Stability and dynamics of the continental tectosphere

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Abstract

Continental cratons overlie thick, high-viscosity, thermal and chemical boundary layers, where the chemical boundary layers are less dense than they would be due to thermal effects alone, perhaps because they are depleted in basaltic constituents. If the continental tectosphere is the same age as the overlying Archaean crust, then the continental tectosphere must be able to survive for several billion years without undergoing a convective instability, despite being both cold and thick. Since platforms and shields correlate only weakly with Earth’s gravity and geoid anomalies, acceptable models of the continental tectosphere must also satisfy this gravity constraint. We investigate the long-term stability of the continental tectosphere by carrying out a number of numerical convection experiments within a two-dimensional Cartesian domain. We initiate our experiments with a tectosphere thermal and chemical boundary layers immersed in a region of uniform composition, temperature, and viscosity, and consider the effects on the stability of the tectosphere of 1) activation energy used to define the temperature dependence of viscosity, 2) compositional buoyancy, and 3) linear or non-linear rheology. The large lateral thermal gradients required to match oceanic and tectosphere structures initiate the dominant instability, a ‘‘drip’’ which develops at the side of the tectosphere and moves to beneath its center. High activation energies and high background viscosities restrict the amount and rate of entrainment. Compositional buoyancy does not significantly change the flow pattern. Rather, compositional buoyancy slows the destruction process somewhat and reduces the stress within the tectosphere. With a non-Newtonian rheology, this reduction in stress helps to stiffen the tectosphere. In these experiments, dynamical systems that adequately model the present ocean-continent structures have activation energy $E_U \geq 180$ kJ mole$^{-1}$ — a value about one third the estimate of activation energy for olivine, $E_U = 520$ kJ mole$^{-1}$. Although for $E_U = 520$ kJ mole$^{-1}$, compositional buoyancy is not required for the tectosphere to survive, the joint application of longevity and gravity constraints allows us to reject all models not containing compositional buoyancy, and to predict that the ratio of compositional to thermal buoyancy within the continental tectosphere is approximately unity. © 1999 Published by Elsevier Science B.V. All rights reserved.

Keywords: Continental tectosphere; Dynamics; Conductive cooling; Mantle convection; Mantle rheology

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1. Introduction

The weak association between platforms and shields and long-wavelength geoid height anomalies (Shapiro et al., 1999), as well as the near constancy of continental freeboard (e.g., Wise, 1974) suggest that the continental tectosphere did not form simply by conductive cooling. Jordan (1975, 1978) proposed that the thick, cold, continental chemical boundary layer (CBL) inferred from the analysis of seismic data (see also, for example, more recent studies by Grand, 1994; Su et al., 1994; Ekström and Dziewonski, 1995; Masters et al., 1996) is stabilized against convective disruption by compositional variations that yield neutral buoyancy (i.e., the continental tectosphere and neighboring oceanic material have the same density profile). These compositional variations have been attributed to a depletion, relative to that from the source of mid-ocean-ridge volcanism, of the denser basaltic constituents (garnet and clinopyroxene) in the continental tectosphere (Jordan, 1978).

Monte Carlo simulations indicate that the strong association between platforms and shields and upper-mantle shear-wave anomalies is not simply fortuitous (Shapiro, 1995). This global relationship supports the hypothesis (Jordan, 1975) that these thick CBLs translate coherently with continental plate motions. Combining this conjecture with the measured ages of South African diamond inclusions (e.g., Richardson et al., 1984) and ages obtained from rhenium–osmium and other isotope systematics (Walker et al., 1989; Pearson et al., 1995) supports the supposition that the continental tectosphere could remain intact in the convecting mantle for times in excess of a billion years (Jordan, 1978).

If the tectosphere is to survive in a convecting mantle, it must be stable both to double diffusive instabilities resulting from compositional buoyancy (e.g., Stevenson, 1979) and to tractions from the convecting mantle in which it is immersed. In this study, we investigate primarily the former. Our aim is to quantify the role of viscosity and compositional buoyancy in determining whether a tectosphere would undergo spontaneous disintegration. The boundary conditions in our model are chosen to suppress thermal anomalies associated with the background large-scale mantle convection. We consider the effects of activation energy, compositional buoyancy, and dependence of rheology on stress. We further constrain the range of acceptable parameters by requiring that they not only produce long-term CBL stability, but also create density distributions that yield geoid height anomalies consistent with those observed over platforms and shields (Shapiro et al., 1999).

2. Numerical formulation

We use a double-diffusive version of the finite-element program, ConMan (King et al., 1990), to...
solve numerically the advection–diffusion equations for flow of an incompressible, infinite Prandtl number fluid in a two-dimensional Cartesian domain. With two fields affecting density: temperature, \( T \), and composition, \( C \), the relevant (dimensionless) equations are those of momentum balance:

\[
\nabla (\eta \nabla u) = \nabla p - Ra_T (T + BC) \hat{z}
\]

continuity:

\[
\nabla u = 0
\]

and conservation of energy (with no internal heating):

\[
\frac{\partial T}{\partial t} = -u \nabla T + \nabla^2 T
\]

and its compositional analog:

\[
\frac{\partial C}{\partial t} = -u \nabla C + \frac{1}{Le} \nabla^2 C
\]

where \( \eta \) is dynamic viscosity, \( u \) velocity, \( p \) pressure, \( t \) time, \( Le \) Lewis number (ratio of thermal to compositional diffusivity), and \( \hat{z} \) the unit vector in the direction of increasing depth. The thermal Rayleigh number is

\[
Ra_T = \alpha g \rho \Delta T d^3 / \kappa_T \eta_1
\]

where \( \alpha \) is the coefficient of thermal expansion, \( g \) the acceleration due to gravity, \( \rho \) the density, \( \Delta T \) the difference between the temperature at the bottom and that at the top of the domain of depth \( d \), \( \kappa_T \) the thermal diffusivity, and \( \eta_1 \) a reference value of dynamic viscosity we define to be the viscosity corresponding to dimensionless \( T = 1 \). The buoyancy number, \( B = \Delta \rho_C / \rho \alpha \Delta T \), with \( \Delta \rho_C \) the difference in density due to a unit change in composition, describes the ratio of compositional to thermal buoyancy. Unlike thermal gradients, which evolve through the diffusion of heat over geologic time scales, compositional gradients are essentially unaffected by solid-state diffusion; hence \( Le \) is effectively infinite. Owing to numerical constraints, however, we are limited to \( Le \leq 100 \) (Brooks, 1981).

We define a temperature-dependent Newtonian viscosity, \( \eta_N \), using an Arrhenius law (e.g., King, 1990):

\[
\eta_N(T) = \eta_1 \exp \left( \frac{E^*}{T + T_{\text{off}}} - \frac{E^*}{1 + T_{\text{off}}} \right)
\]

where \( E^* \) is the activation energy, \( T_{\text{off}} \) is the offset for dimensionless surface temperature required for dimensionless \( T = 0 \) to correspond to the \( \approx 273 \) K surface temperature of Earth. The background viscosity, \( \eta_1 \), implicitly accounts for pressure variations, including the effects of phase changes. A generic form for non-Newtonian viscosity is

\[
\eta_{\text{NN}} = \eta_N (\tau / \tau_0)^{n-1},
\]

where \( \tau \) is the second invariant of the stress tensor, \( \tau_0 \) is the stress value when the Newtonian and non-Newtonian viscosities are equal, and \( n \), the power law exponent, is assumed to be three for

Fig. 2. Newtonian viscosity profiles based on background viscosities (a) NLO (thick line) and (b) HGPA (thick line), and evaluated using the initial temperature field of the mid-tectosphere and three activation energies: \( E^*_{\text{init}} \) (dash-dotted lines), \( E^*_{\text{init}} / 3 \) (dashed lines), and \( E^*_{\text{init}} / 9 \) (thin lines), where \( E^*_{\text{init}} = 522 \) kJ mole\(^{-1}\). The dotted lines represent the viscosity profile corresponding to the initial mid-oceanic temperature profile. Note: we do not model NLO’s 100-fold increase in viscosity at 670 km depth because its effect on our experiments is insignificant (Shapiro, 1995).
our non-Newtonian experiments. Since strain rates are additive, an effective viscosity, \( \eta_{\text{eff}} \), that in the limit of low stress yields Newtonian creep and in the limit of high stress yields non-Newtonian creep, can be written as \( \eta_{\text{eff}} = \left[ \eta_{\text{N}}^{-1} + \eta_{\text{NN}}^{-1} \right]^{-1} \). We use a value of \( \tau_0 = 1 \text{ MPa} \) (10 bars) to define the transition between Newtonian and non-Newtonian rheology and, to avoid numerical problems, we fix the maximum dimensionless viscosity at \( \eta_{\text{eff}}/\eta_1 \), where \( \eta_{\text{eff}} = 10^5 \). Because this viscosity cutoff reduces the strength of the uppermost mantle, we simulate the effect of this strong layer by imposing no-slip boundary conditions at the surface.

3. Experimental design and parameters

In this study, we address the stability of a CBL exposed to convective and buoyancy stresses while not explicitly including all of the effects of a convecting mantle. Modeling numerically a domain the size of Earth with a realistic Rayleigh number, and temperature and stress-dependent viscosity, is computationally intensive even in two-dimensions because of the high spatial and temporal resolution required to model accurately the flow globally. Our tectosphere is immersed in a hot (and hence reduced viscosity) isothermal and isochemical environment. We obtain basal tractions in the fraction of a megapascal range, comparable to estimates for intraplate sublithospheric tractions on Earth (e.g., Hager and O’Connell, 1981), so this approximation seems reasonable. (Stresses in subduction zone environments are likely somewhat higher, but, as discussed below, regions with preserved tectosphere typically have been far from subduction zones for most of their history.)

We solve our system of equations in a two-dimensional Cartesian domain using a 76 × 38 grid of square elements (see Shapiro (1995) for a discussion of the sensitivity of the results to grid resolution, aspect ratio, and symmetry). We begin all of our experiments with an “oceanic” thermal boundary layer (TBL) of 100 km \([0.13]\) and a “continental” TBL of 400 km thickness \([0.53]\) above a region of uniform temperature (Fig. 1). (To facilitate rescaling our model results to other plausible initial conditions with thinner or thicker TBL’s, we present dimension-

<table>
<thead>
<tr>
<th>( E^* ) (kJ mole(^{-1}))</th>
<th>( B )</th>
<th>( \eta_0 ) ((z))</th>
<th>( \delta N )(m)</th>
<th>( \zeta_{\text{CBL}} )(km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>174</td>
<td>0.0</td>
<td>HGPA</td>
<td>(-37)(\pm)(28)</td>
<td>216</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NLO</td>
<td>(-62)(\pm)(40)</td>
<td>250</td>
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<tr>
<td></td>
<td>1.0</td>
<td>HGPA</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>NLO</td>
<td>(-8)(\pm)(5)</td>
<td>287</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>HGPA</td>
<td>14(\pm)8</td>
<td>297</td>
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<td></td>
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<td>NLO</td>
<td>21(\pm)10</td>
<td>302</td>
</tr>
<tr>
<td>522</td>
<td>0.0</td>
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<td>317</td>
</tr>
<tr>
<td></td>
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<td>NLO</td>
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<td>346</td>
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<tr>
<td></td>
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<td>NLO</td>
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<td>HGPA</td>
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<tr>
<td></td>
<td></td>
<td>NLO</td>
<td>(-3)(\pm)(5)</td>
<td>346</td>
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Fig. 3. Parameters: $E_{tot} = E_{tot}^i/3$, $B = 0$, $n = 3$, $\eta_f(z) = \text{NLO}$. Four frames: $t = (a) 0$, (b) 150, (c) 500, and (d) 1000 My. Each frame contains (left) contours of composition (purely a tracer field having no effect on the dynamics of the fluid) with superposed velocity arrows, (center) the difference, as a function of depth, between the average mid-tectosphere ($x \leq 200$ km) density and the average lateral density (see Eq. 10), and (mirrored — right) contours of temperature with superposed velocity arrows. Contour levels as in Fig. 1.
less parameter values within brackets.) We apply the following boundary conditions along the top: $z = 0$:

$$u_z = 0 \quad (6a)$$

$$u_x = 0 \quad (6b)$$

$$T = 0^\circ \text{C} \quad (6c)$$

$$\frac{\partial C}{\partial z} = 0 \quad (6d)$$

along the bottom: $z = 760 \text{ km} \ 1.0$):

$$\tau_z = 0 \quad (7a)$$

$$u_z = 0 \quad (7b)$$

$$\frac{\partial T}{\partial z} = 0 \quad (7c)$$

$$\frac{\partial C}{\partial z} = 0 \quad (7d)$$

along the sides: $x = 0, 1520 \text{ km} \ 2.0$):

$$\tau_z = 0 \quad (8a)$$

$$u_z = 0 \quad (8b)$$

$$\frac{\partial T}{\partial x} = 0 \quad (8c)$$

$$\frac{\partial C}{\partial x} = 0 \quad (8d)$$

and beneath the oceanic TBL: $x \geq 800 \text{ km} \ 1.05$; $z = 100 \text{ km} \ 0.13$): The thermal boundary condition at the bottom of the domain and at the base of the oceanic lithosphere were designed to suppress hot plumes and small-scale convection beneath oceanic regions, respectively.

Our initial temperature field (Fig. 1) contains two adjoining TBLs of different thicknesses, one representing an ocean and the other a continental tectosphere. We obtain our initial temperature field by solving the heat conduction equation subjected to the above temperature boundary conditions and a fixed temperature, $T = 1300^\circ \text{C} \ 1.0$, along the base ($z = 400 \text{ km} \ 0.53$) and side ($x \leq 800 \text{ km} \ 1.05$) of the tectosphere TBL. We remove these supplementary boundary conditions once the initial temperature field is formed.

The compositional field represents the degree of basalt depletion with respect to the oceanic average, with a higher value of $C$ representing a larger amount of depletion and yielding a lower normative density. Following the hypothesis of Jordan (1978, 1988) that, within the continental tectosphere, contours of constant composition are parallel to isotherms, we create an initial (dimensionless) composition field (Fig. 1) that is a function of the initial (dimensionless) temperature field:

$$C = 1 - T, \quad 0 \leq T \leq T_{\text{CBL}}$$

$$C = 0, \quad T > T_{\text{CBL}}$$

where $T_{\text{CBL}} = 1170^\circ \text{C} \ 0.9$ defines the temperature at the base of the CBL. It is not essential that $T_{\text{CBL}}$ have this particular value. Somewhat larger values lead to initial conductive thickening of the TBL while somewhat smaller values lead to the lower part of the TBL dropping off into the fluid below (see Shapiro (1995)). We investigate buoyancy ratios of 0, 1, and 1.5, where $B = 0$ corresponds to density unaffected by composition, $B = 1$ to the isopycnic hypothesis of Jordan (1988), and $B = 1.5$ to a tectosphere with net positive buoyancy.
For background viscosity profiles $\eta(z)$, we use the models HGPA (Hager, 1991) and NLO (Nakada and Lambeck, 1989) (Fig. 2). We base our selection of activation energies on the estimate for dry olivine of $E^* = E_{\text{ref}} = 522$ kJ mole$^{-1}$ by Ashby and Verrall (1977); we use $E^* = E_{\text{ref}}$, $E_{\text{ref}}/3$, and $E_{\text{ref}}/9$ to explore the sensitivity of tectosphere stability to activation energy (Tables 1 and 2). Fig. 2 shows, for each activation energy used, the corresponding initial mid-oceanic and mid-tectosphere Newtonian viscosity profiles. In all of our numerical experiments, we use representative values for the following quantities: $\alpha = 3 \times 10^{-5}$C$^{-1}$, $g = 9.8$ m s$^{-2}$, $\rho = 3.5 \times 10^3$ kg m$^{-3}$, and $\kappa_r = 10^{-6}$ m$^2$ s$^{-1}$.

4. Buoyancy contrast

One can begin to analyze the stability of a particular thermal and compositional structure by calculating the continent–ocean buoyancy contrast, i.e., by calculating at each depth the difference in density between the average mid-tectosphere ($x \leq 200$ km) value and the laterally averaged value (Fig. 1):

$$\frac{\delta \rho(z)}{\rho} = \alpha \Delta T \left[ \langle T + BC \rangle_{x \leq 200 \text{ km}} - \langle T + BC \rangle_x \right]$$

(10)

For example, a negative value of $\delta \rho(z)/\rho$ indicates that at depth $z$, the central region of the tectosphere is lighter, on average, than the average of the ocean and continent values. Considering $\delta \rho/\rho$ as a function of depth allows one to predict whether a structure might tend to remain near its initial configuration. As one can see from the plots of $\delta \rho(z)/\rho$ for $B = 0$ and $B = 1$ in Fig. 1, a structure with $B = 1$ has a greater chance for survival than a structure with $B = 0$ because, with $B = 0$, the mid-tectosphere region is much denser than the surrounding material. Of course, this plot neither tells us what structures, if any, are stable nor how an unstable structure might disintegrate. In the subsequent discussion, we take 1000 My as the characteristic time at which to assess stability. By this time the CBL has either been destroyed or the fluid flow is sufficiently regular that one can reliably make predictions concerning long-term stability.

5. Compositional buoyancy: effect on boundary layer stability

To illustrate the effect of composition-induced buoyancy on boundary-layer stability, we discuss in detail the evolution of two cases that differ only in the value of $B$. Since an acceptable model of the continental tectosphere must satisfy both the longevity and the gravity constraints, at 1000 My we calculate the depth to the base of the CBL, $z_{\text{CBL}}$, and estimate the associated geoid height anomaly, $\delta N$ (Tables 1 and 2). In the first example, we take $E_{\text{ref}}/3$, $B = 0$, and $n = 3$, which leads to very rapid destruction of the tectosphere CBL. With $B = 0$, composition is simply a tracer field — it has no effect on the motion of the fluid. At 1000 My, the CBL is essentially gone, having been washed away by the flow driven by the lateral variations in density (Fig. 3). The tectosphere–ocean buoyancy contrast, $\delta \rho(z)/\rho$, decreases in magnitude and, as the tectosphere CBL disappears, becomes non-zero only at shallow depths.

The average geoid height anomaly associated with the beginning of the experiment is much greater in amplitude than those observed over platforms and shields (e.g., Shapiro et al. (1999)) (Fig. 4a). Of course, as the source of the density contrast, the tectosphere TBL disappears, the average geoid height anomaly decreases accordingly. Similarly, the variation in dynamic topography is unreasonably large when compared with the near constancy through
time of Earth’s continental freeboard (e.g., Wise, 1974) (Fig. 4b). The initial viscosity and stress fields (Fig. 4c) show that the transition between Newtonian and non-Newtonian rheology occurs near the base of the tectosphere — much of the flow occurs in the Newtonian regime, although a substantial amount also occurs in regions of high stress (Fig. 5a and b). At 1000 My, the tectosphere is gone, so the size of
Fig. 6. Parameters: $E_s = \frac{E^*_s}{3}$, $B = 1.0$, $n = 3$, $\eta(s) = \text{NLO}$. Four time frames: $t = (a) 0$, (b) 150, (c) 500, and (d) 1000 My. Frames as described in Fig. 3.
Fig. 7. Parameters as in Fig. 6. Frames as described in Fig. 4.
the region of high stress is significantly reduced, as is the region of high viscosity (Fig. 4d). Subtracting the initial from the final composition (and temperature) fields illustrates which regions have gained or lost composition (and heated up or cooled down) and by how much (Fig. 4e).

By plotting the area, \( A \), of the \(-0.1\) composition difference contour (normalized to the combined area of the initial oceanic and continental CBLs) as a function of time (Fig. 4f), we see that most of the composition is removed from the tectosphere CBL within the first 50 My. After this time, the rate of removal diminishes dramatically, decreasing slowly to zero at about 600 My. After the initially weak (i.e., easily deformed) material is quickly washed away, the remnant continental CBL weakens gradually as a result of the combination of the hot material flowing along its base and the strong temperature-dependence of the material’s viscosity. We interpret this temporal pattern as indicating the presence of two instabilities, a mechanical mode \((t < 50\, \text{My})\) and a thermal ablation mode \((t > 50\, \text{My})\). The change in the depth to the base of the CBL, \( \Delta z_{\text{CBL}} = z_{\text{CBL}}(0) - z_{\text{CBL}}(t) \), expressed as a percentage of the initial CBL thickness \((z_{\text{CBL}}(0))\), varies with time in a manner similar to \( A(t) \) (Fig. 4f). Such a correspondence is reassuring since both of these quantities were devised independently to estimate, as a function of time, the condition of the CBL.

The conductive heat flux through the continental surface (Fig. 4g) increases by only a factor of about two \((10\, \text{mW m}^{-2})\) throughout the calculation. The advective heat flux through the base of the domain is more variable. The intervals of large fluxes coincide with part of the tectosphere’s TBL falling off and sinking through the base of the domain. Such occurrences are episodic. (Note that the time interval at which we stored the fields needed to compute regional fluxes is too coarse to fully resolve the peaks in the flux-time curve.)

When we repeat the above experiment with \( B = 1 \), the results differ significantly. The CBL remains largely intact after 1000 My, although its initially horizontal base becomes rounded (Fig. 6). The function \( \delta \rho(z)/\rho \) changes shape, gradually forming a profile which indicates that a cold, dense upper section of the tectosphere is being partially supported from below by a light CBL — opposite to our initial condition where the TBL extends beneath the CBL, causing the small positive deviation from zero in \( \delta \rho(z)/\rho \). This dense upper region is caused by the conductive cooling of the upper section of the tectosphere — with composition effectively unable to diffuse, the isopycnic condition is no longer satisfied in this region. Throughout the experiment, both the average geoid height anomaly and the dynamic topography associated with the tectosphere CBL have geophysically reasonable values (Fig. 7a and b).

Due to the addition of compositional buoyancy, the second invariant of the stress is reduced relative to the above experiment \((B = 0)\). This lower stress yields a correspondingly higher viscosity field (Fig. 5c, Fig. 6b, Fig. 7c and Fig. 7d) which leads to a more stable CBL. The changes in the composition and temperature fields after 1000 My are confined to a much smaller area than in the previous experiment (Compare Fig. 4e and Fig. 7e). Again, the initially weak material is removed quickly from the tectosphere, although with the stabilizing effect of compositional buoyancy more material remains (Fig. 7d). Further, the higher viscosity due to the lower stress than in the \( B = 0 \) case inhibits the thermal ablation process so that it is barely observable in \( A(t) \). Here, \( A(t) \) and \( \Delta z_{\text{CBL}}(t) \) diverge at about 200 My. After this time, \( A(t) \) is essentially constant whereas \( \Delta z_{\text{CBL}}(t) \) decreases slightly, indicating that the CBL is thickening. From Fig. 6c and d, one can see that this thickening occurs in the center of the tectosphere CBL as a consequence of the cold “blobs” detaching from the tectosphere TBL. The plot of heat flux through the base of the domain (Fig. 7g) shows some of these instabilities. The heat flux through the top of the tectosphere, however, does not vary noticeably with time, holding at a value of about 20 mW m\(^{-2}\).

### 6. Bounds on \( E^*, B \), and \( \eta(z) \) from longevity and geoid constraints

To evaluate the relative effects of (1) activation energy, (2) compositional buoyancy, and (3) rheology on tectosphere stability and the resulting geoid signal, we plot the depth to the base of the CBL and the average geoid height anomaly over the continental tectosphere, at \( t = 1000\, \text{My} \), for each experiment.
listed in Tables 1 and 2 (Fig. 8). We do not plot results for our experiments with $E^*/9$, which are so unstable that the tectosphere is removed in $t \ll 1000$ My, and the calculations terminated.

Fig. 8. The depth to the base of the CBL ($z_{\text{CBL}}$) (ordinate) and the mean geoid height anomaly ($\delta N$) over the tectosphere CBL (abscissa) both at 1000 My. The depth to the base of the CBL is estimated from the median depth to the $C = 0.1$ contour within the tectosphere CBL. Triangles and squares indicate experiments with activation energies of $E^* = E_{\text{act}}$ and $E_{\text{act}}/3$, respectively. White, gray, and black symbols indicate experiments with buoyancy ratios of $B = 0, 1,$ and $1.5$, respectively. Small and large symbols indicate experiments with background viscosity profiles based on HGPA and NLO, respectively. The ordinates of the horizontal dashed lines indicate a somewhat arbitrary upper bound of $z_{\text{CBL}}$ based on the inferred thickness of the continental tectosphere (Shapiro, 1995). The abscissas associated with the vertical dashed lines correspond to $\delta N_{\text{max}} \pm 2 \sigma$ as determined by Shapiro et al. (1999). (a) Newtonian rheology. (b) Non-Newtonian rheology. Circles represent experiments with a background viscosity profile determined by NLO and an activation energy of $E^* = E_{\text{act}}/1.25$. White and gray symbols indicate experiments with buoyancy ratios of $B = 0$ and $1$, respectively.
In both the Newtonian and non-Newtonian experiments, using an activation energy of 522 kJ mole\(^{-1}\), corresponding to dry olivine (Ashby and Verrall, 1977), assures stability — regardless of the amount (within reasonable limits) of compositional buoyancy present. However, not all of these experiments produce geoid height anomalies consistent with those observed for platforms and shields (Shapiro et al., 1999). In fact, by comparing the average geoid height anomalies resulting from the \(E^* = E_{int}^*\) experiments (triangles in Fig. 8), one can see that simply satisfying the isopycnic hypothesis is insufficient to ensure a geoid height anomaly consistent with the observed geoid associated with platforms and shields — one must also consider the dynamics of the flow. For example, the Newtonian experiments with initial \(B = 1.5\), but not with \(B = 1\), satisfy the geoid constraint. From the corresponding, more realistic, experiments with stress-dependent rheology, we estimate that the initial \(B = 1\) is likely between about 0.9 and 1.3, depending on the background viscosity profile (Fig. 8b). (We refer to the "initial" \(B\) because, after 1000 My, both the temperature and composition are modified and the local value of \(B\) changes. For example, for the experiment shown in Fig. 6, at 1000 My, for \(z \leq 400\) km, \(\delta \rho > 0\) implies \(B < 1\), while for \(z \geq 400\) km, \(\delta \rho < 0\) implies \(B > 1\).

With an activation energy one third the above value \((E_{int}^* / 3)\) and a stress-dependent rheology, compositional buoyancy plays a major role in stabilizing the continental tectosphere (compare the open with the shaded squares in Fig. 8b; compare also Figs. 3 and 6). After 1000 My, the CBLs with \(B = 1\) are roughly 200 km thicker than those containing no compositional buoyancy. Compositional buoyancy has a similar effect on stability for the corresponding Newtonian experiments, except for those characterized with an activation energy of \(E_{int}^* / 3\). In this case, \(B = 1\) results in CBLs that are only about 50 km thicker than the corresponding CBLs that contain no compositional buoyancy. By comparing the Newtonian to the non-Newtonian experiments, we see that compositional buoyancy plays a larger stabilizing role with a non-Newtonian rheology. Compositional buoyancy reduces the stress within the boundary layer, which in conjunction with the stress dependence of viscosity, causes an increase in viscosity which stabilizes the CBL.

The two background viscosity models yield continental tectosphere thicknesses that are always within about 50 km of each other. In most, but not all cases, the model with a background viscosity described by NLO is the more stable one.

7. Discussion

The most important question our numerical experiments were designed to address is whether under any circumstances a thick, cold root beneath continents can be stable over time scales of a billion years or more. The alternative is that these roots are parts of massive convective downwellings (e.g., Pari and Peltier, 1996). We find that the effects of temperature dependent viscosity can stabilize these roots, with only slow ablation by small-scale convection. The wisps of cold material sinking into the deeper interior in our models are consistent with the seismic observations of Li et al. (1998), who found that beneath the deep root below the eastern United States there is little perturbation in the depth of the 400 km seismic discontinuity. They conclude that any cold downwellings in the transition zone in the region they studied must be weak (< 50 K) and/or small-scale.

Conclusions drawn from studies of boundary-layer dynamics depend strongly on assumptions concerning the temperature and stress dependence of viscosity and the magnitude of \(\tau_s\), the typical value of background stress in the convecting mantle. From an analytical analysis of the stability of a constant viscosity continental tectosphere described by linear gradients in composition and temperature, assuming \(\eta = 10^{21}\) Pa s, Stevenson (1979) found modes of instability with characteristic growth times as short as about 200 My — an order of magnitude less than that required by the above age constraints. If one considers the temperature dependence of viscosity and the fact that continents are cold, a constant viscosity of \(\eta = 10^{21}\) Pa s is an unrealistically low estimate (e.g., Simons and Hager, 1997). An increase of only one order of magnitude would yield characteristic time constants comparable to the age of Earth. In addition, the strong temperature dependence of viscosity helps to confine the flow to a very
limited region, further stabilizing the tectosphere, as discussed below.

In the high-stress regime, where $\tau_s > 1$ MPa (10 bars), Kincaid (1990) concluded that viscosity, not compositional buoyancy, is responsible for achieving long-term stability. Shapiro (1995) further demonstrated that, with $\tau_s \approx 6$ MPa (60 bars), 10–20 times that considered to be appropriate for most of Earth (e.g., Hager and O’Connell, 1981), a viscosity increase of a factor of about 20 between the TBL and the surrounding mantle is sufficient to maintain stability, regardless of the amount of compositional buoyancy (within reasonable bounds). These studies probably overestimate the role of tractions from mantle convection because, due to numerical constraints, they were carried out at a Rayleigh number lower than appropriate for Earth. Convective stresses scale as $\tau_0 \propto g\rho\alpha\Delta T Ra^{2/3}$, since the value of $\Delta T$ assumed in these studies is a realistic one, higher Rayleigh numbers, corresponding to lower viscosities, result in thinner boundary layers, and lower stresses.

Our transition stress value is based on a small grain size — about one mm (Ashby and Verrall, 1977). As grain size increases, the transition stress decreases, so our choice of $\tau_s = 1$ MPa (10 bars) is a conservative one in the following sense: If we had assumed a larger grain size, the fluid within and beneath the tectosphere would have been more viscous, and the tectosphere would have been more likely to survive.

In the low-stress regime, we find that destruction is achieved in two ways: (1) through a mechanical removal of material and (2) via a thermal ablation process in conjunction with the mechanical process. The initially weak regions of the tectosphere are washed away quickly by convective processes; the remaining material is removed more slowly. As the base warms through conduction, it becomes weaker due to viscosity’s inverse dependence on temperature. Then, in its weakened state, it is swept away by the convection currents. To estimate the viscosity required to prevent this mechanical removal, we consider the viscosity corresponding with this “ablation front”. For the experiments which yield CBLs that are stable over a billion year time scale (Fig. 8), the viscosity corresponding to the edge of the ablation front, where velocities become negligible, is approximately $10^{20.5}$ Pa s (for a representative example, see Fig. 9). Once the viscosity of the tectosphere

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**Fig. 9.** Model parameters as in Figs. 6 and 7. Viscosity field ($\eta$) (solid lines) with velocity arrows and the change in the composition field (see Fig. 7) (dashed lines) superposed; $t = 1000$ My. Velocity scaling is clipped at $v_{\text{clip}} = 1$ cm/year to exhibit lower values more clearly. Contour intervals as in Fig. 4.
material is about a factor of ten greater than that of the underlying mantle, negligible flow occurs — similar to the result from high-stress models (Shapiro, 1995).

Further support for this interpretation comes from the work of Conrad and Molnar (1997). They used a semi-analytic approach to investigate Rayleigh–Taylor-type instabilities of a more dense fluid layer of thickness, $h$, overlying a uniform halfspace. Within the layer, density increases linearly with distance above the boundary, as $\Delta \rho z/h$, and viscosity varies exponentially, as $\eta_0 \exp(z/L)$. Their results show that flow is essentially confined to a layer of thickness $z = 2L$, corresponding to a depth range over which the viscosity varies by a factor of 10. As is the case with our numerical experiments, there is negligible flow excited in regions with higher viscosity. For a ratio $L/h \sim 0.03$, typical Rayleigh–Taylor growth times are of order $30 \eta_0/\Delta \rho g L$. Assuming representative values of density contrast, $\Delta \rho = 100$ kg/m$^3$; tectospheric thickness, $h = 300$ km; reference viscosity, $\eta_0 = 10^{20}$ Pa s; and viscosity e-folding distance, $L = 10$ km; the characteristic growth time is $\sim 50$ My. Such a short growth time is consistent with the results of our numerical experiments, which show a subtectospheric drip developing on a comparable time scale.

Once this sublithospheric drip strips away the initial layer of thickness $2L \sim 20$ km, the gradient in viscosity at the base of the tectosphere is much sharper, resulting in a situation where $L$ is of order 1 km. Since the growth time is inversely proportional to $L$, the rate at which the tectosphere ablates decreases by a comparable factor, with a characteristic growth time $\sim 1$ Gy.

Fleitout and Yuen (1984) demonstrated that the combination of pressure- and temperature-dependent viscosity can help to stabilize a thick thermal boundary layer from convective disturbance. The temperature-dependence of viscosity stabilizes the cold (shallow) part of the TBL while the pressure-dependence of viscosity stabilizes the warmer (deeper) part. Our numerical representation of the continental tectosphere is stable even without the stabilizing effects of pressure-dependent viscosity.

Doin et al. (1997) have extended these calculations, investigating the effects of rheology dependent on temperature, pressure, and composition, in a domain of width 2680 km and depth 670 km. They enforce a tectonic regionalization using velocity boundary conditions, with one third to one half of the surface, representing a continent, pinned to the surface, and the remaining domain driven by imposed horizontal velocities, representing oceanic plates. The velocity boundary conditions change with time such that the subduction zone is periodically adjacent to either edge of the tectosphere region, with one side or the other of the craton experiencing subduction every 90 My. They find that viscosity is the most important parameter in determining whether the tectosphere is stable, and that compositional buoyancy is not a primary contributor for the value of $E^\ast$ that they use, 310 kJ mole$^{-1}$ (corresponding to 0.6 $E_{\text{mt}}^\ast$). For models with strong pressure dependence of viscosity, there are two preferred lithospheric thicknesses, one corresponding to mature oceanic lithosphere and the other to the thickness of the subcontinental tectosphere. Doin et al. (1997) find somewhat higher rates of tectosphere ablation than do we. One reason is that the activation energy that they used, 310 kJ mole$^{-1}$, is less than the value that we used, 522 kJ mole$^{-1}$, the estimate for dry olivine (Ashby and Verrall, 1977). Also, their models have flow confined to the upper mantle, so there is strong shear between the base of their continental TBL and the bottom of their computational domain. Perhaps most importantly, their “continents” are narrow (900–1350 km wide) and are bounded by subduction zones at intervals of $< 100$ My. In contrast, on Earth, tectosphere is only rarely adjacent to a subduction zone. For example, Bostock (1998, Bostock, 1999) interprets the seismic structure of the Slave craton as resulting from a fossil subduction zone that has been preserved for $> 1.9$ Gy since the suturing of the Fort Simpson terrain and the Slave province.

Forte et al. (1995) suggested that the buoyancy profile beneath continents reverses in sign at about 250 km depth. Specifically, Forte et al. (1995) proposed that the negative buoyancy in the upper 250 km of the subcontinental mantle is partially supported by underlying lighter material. Interestingly, some of our acceptable models exhibit this same buoyancy reversal (see, for example, Fig. 6). As we discussed above, this buoyancy reversal is caused by the conductive cooling of the tectosphere. This cool-
ing is dependent on the convecting system in which it is placed. In fact, the existence of a buoyancy reversal is very sensitive to several of our assumptions, with some acceptable models showing this reversal, and others not [see, for example, Fig. B.21 of Shapiro (1995)]. Hence, our study cannot be used to predict accurately whether such a reversal actually exists.

For models with viscosity sufficiently high that the tectosphere survives, there is negligible deformation of the tectosphere. Thus, heat transport through the tectosphere in our models is controlled by conduction. The predicted mantle heat flow decreases from the suboceanic region to the interior of the tectosphere, as the depth to a given isotherm increases. Such variation is consistent with the careful analysis of heat flow observations by Jaupart and Mareschal (1999), who estimated variations in mantle heat flow and lithospheric thickness after stripping off the effects of near-surface heat production. They estimate that the conductive part of the subcratonic thermal boundary layer is 200–330 km thick. Only beneath that depth does advective heat transport become important. In contrast, Lenardic (1997) used the results of convection models that, like ours, include the effects of compositional buoyancy, to challenge this interpretation. In Lenardic’s models, the viscosity of the material comprising the subcratonic regions is sufficiently low that recirculation within the crust and underlying buoyant material is driven by the convecting mantle. This recirculation leads to a substantial advective contribution to the heat flux. In his models, there is little variation in mantle heat flux between “cratonic” and surrounding regions. In our view, if the tectosphere is to survive, its viscosity must be so high that deformation is negligible; heat transport results from simple conduction, with negligible advective contribution. The results of Jaupart and Mareschal (1999), who explain observed heat flow variations of both short and long wavelengths in terms of observed crustal heat production and a conductive model of heat transport through the tectosphere, support our interpretation.

In summary, the joint application of longevity and gravity constraints allows us to evaluate the importance of specific properties of a continental tectosphere in the low-stress regime. High viscosity is crucial for the long-term survival of the tectosphere. Flow models characterized by the activation energy for dry olivine, 522 kJ mol\(^{-1}\), yield stable boundary layers that, once established, are stable, even with no compositional buoyancy present. However, activation energies, say tenfold smaller, are too low; they lead to a rapid (of order 10 My) destruction of the tectosphere. With an activation energy about 20% less than that estimated for olivine, temperature-dependent viscosity alone is sufficient to assure stability (Fig. 8). With lower values of activation energy, stability of an existing tectosphere can be achieved with the inclusion of compositional buoyancy. Compositional buoyancy plays a dual role within a thermal (and chemical) boundary layer: It (1) reduces the stress within the boundary layer and (2) counteracts the thermally-induced density increase. With a stress-dependent rheology, this reduction in stress results in an increase in viscosity which, in turn, inhibits a greater region of the boundary layer from deforming. Removal of volatiles by depletion would also increase the viscosity, providing a plausible mechanism contributing to the stabilization of the tectosphere (Pollack, 1986).

If, for realistic activation energies, compositional buoyancy is not required to maintain a stable tectosphere, it is interesting to ask why the geoid observations (Shapiro et al., 1999) indicate that \( \beta = 1 \)? If the tectosphere formed by advective thickening, the results of our numerical experiments provide a plausible answer. Formation via advective thickening requires that the material that now constitutes the rigid tectosphere was ductile enough to deform and thicken at the time of formation of the proto-tectosphere. Thus, the proto-tectosphere was likely to have been somewhat warmer than mature tectosphere is now in order that it had a sufficiently low viscosity to deform. Since it is the ratio of \( E^*/T \) that governs viscosity, a higher \( T \) is equivalent to a lower \( E^* \). As our numerical experiments demonstrate, compositional buoyancy is required to stabilize the tectosphere at lower \( E^* \). Thus it seems likely that compositional buoyancy would also be important in stabilizing a somewhat hotter proto-tectosphere, formed under advective thickening, that later completely stabilized by moderate cooling. Such moderate cooling would be consistent with the estimate from the geoid signal associates with cratons.
that the present-day $B$ is slightly less than unity (Shapiro et al., 1999).

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