reminded of how difficult it currently is to extract strain-rate information from pluton aureoles. Because these aureoles are not characterized by steady-state thermal and dynamic conditions, applying experimental rock deformation studies to microstructural observations therein is currently tenuous. On the bright side, this study points to an area of research—rates of geological processes—in which more progress is becoming essential for modern geological studies.

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