Resolving fault plane ambiguity for small earthquakes

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SUMMARY
We have developed an automated procedure to resolve fault-plane ambiguity for small to medium-sized earthquakes (2.5 ≤ ML ≤ 5) using synthetic Green’s tensors computed in a 3-D earth structure model and applied this procedure to 35 earthquakes in the Los Angeles area. For 69 per cent of the events, we resolved fault plane ambiguity of our CMT solutions at 70 per cent or higher probability. For some earthquakes, the fault planes selected by our automated procedure were confirmed by the distributions of relocated aftershock hypocentres. In regions where there are no precisely relocated aftershocks or for earthquakes with few aftershocks, we expect our method to provide the most convenient means for resolving fault plane ambiguity. Our procedure does not rely on detecting directivity effects; therefore it is applicable to any types of earthquakes.

Key words: Earthquake source observations; Seismicity and tectonics; Computational seismology.

INTRODUCTION
A fundamental ambiguity of the most general point-source representation of an earthquake, the centroid moment tensor (CMT), is that it does not specify which of the two nodal planes is the actual fault plane (Aki & Richards 2002). To identify the fault plane, additional information about the seismic source is needed.

The lowest-order representation of a finite rupture is the finite moment tensor (FMT, Chen et al. 2005), which contains the second-order polynomial moments of the source–space–time function in addition to the zeroth- and first-order polynomial moments included in the CMT parameters (Backus & Mulcahy 1976; McGuire et al. 2001). The FMT representation resolves fault-plane ambiguity from the CMT representation and gives the characteristic duration (Silver & Jordan 1983) and dimensions of the faulting as well as the directivity vector of the fault slip (Ben-Menahem 1961).

In previous studies that derive finite source properties for small to medium-sized earthquakes, empirical Green’s functions (EGF) were often employed to account for propagation path-effects (Mori 1996; Hellweg & Boatwright 1999; McGuire 2004). In Chen et al. (2005) we demonstrated the feasibility of full FMT inversion for regional medium-sized earthquakes (4.5 ≤ ML ≤ 6) using high-frequency (~2.5 Hz) waveform data. We corrected the propagation path-effects in the observed data using synthetic Green’s functions computed in path-averaged 1-D Earth structure models and a ‘denoising’ technique, which is similar to, but more flexible than traditional EGF techniques.

With the advancement of computing technology and numerical methods, synthetic Green’s functions can be calculated accurately in 3-D Earth structure models using finite-difference (FD, Olsen 1994; Graves 1996; Olsen et al. 2003), finite-element (Bao et al. 1998; Akcelik et al. 2003) and spectral-element methods (Komatitsch et al. 2004). In regions such as Southern California, preliminary 3-D Earth structure models are already available (Magistrale et al. 2000; Kohler et al. 2003; Süss & Shaw 2003), and efficient numerical methods have been adapted to simulate seismic wave propagation in these 3-D Earth structure models (Komatitsch et al. 2004; Olsen et al. 2006). Our automated procedure for CMT inversion and fault-plane ambiguity resolution is built upon these previous achievements. In this paper, we demonstrate the feasibility of resolving fault-plane ambiguity for local small to medium-sized earthquakes (2.5 ≤ ML ≤ 5) using FD synthetic Green’s functions computed in a 3-D structural model, the Southern California Earthquake Center (SCEC) Community Velocity Model Version 3.0 (CVM3.0). For most local small events, the low-frequency data (<1 Hz) that can be accurately modeled by FD synthetic Green’s functions are usually insufficient for precise determination of all FMT parameters. However, we show that the low-frequency data can be used to detect the constructive/destructive interference effects caused by the second-order spatial moment and to establish the probability that one of the CMT nodal planes is the actual fault plane. This technique does not rely on detecting directivity effects therefore it is applicable to any type of seismic source, including bilateral earthquakes with no directivity.
FORMULATION

Receiver Green’s tensor (RGT)

Our procedure relies on the use of receiver Green’s tensors (RGTs) \( G_{ik}(r, r_S; t) \) and seismic reciprocity principle (Aki & Richards 2002). The RGT \( G_{ik} \) is the \( i \)th component wavefield at position \( r \) and time \( t \) for a \( k \)th component impulsive point-force at the receiver position \( r_S \) (Zhao et al. 2005). Seismic reciprocity implies that the synthetic seismogram at \( r_S \) excited by a point source with moment tensor \( M_j \) at position \( r_S \) at time \( t_S \) is

\[
\hat{u}_k(t) = \sum_{ij} M_{j} \partial^S \delta G_{ik}(r_S, r_S; t - t_S) \\
= \sum_{ij} M_{j} \partial^S G_{ik}(r, r_S; t - t_S),
\]

where \( \partial^S \) denotes source-coordinate gradient with respect to \( r_S \). We calculated \( G_{ik} \) for 48 broad-band stations of the California Integrated Seismic Network (CISN) in the Los Angeles region (Fig. 1) using the SCEC CVM3.0 (Magistrale et al. 2000; Kohler et al. 2003), and the fourth-order staggered-grid finite-difference code (Olsen 1994). These receiver Green’s tensors were sampled on a regular mesh of 36 million gridpoints with a grid-spacing of 200 m and archived on a RAID storage system at the High-performance Computing Center in University of Southern California, where they occupied a total data volume of about 20 TB (TB \( = 10^{12} \) bytes). Point-source synthetic seismograms were calculated by retrieving \( G_{ik} \) on a small source-centred grid and calculating the source-coordinate gradient \( \partial^S \) using a fourth-order numerical differentiation scheme (Press et al. 1992). Details about the construction of our RGT database as well as our automated CMT inversion procedure are documented in a separate paper (Zhao et al. 2006).

Generalized seismological data functional (GSDF)

The waveform differences between observed seismograms and point-source synthetics generated from a CMT solution and the RGTs by applying the reciprocity principle (eq. 1) can be parametrized using the generalized seismological data functionals (GSDF) of Gee & Jordan (1992). In the frequency domain, we can map the synthetic waveform

\[
\hat{u}_k(\omega) = \exp[i\omega(\delta \tau_p(\omega) + i \delta \tau_q(\omega))]
\]

into the observed waveform using two frequency-dependent, time-like quantities \( \delta \tau_p(\omega) \) and \( \delta \tau_q(\omega) \)

\[
u_k(\omega) = \hat{u}_k(\omega) \exp[i\omega(\delta \tau_p(\omega) + i \delta \tau_q(\omega))].
\)

In GSDF analysis, we estimate \( \delta \tau_p(\omega) \) by measuring frequency-dependent phase-delay time \( \delta \tau_p(\omega) \) and amplitude-reduction time \( \delta \tau_q(\omega) \) at a set of discrete frequencies \( \omega_n \). The measured \( \delta \tau_k(\omega_n) \) \( (x = p, q) \) are weighted averages of \( \delta \tau_x(\omega_n) \) in narrow frequency bands centred around \( \omega_n \) (Chen 2005; Chen et al. 2007a). The phase-delay time \( \delta \tau_p \) is the frequency-dependent generalization of differential travel time and the amplitude-reduction time \( \delta \tau_q \) is the frequency-dependent generalization of differential \( r^\ast \).

Finite moment tensor (FMT)

Any indigenous seismic source can be represented using a space–time distribution of stress glut \( \Gamma(r, t) \) (Backus & Mulcahy 1976). If we assume the stress glut is everywhere proportional to

\[ \text{Figure 1. Focal mechanisms with fault-plane ambiguity resolutions for 35 small to medium-sized earthquakes in the Los Angeles area. The preferred fault planes picked by our automated procedure are highlighted in thick black lines. Left-lateral events have blue extensive quadrants; right-lateral events have red extensive quadrants. The numbers near the beachballs indicate the likelihood that the preferred nodal planes are the actual fault planes based on a bootstrap procedure (see text). Epicentres of these 35 earthquakes are indicated by the black dots. Background seismicity in this area is shown in grey points. CISN stations used in this study are shown as grey triangles. Major faults in this area are plotted as black solid lines. The two seismicity anomalies discussed in the text, the Yorba Linda cluster and the Fontana trend, are indicated with the two black ellipses.} \]
a constant seismic moment tensor, $\mathbf{\Gamma}(\mathbf{r}, t) = \mathbf{M} f(t) f(t') d t'$, the seismic wavefield is proportional to the source space–time function $f(t)$, which we expand in terms of its space–time polynomial moments $\mu^{(m,l)}$. Here, $m$ and $l$ represent the order of spatial and temporal moments. The zeroth-order moment is by definition unity, $\mu^{(0,0)} = \int f(\mathbf{r}, t) dV dt = 1$, and the first moments yield the source centroid:

$$\mu^{(1,0)} = \int f(\mathbf{r}, t) d\mathbf{r} dV dt \equiv \mathbf{r}_1, \quad \mu^{(0,1)} = \int f(\mathbf{r}, t) d\mathbf{v} dV dt \equiv \mathbf{t}_1.$$  

(4)

The tensor $\mathbf{M}$, vector $\mathbf{r}_1$, and scalar $\mathbf{t}_1$ form the CMT (point-source) representation. The lowest order description of source finiteness is given by the second moments. The second central moment in time gives the characteristic duration of the event (e.g. Silver & Jordan 1983),

$$\mu^{(2,0)} = \int f(\mathbf{r}, t) (t - t_1)^2 d\mathbf{r} dV dt \equiv (T_C/2)^2.$$  

(5)

The second central moment in space is a symmetric second-order tensor

$$\mu^{(2,0)} = \int f(\mathbf{r}, t) (\mathbf{r} - \mathbf{r}_1)^2 d\mathbf{r} dV dt = \mathbf{U} \mathbf{U}^T.$$  

Here $\Lambda$ is a diagonal matrix of eigenvalues, and $\mathbf{U}$ is an orthogonal matrix of eigenvectors. For the special case of a planar rectangular dislocation of length $L$ and width $W$, the two non-zero eigenvalues will be $(L/2)^2$ and $(W/2)^2$, where $L_C = L/3$ is the characteristic length and $W_C = W/3$ is the characteristic width of the source. For the 2002 September 3 Mt. 4.8 Yorba Linda earthquake investigated in Chen et al. (2005), the second central moment in space is given by

$$\Lambda = \text{diag} \left[ \frac{0.74}{2}, \frac{0.4}{2}, 0 \right].$$

$$\mathbf{U} = \begin{bmatrix} \cos(-20^\circ) \cos(30^\circ) & \cos(-110^\circ) \cos(30^\circ) & \sin(30^\circ) \\ \cos(-20^\circ) \sin(30^\circ) & -\cos(-110^\circ) \sin(30^\circ) & -\cos(30^\circ) \\ \sin(-20^\circ) & -\sin(-110^\circ) 
& 0 \end{bmatrix},$$

where diag[ ] denotes a diagonal matrix, the planar rupture has a characteristic length of 0.74 km, width 0.4 km and the first principle axis has a strike of 30° and dips 20° upwards ($x$-north, $y$-east and $z$-down). The mixed space–time moment yields the directivity vector (Ben-Menahem 1961; McGuire et al. 2001),

$$\mu^{(1,1)} = \int f(\mathbf{r}, t) (\mathbf{r} - \mathbf{r}_1)(t - t_1) d\mathbf{r} dV dt \equiv \mathbf{v}_2 \cdot \mu^{(2,0)}.$$  

(7)

The directivity velocity vector $\mathbf{v}_2$ will lie in the plane of a simple dislocation. For a perfect bilateral rupture, the directivity parameter $D \equiv |\mathbf{v}_2| T_C/L_C$ will be zero, and for a perfect unilateral rupture, it will be unity. In previous studies, McGuire et al. (2002) inverted low-frequency teleseismic waveform data for FMT parameters and showed that global large earthquakes ($M_w \geq 7.0$) are predominantly unilateral. McGuire (2004) and Chen et al. (2005) inverted broad-band waveform data for FMT parameters of local medium-sized earthquakes (4.0 $\leq M_w \leq 5.0$) and showed that the symmetry breaking process also happens at a much smaller scale.

In general, the FMT representation will comprise the 10 parameters of the CMT plus 10 additional parameters that specify the second central moments of $f(\mathbf{r}, t)$. In the analysis of small earthquakes described below, we have generally assumed a planar dislocation (double-couple) source, which reduces the number of CMT parameters to 8 and the total number for the FMT to 13.

The partial derivatives of $\tilde{\mathbf{S}}(\omega)$ with respect to FMT parameters $\mu^{(m,l)}$ ($m + l \leq 2$) have been derived in McGuire et al. (2001). The linearized forward problem that relates the perturbation $\delta \tilde{\mathbf{S}}(\omega)$ to $\mu^{(m,l)}$ can be expressed as

$$\delta \tilde{\mathbf{S}}(\omega_R, \omega) \approx \nabla \tilde{\mathbf{S}} \cdot \mu^{(1,0)} + \frac{\partial}{2} \mu^{(0,2)} + \omega \nabla \tilde{\mathbf{S}} \cdot \mu^{(1,1)} + \frac{1}{2} \nabla \cdot \mu^{(2,0)},$$  

(8a)

$$\delta \tilde{\mathbf{p}}(\omega_R, \omega) \approx \mu^{(1,1)} + \nabla \tilde{\mathbf{S}} \cdot \mu^{(1,0)} + \omega \nabla \tilde{\mathbf{S}} \cdot \mu^{(1,1)} + \frac{1}{2} \nabla \cdot \mu^{(2,0)}.$$  

(8b)

Here, $\nabla \tilde{\mathbf{S}}$ is the gradient operator with respect to source coordinate $\mathbf{r}_s$, which is approximated by finite-difference in our automated procedure, and the contributions from the second-order spatial moment $\mu^{(2,0)}$ are proportional to the second-order spatial gradients

$$\tilde{\mathbf{X}} = \nabla \tilde{\mathbf{S}} \cdot \tilde{\mathbf{S}} - \omega \nabla \tilde{\mathbf{S}} \cdot \tilde{\mathbf{S}} - \omega \tilde{\mathbf{S}} \cdot \nabla \tilde{\mathbf{S}}.$$  

(9a)

$$\tilde{\mathbf{Y}} = \nabla \tilde{\mathbf{S}} \cdot \tilde{\mathbf{S}} - \omega \nabla \tilde{\mathbf{S}} \cdot \tilde{\mathbf{S}} - \omega \tilde{\mathbf{S}} \cdot \nabla \tilde{\mathbf{S}}.$$  

(9b)

For a finite source with strong directivity, the fault-plane ambiguity can be effectively resolved by examining the azimuthal variation of $\delta \tilde{\mathbf{S}}$ at high frequency, which usually has a minimum in the direction of rupture propagation (i.e. a maximum in the amplitude ratio between the observed waveform and the point-source synthetic waveform). This azimuthal variation is mainly due to the contributions from the mixed space–time moment $\mu^{(1,1)}$, whose sensitivity derivative $\omega \nabla \tilde{\mathbf{S}} \cdot \tilde{\mathbf{S}}$ has a $\cos(\theta)$-dependence on azimuth $\theta$ and increases with frequency (McGuire et al. 2001; Chen et al. 2005).

Resolving fault-plane ambiguity

Most previous studies that resolved fault-plane ambiguity for small to medium-sized earthquakes have relied on detecting directivity effect using empirical Green’s function techniques (Mori & Hartzell 1990; Mori 1996; Hellweg & Boatwright 1999; Okada et al. 2001; McGuire 2004). For a finite source without significant directivity, it is possible to resolve fault-plane ambiguity by detecting the constructive/destructive interference effect due to source finiteness, which is characterized by the contributions to the waveform from the second-order spatial moment $\mu^{(2,0)}$. In particular, the interference effect due to contributions from $\mu^{(2,0)}$ can be detected by examining the azimuthal and frequency variations of the GSDF measurements. For strike-slip events, the amplitude-reduction time measurements usually have a $\cos(2\theta)$-dependence on azimuth $\theta$ and increases with frequency. To demonstrate this point, we give an example in the Appendix.

To resolve fault-plane ambiguity using the contributions from $\mu^{(2,0)}$, we systematically compare the actual GSDF measurements $\delta \tilde{\mathbf{S}}(\omega_R)$ with model-predicted measurements $\delta \tilde{\mathbf{S}}(\omega_R)$ calculated for the two nodal planes of the CMT solution. The nodal plane that provides better prediction to the actual measurements is picked as the preferred fault plane. To make the GSDF measurements, the observed and point-source synthetic seismograms were first low-pass filtered using a fourth-order Butterworth filter with the corner frequency at 0.5 Hz; GSDF analysis were then applied on selected $P$ and $S$ waveforms to obtain $\delta \tilde{\mathbf{S}}(\omega_R)$ at 0.1, 0.2, 0.3, 0.4 and 0.5 Hz. The model-predicted $\delta \tilde{\mathbf{S}}(\omega_R)$ were computed using eq. (8) for the two nodal planes of our CMT solution. The model-predicted $\delta \tilde{\mathbf{S}}(\omega_R)$ can be obtained by integrating $\delta \tilde{\mathbf{S}}$ against a seismogram perturbation kernel $I_\phi(\omega, \omega_0)$,

$$\delta \tilde{\mathbf{S}}(\omega_R) = \int d\omega I_\phi(\omega, \omega_0) \delta \tilde{\mathbf{S}}(\omega_0).$$  

(10)
Figure 2. CMT solutions with fault-plane ambiguity resolutions for the earthquakes studied in this paper. Column 1, the event ID number, which is unique for each earthquake in Southern California Seismic Network (SCSN) catalogue; columns 2–4, longitude, latitude and depth of events’ centroid locations; column 5, local magnitude; columns 6–8, strike, dip, and rake of the best-fit-double-couple (bfdc) solutions; column 9, likelihood of the picked nodal plane to be the actual fault plane based on a Bootstrap procedure; column 10, beachball plots of the bfdc solutions with the preferred fault planes highlighted in black thick lines.

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<td>68</td>
<td>175</td>
<td>98%</td>
<td></td>
</tr>
<tr>
<td>9639729</td>
<td>-117.5806</td>
<td>34.0349</td>
<td>9.91</td>
<td>3.35</td>
<td>325</td>
<td>86</td>
<td>-174</td>
<td>95%</td>
<td></td>
</tr>
</tbody>
</table>
Here $I_\omega(\omega_n)$ is a bell-shaped function centred around $\omega_n$ with its width controlled by the windowing and filtering parameters chosen during the GSDF processing (Chen 2005 chapter 2; Chen et al. 2007a).

In this study, only amplitude-reduction time $\delta t_q(\omega_n)$ were used to distinguish the two nodal planes of the CMT solution. For the frequency range used in this study, we found that the sensitivity of $\delta t_q$ with respect to the directivity effect $\mu^{(1)}$ is only about one tenth of the sensitivity with respect to the source finiteness $\mu^{(2)}$ (Chen et al. 2005); therefore we ignore the contributions from $\mu^{(1)}$ in calculating the model-predicted $\delta t_q$. In our automated procedure, we first estimate the rupture dimension from the scalar moment; then calculate the model-predicted $\delta t_q(\omega_n)$ using eqs (8a) and (10). The correlation coefficient between the actual measurements $\delta t_q^i$ and model predictions $\delta t_q$ are calculated as

$$C = \frac{\sum_{l=1}^{n} [\delta t_q^i - E(\delta t_q^i)] [\delta t_q^i - E(\delta t_q^i)]}{(n-1) \text{std}(\delta t_q^i) \text{std}(\delta t_q^i)}. \quad (11)$$

Here $E()$ denotes the mean and $\text{std}()$ denotes the standard deviation, $l$ is the index for measurements made at different azimuth $\theta$, frequency $\omega$, distance $r$ and component $k$, $n$ is the total number of measurements used to calculate the correlation coefficient $C$. The nodal plane that provides a higher correlation coefficient is identified as the actual fault plane by our automated procedure.

To quantify the likelihood that the automatically picked nodal plane is the actual fault plane, we adopted a bootstrap method. In each iteration of our bootstrap process, we randomly select half of our measurements and compute the correlation coefficients for the two conjugate nodal planes of our CMT solution using eq. (11). The nodal plane that provides a larger correlation coefficient for more than half of our bootstrap iterations is identified as the actual fault plane. The likelihood that the picked fault plane is the real fault plane is computed as the ratio between the number of bootstrap iterations for which the pick fault plane provides a larger correlation coefficient and the total number of bootstrap iterations.

**RESULTS**

The geographic area of our study is shown in Fig. 1. We constructed a RGT database for the 48 CISN stations located in this area using the finite-difference method. The calculated RGTs are accurate up to 1 Hz. Synthetic seismograms at these 48 stations from an arbitrary point source located in our modelling volume can be easily computed by retrieving a small source-centred volume from our RGT database and calculating the source-coordinate gradient (eq. 1). At the same time, we also calculate the partial derivatives needed for CMT inversion (Zhao et al. 2006) and the higher-order gradients in eq. (8) needed for testing the two nodal planes of our CMT solution.

The fault planes picked by our automated procedure as well as the likelihood computed from 1000 bootstrap iterations are shown in Figs 1 and 2. In general, the likelihood calculated from the bootstrap method is higher for earthquakes with good receiver coverage. In Fig. 3, we show the distributions of the differences between the correlation coefficients calculated for the two nodal planes as well as the azimuthal coverage of the receivers used to resolve fault plane ambiguity for three earthquakes: the 2002 September 3 M 4.8 Yorba Linda earthquake, the 2001 September 9 M 4.2 Hollywood earthquake, and the 2005 January 6 M 4.4 Fontana earthquake. For these three events and several other smaller events, the fault planes picked by our automated procedure were confirmed by the distributions of relocated aftershock hypocentres (Fig. 3c). The Yorba Linda earthquake, which has the best receiver coverage among the three earthquakes, also has the highest likelihood computed from the bootstrap method, while the Fontana earthquake, which has the
worst receiver coverage among the three earthquakes, has the lowest likelihood.

Among the 35 earthquakes that we have resolved fault-plane ambiguity for, there are two clusters. One is the ‘Fontana trend’ (Hadley & Combs 1974; Cramer & Harrington 1987; Sheridan 1997; Plesch et al. 2003) located to the southwest of the right-lateral San Jacinto fault and the north-dipping Cucamonga fault (Figs 1 and 4). The Fontana trend is not associated with any mapped surface fault traces (Plesch et al. 2003). The other seismicity anomaly is the ‘Yorba Linda cluster’ (Walls et al. 1998; Plesch et al. 2003; Chen et al. 2005) located about 20 miles to the southwest of the ‘Fontana trend’ and near the right-lateral Whittier fault. The CMT solutions for the earthquakes in the Fontana trend are similar to those in the Yorba Linda cluster (Fig. 1), but the fault planes picked by our automated procedure show that the earthquakes in these two clusters have very different source mechanisms (Figs 1 and 4).

Most of the earthquakes in the Yorba Linda cluster show left-lateral faulting conjugate to the nearby right lateral Whittier fault, while the earthquakes in the Fontana trend show right-lateral faulting parallel to the San Jacinto fault (Fig. 4). The underlying causes for these two anomalous clusters are still under investigation, but this example demonstrates that our new technique are potentially useful for providing additional constraints to regional tectonics.

**DISCUSSION**

The methodology discussed in this paper is generally applicable. It does not rely on detecting the directivity effect; therefore it can be applied to any seismic sources including bilateral earthquakes with zero directivity. In regions where there are no precisely relocated aftershocks or for earthquakes with few aftershocks, we expect our methodology to be the most convenient means for resolving fault-plane ambiguity.

By removing fault-plane ambiguities from CMT solutions in a routine manner, we can provide clearer pictures on regional tectonics. Resolving fault-plane ambiguity from CMT solutions can also reduce the uncertainties in regional stress inversions based on earthquake focal mechanism data (Gephart 1985; Michael 1987; Gephart 1990; Yin 1996). The identified fault planes can also be used in calculating Coulomb stress changes and provide a more robust description of probabilistic seismic hazard based on the stress transfer model (Harris 1998; McCloskey et al. 2003; Parsons 2005; Steacy et al. 2005).

This automated procedure for picking the fault plane from the two conjugate nodal planes of our CMT solutions is a generic module of our computational framework for unified seismic data processing. The same types of GSDF measurements used to improve our seismic source models were also used to refine our seismic structure models iteratively in linearized tomographic inversions (Chen et al. 2007a). The RGT database used in our seismic source inversion procedures were also used to compute the sensitivity kernels of the GSDF measurements with respect to the seismic structure model using the scattering-integral method (Zhao et al. 2005; Chen et al. 2007b). We expect this unified seismic data processing framework, which is based on the GSDF measurements and RGT database, will provide seismologists a general platform for seismic waveform analysis and inversion at different geographic scales.

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Figure 4. Projection of a 3-D rendering of the focal mechanisms for earthquakes in the Yorba Linda cluster and the Fontana trend (Fig. 1). Focal mechanisms are represented as two intersecting disks, whose sizes are proportional to the likelihood of being the actual fault plane as determined in this study. Extensive quadrants are painted in yellow and compressive quadrants are painted in red. Major faults in this area are represented as coloured transparent surfaces. The surfaces denoted as ‘Fontana Seismicity Bestfit’ (FSB) and ‘Yorba Linda Extruded’ (YLE) are linear least-square fit to the hypocentres of the earthquakes in the Fontana trend and the Yorba Linda cluster, respectively. Locations and orientations of the faults as well as the FSB and YLE surfaces are obtained from the SCEC Community Fault Model (CFM-A). Background seismicity is shown as green spheres. Major highways in this area are plotted as white solid lines and the interstate freeways I-10 and I-15 are labelled in red. The 3-D rendering was done using the Java3D-based interactive 4-D visualization software SCEC-VDO (http://scecdata.usc.edu/scecinterns/index.php?title=UseIT:_SCEC-VDO_User%27s_Page) developed by the SCEC Undergraduate Studies in Earthquake Information Technology (UseIT) interns under the supervision of Sue Perry (perry@usc.edu).

REFERENCES


APPENDIX

We consider a circular crack with symmetric slip distribution and the slip rate on the fault plane is (Sato & Hirasa 1973)

$$\Delta v(r_s, t) = \frac{24 \Delta \sigma}{7\pi \mu} \sqrt{\frac{V^2}{(V^2 - r_s^2)} H(Vt - r_s)} H(R - Vt). \quad (A1)$$

Here, $\Delta \sigma$ is stress-drop, $\mu$ is the shear modulus in the source region, $V$ is the rupture propagation velocity, $R$ is the final radius of the circular crack and $H(t)$ is the Heaviside function.

The scalar moment for such a kinematic source model is $M_0 = (16/7) \Delta \sigma R^3$. We set up our coordinate system as shown in Fig. A1, we have the centroid location $\mu^{(0)} = 0$ and the centroid time $\mu^{(1)} = (3/4) R/V$. The mixed space–time moment that accounts for the directivity effect is exactly zero $\mu^{(1,1)} = 0$ for such a symmetric source. The second-order temporal moment is $\mu^{(0,2)} = (3/5) (R/V)^2$. The second-order spatial moment is $\mu^{(2,0)} = (R^2/5) \text{diag}[1, 0, 1]$, if the rupture lies in the $x$–$z$ plane, and $\mu^{(2,0)} = (R^2/5) \text{diag}[0, 1, 1]$, if the rupture lies in the $y$–$z$ plane. Here diag[] denotes a diagonal matrix.

To simplify discussion, we consider the far-field term in the whole-space Green’s function (Aki & Richards 2002)

$$\tilde{u}(r, \omega) = \frac{P(\theta, \phi)}{4\pi \rho c^2 r} \exp(i \omega r/c). \quad (A2)$$

Figure A1. Source–receiver configuration for the numerical experiments given in the Appendix. We consider two circular cracks, one lies in the $x$–$z$ plane, the other lies in the $y$–$z$ plane. The one lying in the $x$–$z$ plane is not shown here. The radius of the circular cracks is 1 km, rupture propagation velocity is 2 km s$^{-1}$. The receiver array is circular, with radius 10 km and lies 5 km above the source. The shear modulus in the source region is 300 kbar and stress drop is 30 bars.
where, $c$ is $P$- or $S$-wave speed, $\rho$ is density, $r = r_R - r_S = r\hat{r}$ and $P(\theta, \phi)$ denotes the radiation pattern. If we bring eq. (A2) into eq (8), use spherical decomposition $\nabla = \hat{r}\partial_r + \nabla_1/r$ (Dahlen & Tromp 1998), for the circular crack source model, we obtain

$$\delta\tilde{\tau}_q(r, \omega) = \frac{\omega^2}{2} \mu^{(0,2)} + \frac{\omega}{2c^2} \hat{r} \cdot \mu^{(2,0)} \cdot \hat{r} - \frac{\mu^{(2,0)}}{2\omega r^2} \cdot \nabla_1 P \frac{P}{r}, \quad (A3)$$

$$\delta\tilde{\tau}_p(r, \omega) = \frac{\mu^{(0,1)}}{cr} \cdot \mu^{(2,0)} \cdot \nabla_1 P \frac{P}{r}. \quad (A4)$$

If we bring the FMTs into eq. (A3), for the source–receiver configuration shown in Fig. (A1), the model predicted azimuthal variations of $\delta\tilde{\tau}_q$ are shown as the solid line for a rupture lies in the $x$–$z$ plane and as the dash line for a rupture lies in the $y$–$z$ plane. Since the contributions from $\mu^{(1,0)}$ and $\mu^{(1,1)}$ are zero and contributions from $\mu^{(0,1)}$ and $\mu^{(2,0)}$ do not have azimuthal dependence, the azimuthal variation of model predicted $\delta\tilde{\tau}_q$ is caused by the second-order spatial moment $\mu^{(2,0)}$ only. As shown in Fig. A2, the contribution from $\mu^{(2,0)}$ has a $\cos(2\phi)$-dependence on azimuth. If the rupture lies in the $x$–$z$ plane, $\delta\tilde{\tau}_q$ has two minima (i.e. maxima in waveform amplitude) at $0^\circ$ and $180^\circ$, which is caused by destructive interference. For a rupture lies in the $y$–$z$ plane, the minima are at $90^\circ$ and $270^\circ$, while the maxima are at $0^\circ$ and $180^\circ$.

For a fixed receiver location, $\delta\tilde{\tau}_q$ also has different frequency-dependences for the two ruptures lying in the $x$–$y$ and $y$–$z$ planes as shown in eq. (A3).

An important issue in detecting second-order source effects is the signal-to-noise ratio. The contribution to $\delta\tilde{\tau}_q$ from the second-order spatial moment $\mu^{(2,0)}$ is

$$\left\| \frac{\hat{X}}{2} : \mu^{(2,0)} \right\| \leq \frac{\omega}{10} \left( \frac{R}{r} \right)^2 + \frac{1}{10\omega} \left( \frac{R}{r} \right)^2. \quad (A5)$$

The first term on the right-hand side of eq. (A5) does not depend on source–receiver distance; therefore, it is possible to detect the contribution from $\mu^{(2,0)}$ in the far field when the frequency is sufficiently high. For shear wave speed $\beta \sim V$, this term is roughly one third of the contribution from $\mu^{(0,2)}$. The second term on the right-hand side of eq. (A5) is significant in the near field and increases as we decrease frequency. For local small to medium-sized earthquakes (2.5 $\leq M_L \leq 5$), we found that the contribution from $\mu^{(2,0)}$ is statistically significant for source–receiver distance ranging from 10 to 50 km and for frequency ranging from 0.1 to 0.5 Hz.