1. Introduction

Plate tectonics, the fundamental process of earth science, is physically very poorly understood. The theory was first proposed to explain geological observations, with matching rock units and land dwelling fossils found across oceans from each other, and later magnetic stripes on the sea floor. In fact, the original theory of continental drift was discredited because the mechanism of continents plowing through the ocean floor was physically impossible. The modern theory of plate tectonics assumes that thermal convection of the mantle drives plate motion on the surface. In fact, plate tectonics arises as the top thermal boundary layer of a convecting mantle, such that the plates are themselves part of mantle convection and not passive objects being pulled around by mantle convection patterns. Geophysicists have an understanding of how a rigid surface forms, through temperature dependent viscosity as will be described later, but how deformation localizes and causes weakening at discrete boundaries, and the wavelength patterns of motion on the Earth are poorly understood [Bercovici et al, 2000; Bercovici 2003a; Tackley 1999].

Furthermore, most researchers assume the whole mantle convects, due to the tomographic evidence of slabs penetrating to near the core-mantle boundary (CMB) and the theoretical problems associated with any layered convection model [Bercovici and Karato, 2003b; Puster and Jordan, 1997; Grand et al, 1997]. Since plates are themselves part of mantle convection, their presence sinking through the whole mantle is by definition whole mantle convection. As a result, one of the most effective ways to understand the physics of plate
tectonics is to model convection in a variable viscosity fluid and see if plate-like features can be reproduced. The first thing to be learned is that standard Rayleigh-Bernard convection does not form most of the features of plate tectonics: downwellings are tubular, not linear slabs due to the lack of viscosity variations; all motion is poloidal, moving directly from an upwelling to a downwelling, without any rotational, or strike-slip, motion due to the lack of lateral viscosity variations; the surface is weak and deforms internally instead of forming rigid plates with deformation localized to discrete boundaries; and the planform of convection is a symmetrical, evenly spaced pattern of upwellings and downwellings, or just downwellings if the model is purely internally heated. However, when the Rayleigh number is increased past a critical point, \( \sim 10^7 \), the planform breaks down into a time-dependent, asymmetric pattern [Davies 1999].

Many models have attempted to recreate plate tectonics, but the most successful have used a visco-plastic rheology. In these models, a temperature-dependent viscosity is paired with a plastic rheology with a prescribed yield stress. Material undergoes a viscosity reduction when the yield stress is met or exceeded, and otherwise its viscosity is determined by the temperature-dependence. These models produce sharp, long, linear subduction zones and reasonably sharp, linear spreading ridges. However, these models fail to produce strike-slip faults, one-sided subduction zones, and unrealistic yield stress values must be used to produce surface motion [Tackley, 2000a,b].

These models can be improved by using the visco-plastic rheology in spherical geometry, and that is the main focus of this study. I performed numerical experiments in two phases: first, models reproducing, in part, results from Tackley [2000a] were used as a test, and then modification such as varied heating modes and varied Rayleigh number were added; second, the same rheology was used in spherical geometry, with varying Rayleigh number and yield stress
values. Spherical geometry should increase the amount of toroidal motion, ie provide more opportunity for forming strike-slip faults, and perhaps allow for higher yield stress values to be used. These effects were all analyzed, and a phase plot of Ra and yield stress was constructed illustrating where plate-like motion occurs in the parameter space. Also, the effect of Rayleigh number was explored. In previos Cartesian domain studies, the effect of Rayleigh number was not tested [Tackley, 2000a,b; Stein et al 2004], but in spherical geometry there is a strong dependence. The spherical harmonic degree of the convection pattern, ie the wavelength of motion, is determined by the Rayleigh number, and these results show how the Rayleigh number of the Earth causes the unique wavelength of plate tectonics.

The effect of heating mode was also examined in both Cartesian and spherical domains, and find that plate tectonics forms only when the convective planform is dominated by downwellings. My results show that plumes break up surface plates and yield a behavior more consistent with the mobile regime, so there exclusion from the planform with pure internal heating improves the plate-like nature of the system in these models. This may be an appropriate simplification for Earth, where the force supplied by subduction is an order of magnitude greater than that provided by ocean ridges [Davies 1999]. However, visco-plastic convection models with pure bottom heating also produce plates [Stein et al, 2004], and plumes should have an effect on surface behavior on Earth, so ideally they should be included in models. Though I don’t provide a clear answer here, if nothing else I show that this topic one needing more research.

2. Background Theory

As stated above, plate tectonics forms as the top thermal boundary layer of the convecting mantle, though isoviscous, bottom heated convection does not produce surface
behavior reminiscent of plate tectonics, indicating mantle convection on Earth is more complicated. One of the most obvious physical differences between the Earth and this simple convection is that the mantle does not have a uniform viscosity. The surface of the Earth is much stronger than the deep interior, and mineral physics confirms that mantle rock has a strong temperature dependent viscosity [Davies 1999]. Adding a strong temperature-dependent viscosity, meaning models with a 3 or more order of magnitude difference between cold and hot material, to thermal convection models produces a rigid surface, but this surface never deforms and subducts [Christensen, 1984; Solomatov, 1995]. Convection occurs underneath an immobile lid, in the form of small, drip-like downwellings, while upwellings become entrenched in the upper mantle and never cause motion on the surface. This behavior is known as the stagnant lid mode of convection [Davies 1999, Solomatov, 1995; Zhong et al 2000].

Clearly, some mechanism is required to break the stagnant lid and allow for slabs to subduct back into the mantle. Adding a plastic component to the rheology allows the top boundary layer to stay rigid, replicating the lithosphere, while it can readily deform along sharp zones where the material is stressed at or surpassing the specified yield stress. This type of rheology was tested in three-dimensional Cartesian domains and found to reproduce many features of plate tectonics, namely sheetlike downwellings, sharp spreading ridges, and strong plates that are mostly outlined by sharp deformation zones [Tackley, 2000a,b]. The visco-plastic rheology not only allows for rigid surface plates to subduct and participate in mantle convection, it also causes the system to self-organize in a way that deformation only occurs on narrow zones forming sharp, linear subduction zones and spreading ridges. However, there are still many problems: subduction zones are double-sided, in that both “plates” subduct, instead of one-sided like they are on Earth; no strike-slip faults form, resulting in less toroidal motion (approximately
30% of all motion, as opposed to approximately 50%) than is found on Earth [Tackley, 2000a, Stein et al, 2004]; and that the range of yield-stress values required to produce plate-like motion are approximately an order of magnitude lower than experimentally determined yield stress values in mantle rocks [Tackley, 2000a, Stein et al, 2004].

The final major issue is how convection is driven in the mantle; whether most of the thermal energy comes from bottom heating through the core, or internal heating. This topic is crucial to plate tectonics because the mode of heating strongly affects the convective planform for all types of convection. When Rayleigh-Bernard convection is purely bottom heated, the planform is a pattern of evenly spaced upwellings and downwellings, where material moves from upwelling to downwelling in a convective cell. When heating is completely internal, the upwellings disappear, because they can not form without a bottom thermal boundary layer. The mantle is known to have both internal heating, from radioactive decay of elements within the mantle, and bottom heating from the core. Though the exact ratio of the two is unknown, most studies show that internal heating provides the mantle with the majority of its energy [Lay et al, 2008]. This provides some support to the assumption of pure internal heating in my models.

This assumption is also supported by examining plate motion data, where plates connected to down going slabs move about one order of magnitude faster than plates not connected to down going slabs. This effect is caused by the fundamental forces of plate tectonics, ridge-push and slab-pull. The ridge-push force stems from a pressure gradient formed as lithosphere cools and spreads from a mid-ocean ridge, so in a sense the ridge “pushes” the plates away. The slab-pull force is the force exerted by a subducting slab on the rest of the plate, and is essentially the slab pulling the plate as it sinks. Simple calculations of these forces show that slab-pull is approximately one order of magnitude stronger than ridge push, hence the
velocity discrepancy among tectonic plates [Davies 1999]. This has classically been interpreted to mean that subduction is the dominant force of plate tectonics, but recent work suggests plume may be important [Lay et al, 2008].

Finally, tomography shows that spreading ridges are not underlined by slow velocity anomalies, indicating that spreading ridges are not the products of plume activity, but of passive mantle upwelling [Bercovici and Karato, 2003b]. This passive motion is by subducting slabs plunging into the deep mantle and essentially pushing the displaced mantle material up. As a result, some models haven chosen to use purely internal heating, therefore removing plumes and allowing the dominant forces of downwellings to act uninhibited. However, recent studies of CMB heat flux show a higher amount of heat input from the core, and indicate that plumes may be more important [Lay et al, 2008]. Using purely internal heating may be too unrealistic an assumption, therefore motivating the heating mode tests presented here. These tests are preliminary, however, and much more work is required to understand the role of plumes in plate formation.

3. Methods

The finite element convection code Citcom, developed and maintained at the California Institute of Technology was used in both its Cartesian (CitcomCU) and spherical (CitcomS) forms [Moresi and Solomatov, 1995; Zhong et al, 2000; Tan et al, 2006]. This code numerically solves the equations of motion for a viscous, incompressible fluid starting from an initial temperature field. The temperature field is used to solve the momentum equation, with the solution being used by the advection-diffusion equation to solve for a new temperature field. This process is then repeated iteratively. The governing equations are as follows:

$$u_{i,i} = 0$$
\[-P_{i,j} + (\eta u_{i,j} + \eta u_{j,i})_j + RaT \delta_{it} = 0\]

\[T_{,t} + u_i T_{,i} = T_{,ii} + H\]

Where \(u\) is velocity, \(P\) is the dynamic pressure, \(\eta\) is the viscosity, \(Ra\) is the Rayleigh number, \(g\) is the acceleration due to gravity, \(T\) is the temperature, and \(H\) is the heat production rate. The term \(X_{,y}\) means the derivative of \(X\) with respect to \(y\), with \(i\) and \(j\) representing spatial indices, \(r\) representing the radial direction, and \(t\) representing time. The Rayleigh number, \(Ra\), is calculated as follows:

\[Ra = \frac{\rho_0 g \alpha \Delta T R_0^3}{\eta_0 \kappa}\]

Where \(\kappa\) is the thermal diffusivity, \(\eta_0\) is the reference viscosity, \(\rho_0\) is the reference density, \(\Delta T\) is the temperature difference from the CMB to surface, and \(R_0\) is the layer thickness, which in CitcomS is taken as the radius of the Earth (not the thickness of the mantle). On top of this, a temperature-dependent viscosity was used, following the Arrhenius type law:

\[\eta = \eta_0 \times \exp \left(\frac{\eta_e}{T + \eta_T} - \frac{\eta_e}{1 + \eta_T}\right)\]

where \(\eta_0 = 1\), \(\eta_e = 18.43\) for spherical cases and 23.03 for Cartesian cases, and \(\eta_T = 1\). This gives a four order of magnitude viscosity contrast from non-dimensional temperatures of 0 to 1 in spherical models and a 5 order contrast in Cartesian models, as the spherical code could not feasibly run the higher viscosity contrast. Since internally heated models can exceed a temperature of 1, especially in the stagnant lid case, the exponential becomes negative and the \(\log(\eta)\) can range from -1 to 4 in spherical runs, or -1 to 5 in Cartesian runs. To ensure that viscosity contrasts do not become too large for the code to handle, viscosities are clipped at \(\log(\eta) = -1\) and 4 for both spherical and Cartesian models. The final piece of the rheology used is plasticity, governed by a Byerlee type yield stress relation:
\[ \sigma_y = \min (a + b(1 - r), y) \]

Where \(a\) is the cohesion term and \(b\) is the friction multiplied by hydrostatic pressure, giving the Byerlee plasticity for brittle behavior. For ductile behavior, \(y\) is used to prescribe a constant yield stress throughout the layer. Experiments combining ductile and brittle behavior are termed composite yield stress experiments, while those with just ductile behavior are termed constant yield stress experiments. Viscosity is then recalculated as follows:

\[ \eta_y = \frac{\sigma_y}{2\dot{\varepsilon}_H} \]
\[ \eta_{eff} = \frac{\eta \times \eta_y}{\eta + \eta_y} \]

Where \(\eta\) is the previously calculated viscosity, and \(\dot{\varepsilon}_H\) is the second invariant of the strain-rate tensor.

Following Tackley [2000a,b] models in Cartesian geometry, using CitcomCU, were run with an aspect ratio of 8, and resolution of 128 in the \(x\) and \(y\) directions and 32 in the vertical, or \(z\) direction, with tests of double the above resolution showing no significant differences. An internal heating rate of \(H = 10\) was used unless otherwise noted, giving a mean mantle temperature of approximately 1. Yield stress and Rayleigh number were varied as shown in the results. Three main groups of models were run: constant yield stress runs, where both yield stress and Rayleigh number were varied in order to fill in a two dimensional phase space of these parameters; composite yield stress runs, where varying yield stress parameters were tested at various Rayleigh numbers, starting with \(10^5\), and heating mode runs, where different bottom boundary conditions and values of \(H\) were used with various yield stress parameters, and a constant Rayleigh number of \(10^5\).
Models in spherical geometry, using CitcomS, were run at a resolution of $32 \times 32 \times 32$ in each of the 12 caps the spherical shell is broken into when the Rayleigh number is $2 \times 10^6$ or lower, and a resolution of $64 \times 64 \times 64$ otherwise. A constant internal heating rate, $H$, of 60 was used because CitcomS uses the radius of the earth instead of layer thickness for all scaling. Constant yield stress and Rayleigh number terms were varied, mapping out the two dimensional phase space of these parameters, with the starting point for the Rayleigh number being $10^6$, since the different scaling in CitcomS requires an approximately order of magnitude larger Rayleigh number than in CitcomCU.

3.1 Analytical Methods

Model results were analyzed in a number of different ways: examining the planform of convection by analyzing visualizations of the temperature, viscosity, and velocity fields; plotting the surface viscosity and velocity in map view using GMT; plotting horizontal averages of RMS velocity and heat flux through time; decomposing the velocity field into toroidal and poloidal components; and plotting horizontal averages of viscosity at given times. All models were run until they reached a steady-state, which is seen when the RMS velocity and heat flux trends reach a constant value, or oscillate around a constant value. The planform of convection should also reach a steady style (ie number and size of downwellings) but steady-state can not be used to conclusively determine steady-state. The velocity and heat flux plots are more reliable and complete.

The last tool is a temperature power spectrum that shows the spherical harmonic degree that the temperature exhibits as a function of depth (horizontally averaged) and time. This is somewhat useful in determining the differences between the four regimes of model behavior, but it is most useful in quantifying the wavelength of the pattern of convection. Convective
wavelength is often classified in terms of spherical harmonic degrees, with degree-1 convection corresponding to the longest wavelength possible in a sphere: two hemispherical cells converging at a pole and diverging at the opposite pole. The power spectrum shows what degree of convection is dominant in the system, and how this changes with time. This will be discussed in more detail in the results section.

3.2 Explanation of Regimes of Motion

Before showing the results, the terminology used to characterize and categorize the results should be explained. All of the results, in both spherical and Cartesian models, are broken into 4 groups as yield stress goes from higher values to lower values: the stagnant lid regime, the episodic lid regime, the plate-like regime, and the mobile regime. The stagnant lid regime is characterized by the formation of a strong top boundary layer that never reaches the yield stress, and therefore remains essentially undeformed and nearly completely immobile. The planform of convection will show no substantial subduction zone type downwellings that penetrate to the core-mantle boundary, only drip-like downwellings that emanate from the bottom of the top boundary layer. All downwellings are visualized by temperature isosurfaces that are half the temperature of the ambient mantle. Stagnant lid convection should not have any downwellings this cold, and I define this as a diagnostic feature for determining when a model falls into this regime. Since plate motion in these models is driven by subduction there is no surface motion, and therefore no plate tectonics stagnant lid convection. Surface velocity plots will show zero or near zero velocities, while RMS velocity plots of the whole mantle will show a curve that smoothly reaches a constant value with time; there are no oscillations with time because there are no overturns in the system. Thus the name stagnant lid.
As yield stress is lowered, the episodic lid regime is reached, and this mode of convection is characterized by periods of stagnant lid behavior separated by sudden overturns. The yield stress is too high to consistently deform the surface, but low enough to be reached periodically, resulting in surface deformation and downwellings of the whole top boundary layer. A model is defined to be in the episodic lid regime if it contains more than one overturn (as stagnant lid models sometimes have one, initial overturn), but also contains period of stagnation where downwellings are absent and surface motions go to zero. The RMS velocity plot will show periods of low velocity with episodic peaks where overturns take place. The velocity spikes can evolve into longer, bursts of motion, but there must be clear velocity lows between the periods of motion for the model to be classified as episodic lid.

The plate-like regime is found for yield stresses lower than the episodic lid regime, and it is characterized by a constantly subducting top boundary layer when the system has reached steady state. To be called plate-like, a model must show no extended periods (~50 Ma or longer) of low RMS velocity, and show rigid plates that move at a uniform velocity, along with sheet-like downwellings and narrow weak zones outlining the rigid plates. The RMS velocity plot of plate-like models will show a time consistent oscillation around a uniform value after the model has reached steady-state. The oscillation indicates the time-dependent pattern of consistent downwellings. A characteristic heat flux plot of plate-like convection will show oscillations of +/- 5 around a constant value in a similar, sinusoidal type pattern as the RMS velocity. In fact, the two plots should mirror each other considering overturns will cause more heat flux and higher velocities. Essentially the model results must show realistic, Earthlike features to be classified as plate-like.
The mobile regime is the most arbitrary to separate from the others, as stagnant lid and episodic lid behavior can clearly be separated by RMS velocity and heat flux, and movies of the convective planform that show the surface “freezing” up. A mobile lid model will show tubular downwellings, and broad, diffuse zones of deformation, where velocity gradients can be seen over a significant distance (5 + elements). Characteristic RMS velocity and heat flux data will show much larger variability, as the surface is weak enough to allow strong internal deformation and more frequent, time-dependent downwellings. Another important feature, seen through the surface viscosity field, is that plates can no longer be identified, because the whole surface is much weaker (the yield stress is reached almost everywhere). Spreading ridges and subduction zones are present, but they don’t connect to form discrete plates, as happens in plate-like models. The main diagnostic features used to determine plate-like models from mobile models are this loss of defined plates, and the high variability in plots of observables.

4. Cartesian Results

The Cartesian models are broken 3 main groups: models with depth-constant yield stress, where the yield-stress Rayleigh number phase space is explored; models with composite yield stress, where differing values of composite yield stress are tested with a constant Rayleigh number; and heating mode tests, where the affects of using different modes of heating were explored for varying depth-constant yield stress with a constant Rayleigh number. The results of each section will be presented below.

4.1 Constant Yield Stress Models

The results for these models fall into the four categories as outlined in the methods section: mobile regime, plate-like regime, episodic lid regime, and stagnant lid regime. The non-dimensional yield stress values that produce the four regimes at Ra = 10^5 for my results are as
follows: 0-1000 produces mobile lid behavior; 1500-2800 produces plate-like behavior; 3000-
3300 produces episodic lid behavior, and 3500 + produces stagnant lid behavior. This scales to
the following dimensional ranges: 0-12 MPa for mobile lid; 18-34 MPa for plate-like lid; 36-40
MPa for episodic lid; and 42 + MPa for stagnant lid. The range that produces plate-like behavior
should correspond to known yield stress values in mantle rocks, but these ranges are dramatically
lower than the approximately 700-1000 MPa measured in laboratory experiments for mantle
rocks at mantle conditions. Not only are the values much too low, but the range of usable values
is also quite small. The plate-like range is also well below the range Tackley found using the
same rheology. His non-dimensional yield stress values of 5700 to 8500 produced plates, and
the stagnant lid regime was not found until a non-dimensional yield stress of 14000. When
scaled to dimensional numbers, this gives a plate-like range of 68 MPa to 102 MPa, with
stagnant lid behavior forming for yield stress values greater than 168 MPa [2000a]. The reason
for this discrepancy between Tackley’s models and my models will be addressed the Discussion
and Conclusions section.

The Rayleigh number used, $10^5$, is scaled to a Rayleigh number of $10^6$ because of internal
heating, which changes the Rayleigh number by $Ra \times H$. However, $10^6$ is on the low end
of estimates for Earth’s mantle, which range from $10^6$ to $10^8$. Because of this some higher
Rayleigh number models were tested. With increased Rayleigh number, the four regimes shift
towards higher yield stress values, because the system is convecting more vigorously and
generating more stress. Though a thorough map of the Rayleigh number-yield stress phase space
has not been completed (because the main focus is on the spherical models), an approximate
result is presented.
Figure 4.1: Rayleigh Number-Yield Stress Phase Space

4.2 Analysis of Plate-like Models

The results of the constant yield stress experiments produce some plate-like models, though they do not exhibit the Earth-like characteristics of the composite yield stress and spherical models discussed later. These models show the formation of time-dependent, slab-like downwellings, and the formation of spreading ridges at a distance roughly equal to the thickness of the layer, with some forming farther away (see Figure 4.2). This is a typical effect of Rayleigh-Bernard convection, though temperature-dependent viscosity usually produces longer
Figure 4.2: Viscosity Field and Surface Velocity, With Cold (1/2 the Ambient Mantle Temperature) Temperature Isosurface for Plate-like Models With Constant Yield Stress (After Tackley, 2000a,b)

a),b): Yield Stress = 18 MPa
The surface is weak, resulting in a short wavelength pattern on the surface. Downwellings span the entire domain, resulting in surface mobility throughout.

c),d): Yield Stress = 24 MPa
The surface is stronger, and plates that form are more coherent. However, there are now fewer downwellings and surface mobility is found for only ~ 2/3 of the domain.

e),f): Yield Stress = 30 MPa
The surface continues to become stronger and lose mobility. Now only small downwellings form, resulting in localized motion.

g),h): Yield Stress = 36 MPa
This yield stress produces episodic lid motion, seen here during a period of motion. Coherent plates form only at preferred distances from the downwellings, leaving the rest of the domain stagnant.
wavelength cells to form [Davies 1999]. These more symmetrical cells are common at higher yield stresses (~30 MPa) and a longer wavelength pattern forms at lower yield stresses (~20 MPa). Back-arc spreading, ridges formed just behind the arc of a downwelling, form for all yield stresses, though they are more focused for higher yield stress models that have smaller, isolated downwellings.

The amount of surface mobility is drastically different between the higher yield stress models and the lower yield stress models. In low yield stress models, the entire surface is mobile: subduction zones form across the entire surface and this results in the formation of ~15 plates, each about 1 to 1.5 times as large as the layer thickness. A few plates form isolated from subduction zones, and show little motion, but most are connected to downdgoing slabs, and are mobile. However, in higher yield stress models, only about half of the surface is mobile. Downwellings form sparsely, and do not connect across the entire surface, ie they have a finite beginning and end within the domain. Motion is then localized at the downwellings, forming two plates on either side of the subduction zone. The plates that form are roughly symmetrical, approximately as long as the layer thickness of the domain. Outside of these downwellings and the spreading ridges they form, the rest of the surface is immobile. This geometry should create strike-slip faults connecting the spreading ridges with the subduction zones, but this is not observed. Instead, the velocities from the plate motion decay to zero over a broad, diffuse zone that shows only minimal weakening due to yielding. Also, the arc-like shapes of both features cause them to often link, and leave no place for strike-slip fault formation.

These results can be explained by the way the system forms a balance between the temperature-dependent viscosity and the plastic yielding. The wavelength pattern for the low yield stress forms because the yield stress causes the system to revert back to this Rayleigh-
Bernard pattern. At higher yield stresses, one would expect the longer wavelength pattern to form, but here the short wavelength persists. This is due to the fact that these models are behaving in a near episodic mode, with downwellings only forming at preferentially locations; where the boundary layer instabilities focus together and amplify into a downwelling of the whole boundary layer. The system is only breaking at places where the stress builds to critical levels, which is reached only at the most fundamental convection pattern, the symmetrical Rayleigh-Bernard style cell. This effect is also seen in spherical models that revert to degree-1 convection in the episodic lid regime. Conversely, the longer wavelength of the low yield stress models is formed because the system doesn’t have to concentrate stress into the system’s most preferential form to allow for subduction to initiate. This also explains the surface mobility patterns, where the low yield stress allows downwellings to form more easily, and in turn the entire surface is made into mobile plates. However, this pattern may break down for higher Rayleigh numbers.

4.3 Composite Yield Stress Models

The composite yield stress models show quite different behavior than the constant yield stress models. The range of plate-like behavior expands drastically, from 18-34 MPa with a constant yield stress to 18 to 84 MPa with a composite yield stress (using the non-depth dependent part of the yield stress calculation). This range is now comparable to the range that Tackley found for both depth-constant yield stress and composite yield stress [2000a]. All composite yield stress models used a constant yield stress value, and a depth-dependent yield stress value, where the minimum of the two was used as the yield stress at that depth. The depth-dependent yield stress, ie the slope, is chosen to be 20 times the constant yield stress, so that the depth-dependent yield stress is in effect for the upper 1/20th of the model. This corresponds to
Figure 4.3: Viscosity Field and Surface Velocity, With Cold (1/2 the Ambient Mantle Temperature) Temperature Isosurface for Plate-like Models With Composite Yield Stress (after Tackley, 2000a,b)

a),b): Yield Stress = 24 MPa
At the low end of yield stress, downwellings form broader features than in constant yield stress models because of the extra weakening at the surface. Spreading ridges form sharper features, however.

c),d): Yield Stress = 34 MPa
Deformation is focusing into sharper features in both subduction zones and spreading ridges. Long, coherent slab-like subduction zones now forming.

e),f): Yield Stress = 36 MPa
Coherent plates form, clearly moving at uniform velocities. Subduction zones and spreading ridges are sharp.

g),h): Yield Stress = 78 MPa
Surface is becoming longer wavelength of motion due to the higher yield stress. The main downwellings now break into segments, and this geometry creates some strike-slip motion along a diffuse weak zone.
approximately 140 km, so the assumption is brittle behavior acting up to this depth, and ductile behavior beyond.

4.4 Analysis of Plate-like Models

The results show much more Earth-like behavior than the constant yield stress models, namely complete surface mobility for all plate-like yield stresses. Models consistently form long, sheet-like downwellings, sharp, coherent spreading ridges, and ~5 plates (see Figure 4.3). Plates are strong, usually with about 4 orders of magnitude greater than the ambient mantle, and show essentially no internal deformation; they all move at a uniform velocity. Spreading ridges are often oblique; the diverging plates are usually spreading at oblique angles, controlled by the subduction zone they feed. Subduction is nearly always orthogonal (the downgoing plate moves at a velocity orthogonal to the surface trace of the subduction zone), however when geometry forces an oblique subduction zone to form weakening occurs just before the trench and the velocities swing around to line up orthogonally to the downwelling. Other features noted include back-arc spreading, a common component of all models, and diffuse weak zones where strike-slip motion occurs. This only forms for areas with special geometry, where a subduction zone terminates within the domain. This causes a lateral velocity gradient from the plate connected to the subduction zone and the adjacent plate that is not and therefore is moving much slower. When this happens, the velocity gradient becomes diffuse and only minimal yielding takes place. The result is that no sharp strike-slip faults form. This may be appropriate for Earth if one believes that strike-slip faults, such as the San Andreas, are areas of distributed deformation, with velocities decaying across the fault for hundreds of kilometers.

4.5 Varying Heating Modes
Figure 4.4: Viscosity Field and Surface Velocity, With Temperature Isosurfaces for Visualizing Upwellings and Downwellings

a), b): Yield Stress = 24 MPa, mixed heating. The result is very similar to the purely internally heated model with the same yield stress. The wavelength of motion is slightly shorter.

c), d): Yield Stress = 24 MPa, bottom heating. The result is completely different, with few downwellings penetrating into the deep mantle. The surface is weak, and the plumes form radial flow, not plate-like flow.

e), f): Yield Stress = 30 MPa, bottom heating. Similar result to c and d, but this shows a stronger surface and less downwelling activity. Plumes still interact strongly with the surface.

g), h): Yield Stress = 36 MPa, bottom heating. The planform has reverted back to a Rayleigh-Bernard style convection, with hexagonal shaped cells forming. Plumes cause more weakening than downwellings.
Two types of heating modes were tested, to compare with the purely internally heated models already run. Mixed heating, were the bottom boundary layer was switched from insulating to isothermal, but the same rate of internal heating was used. Bottom heating was also used, where the internal heating rate is set to zero and the bottom boundary layer is set as isothermal, providing all of the heating for the system. It is known from Rayleigh-Bernard convection that changing the heating mode changes the planform of convection: using bottom heating creates symmetric sets of plumes and downwellings, and using pure internal heating creates just downwellings. So the main idea of this test is the effect plumes have on the formation of plates. However changing the heating mode also dramatically changes the average temperature profile with depth, and therefore changes the viscosity of the system as well. As a result it is hard to directly compare purely internally heated models with purely bottom heated models.

Models with mixed heating showed no plume formation, just a slight warming at the bottom boundary due to the presence of the hot isothermal boundary layer (see Figure 4.4). There appears to be a slight wavelength effect, where there are now smaller plates and more of them. The wavelength of motion is now shorter. The effect is minor, and must be related to the bottom heating. Material that subducts now heats up faster, as it must be at a non-dimensional temperature of 1 (1573 °C) at the CMB. This could cause a more vigorous convection, and therefore lead to the smaller wavelengths observed in these models. Overall, the effect is quite minor.

Having a much more dramatic effect is the use of bottom heating in isolation, leading to the formation of approximately as many plumes as downwellings. Now the system does not form a plate-like surface (see Figure 4.4). For high yield stress values, the system forms a quasi-
stagnant lid, where evenly spaced upwellings and downwellings form, and they stay virtually stationary. The downwellings reach to about ¾ the mantle depth, but never reach the CMB. As a result surface motion is very small, and the state resembles a type of stagnant lid convection. The average viscosity of the mantle is about two orders of magnitude higher than in pure internal heating. Presumably higher Rayleigh numbers would break this pattern into a more time-dependent style, but this was not tested due to constraints on computing time. At lower Rayleigh numbers, the system formed a more time-dependent, mobile state where downwellings penetrate to the CMB. Here though, plate-like behavior is not observed. Downwellings are still sheet-like, but they form at much smaller lengths; only about 1/10 of the size of internally heated models. This is caused by the plumes stopping subduction, and forcing the model into this more evenly spaced, symmetrical convection cell style of convection. When the plumes reach the surface, they cause flow outward radially, instead of flow in coherent plates, so the surface is now dominated by radial flow, either away from a plume or towards a downwelling. In this mode as well, the average mantle temperature is 2 orders of magnitude higher than in pure internal heating.

4.6 Conclusions

The constant yield stress and composite yield stress models show the four lid regimes, as expected based on previous work, but they show very different characteristics and different yield stress ranges that produce plate-like behavior. This is also different than Tackley’s results, which showed that composite yield stress only helped to sharpen the surface features, and that the yield stress range was the same for both. My composite yield stress results agree with Tackley’s, while my constant yield stress results do not. This must be cause by a difference in how the two codes calculate viscosity or stress. CitcomCU clearly needs the reduced yield stress
near the surface to form plates the way Stag3D does for a constant yield stress. The exact mechanism is unknown, but the only explanation is that it is code related.

Also, I find that large scale plume generation interferes destructively with plate formation, such that it is not possible to form a plate-like upper boundary layer with plumes as a major component of the planform of convection. This is appropriate for Earth, where plumes interact destructively with the crust as well. The African rift zone is thought to be caused by plume activity. The major difference between the Earth and these models is that the plume interaction with the lithosphere isn’t being focused into a discrete spreading system, like the three-armed rift system in Africa and the Arabian peninsula. This can be explained by the different temperature and viscosity profiles caused by bottom heating. To truly test the effect plumes have on forming plate tectonics in these models, this will have to be corrected for so models with and without plumes can be compared directly. Ideally, models with significant plume activity should also produce plate-like surface motions, since Earth does. Models with pure internal heating [Tackley, 2000a] and models with pure bottom heating [Stein et al, 2004] have shown plate-like nature, but a full analysis of the differences remains undone.

5. Spherical Results

All spherical models were run using a constant yield stress (due to time constraints), and this constant yield stress was varied with Rayleigh number to map out a phase space of these two variables. The results of the spherical cases, fall into the same four regimes as the Cartesian cases: mobile lid, plate-like lid, episodic lid, and stagnant lid. These four regimes fall into relatively linear fields as Rayleigh number varies, such that the range of yield stresses for which plate-like lid behavior occurs increases approximately linearly as Rayleigh number increases (see Figure 5.3). The transitions between the stagnant lid, episodic lid, and plate-like lid regimes are
sharp and dramatic, with only a 10% change in yield stress causing a model to switch from plate-like to episodic or episodic to stagnant. There are virtually no transition zones, the system will settle into one of the three regimes based on the yield stress and Rayleigh number. The border between the mobile lid regime and the plate-like lid regime is more diffuse, and harder to accurately characterize. However, there is still a clear linear trend, with the cutoff point for the mobile lid regime increasing with Rayleigh number. These trends make sense, because increasing the Rayleigh number increases the convective vigor of the system, results in higher stress on the surface, and allows higher yield stresses to be reached.

The range of yield stress that produces plate-like behavior is approximately 50-250 MPa. This is higher than the approximately 25-150 MPa range found in Cartesian experiments.
Figure 5.2: Typical Results for the Four Regimes of Motion in With Ra = 10^5.

a) Mobile: Viscosity field with velocity vectors, showing how most of the surface is weak, and motion only takes place in concentrated areas.

b) Mobile Lid: Temperature isosurface showing the pattern of downwelling. Downwelling is mostly tubular, and the planform resembles uniform viscosity convection.
Yield Stress = 25 MPa

c) Plate-like: Viscosity field shows strong “plates” separated by narrow weak zones where convergent and divergent motion occurs. Velocity vectors show how motion in a “plate” is uniform.

d) Plate-like Lid: Temperature isosurface shows one linear, slab-like downwelling, reminiscent of a subduction zone.
Yield Stress = 125 MPa

e) Episodic Lid: Viscosity field shows similar behavior to figure 1c, though here “plates” are higher viscosity.

f) Episodic Lid: Temperature isosurface shows again a linear, slab-like downwelling, though in this case more boundary layer instabilities are forming, causing some additional drip-like downwellings near the major feature.
Yield Stress = 275 MPa

g) Stagnant Lid: Viscosity field shows all high viscosity; only some minor yielding is occurring. There is also essentially no surface motion.

h) Stagnant Lid: There are no downwelling that penetrate to the CMB. Only drip-like downwellings form as instabilities on the base of the top thermal boundary layer.
Yield Stress = 625 MPa
Figure 5.3: Heat Flux and RMS Velocity for the Four Regimes of Motion With $Ra = 10^5$

**Mobile:** The heat flux and RMS velocity plots show how the system evolves into a highly time-dependent state after some initial large overturns. The horizontal average temperature profile shows how the system has the coolest interior temperatures due to the ease of convection. **Plate-like:** The heat flux and RMS velocity plots show how the system evolves into a steady state with heat flux variations of $\pm 5$ and closely spaced fluctuations in RMS velocity. The horizontal average temperature shows a warmer interior than the mobile case. **Episodic Lid:** The heat flux plot shows a large ($\pm 10$), periodic oscillation corresponding to episodic overturns, and the RMS velocity shows periodic spikes for the same reason. A warmer interior is shown by the horizontally averaged temperature profile. **Stagnant Lid:** Both the heat flux and RMS velocity plots show the system smoothly reaching steady-state, with no oscillation because there are no overturns. This also leads to the very warm interior temperatures.
[Tackley, 2000a], though still much lower than laboratory experiments indicate: 700-1000 MPa. Nevertheless, the higher yield stress range is at least an improvement on Cartesian experiments. Furthermore, spherical geometry gives visco-plastic models higher toroidal to poloidal ratios (~.4 in spherical models to ~.3 in Cartesian models [Tackley, 2000a]).

5.1 Analysis of Plate-like Models

As yield stress increases at a constant Rayleigh number (Ra = 10^6), the models show distinct changes in behavior on a more detailed scale than the overarching regimes already discussed. Within the plate-like lid regime, there are many differences between lower yield stress results and higher yield stress results. At the low end, with a yield stress of 75 MPa, downwellings are sheet like and double sided, but smaller than the near half-diameter size of many of the plate-like models (see Figure 5.4a,b). There are also more downwellings, usually two or three, as opposed to just one large downwelling, though all of the downwellings are clustered on one side of the globe. Also, the downwellings often form triple junctions and back-arc spreading centers. There is a large amount of toroidal motion, which can be seen in the velocity patterns, and the measured T:P ratio of ~0.4. Many diffuse weak zones form that are similar to strike-slip faults, but not focused enough to show convincing faulting. Spreading ridges also form quite easily, and they tend to link with the subduction zones to form rather coherent “plates,” though most still have large, diffuse zones of deformation somewhere along their boundaries.

Overall, the yield stress is too low to form sharp, focused zones of deformation, but it causes the surface to break into more “plates” and show a higher rotational motion component than higher yield stress models which is similar to Earth. Also, subduction zones tend to form back-arc spreading centers, which they don’t form in higher yield stress models. This is because
**Figure 5.4: Viscosity Field and Velocity Vectors, With Cold Temperature Iso-surface for Plate-like Models at Ra = 10^5**

a), b): Yield Stress = 75 MPa, resulting in a weak, easily deformable surface. The model shows many weak zones, resulting in about 10 plates. However the overall surface is too weak, resulting in unplate-like amounts of internal deformation (labeled).

c), d): Yield stress = 100 MPa, resulting in a generally stronger surface. Some earth-like features form, like back-arc spreading (labeled). Downwellings now organize into two major subduction zones, and there is less internal deformation. Still numerous weak zones forming ~10 plates.

e), f): Yield Stress = 125 MPa, resulting in a stronger surface, approximately 0.5 higher log(viscosity) than previous two. Now only one downwelling forms, and the model shows 3 plates that are nearly rigid.

g), h): Yield Stress = 162.5 MPa, the model shows similar surface characteristics to 3e and 3f. Here the downwelling has broken into a triple junction, showing that for this mode of plate-like convection the different planforms that can form.
i), j): Yield Stress = 175 MPa, resulting in the continued formation of one large downwelling, and 2-3 plates. The Surface is now quite strong, log(viscosity) ~ 3.

k), l): Yield Stress = 200 MPa, showing no significant changes in the style of convection. Surface continues to become stronger.

m), n): Yield Stress = 225 MPa, showing similar results to 4k and 4l. The isosurface is showing more boundary layer instabilities near the downwelling showing how characteristics of stagnant lid convection are forming.

o), p): Yield Stress = 250 MPa, the system is now in the episodic regime. The periods of motion show strong similarities to the higher yield stress plate-like models.
Figure 5.5: RMS Velocity and Surface Heat Flux Plots Corresponding to Models in Figure 5.4

Note: 75 MPa model was run using a well mixed, steady-state model as its initial condition.
the surface is weak enough to cause deformation in the back-arc where moderate stress from subduction is causing some spreading. With higher yield stress, this effect disappears because the yield stress is too high to allow deformation from the moderate stresses involved. The pattern of downwellings is also controlled by the low yield stress, as the system can now form...
smaller, separate downwellings like it would prefer for Rayleigh-Bernard convection. The formation of triple junctions is likely the effect of the system attempting to form a tubular downwelling, but the higher viscosity surface will only break into a three-pronged set, similar to continental rifting. This feature is decidedly un-Earthlike, along with the diffuse weak zones and double sided subduction.

With increasing yield stress, this phase transitions to the dominant, degree-1 convection pattern found for plate-like models at this Rayleigh number. With a yield stress of 100 MPa, the downwellings are longer, more linear and sheet-like, but there are still predominately two downwellings, clustered near each other on one side of the globe (see Figure 5.4c,d). The surface is still largely weak, being only ~2 orders of magnitude more viscous than the ambient mantle, and this results in the formation of roughly 4 to 5 “plates.” There is still significant weakening in the back-arc region of subduction zones, though this tends to occur on both sides of subduction zones. There seems to be back-arc spreading on both sides of the subduction zone. Though this doesn’t occur on Earth, it is a product of the models having double-sided subduction (ie both plates subduct, instead of one sliding under the other). Because of this, and the low yield stress, weakening occurs in the back-arc region on both sides of the downwelling. There are also more defined strike-slip style faults, but they are still more diffuse than the downwellings and spreading ridges. Overall, the yield stress is forming more coherent structures, but is still too low to form strong enough plates and sharp boundaries.

As yield stress is increased to 125 MPa, in the middle of the plate-like regime, the system forms the style of convection that it maintains for the rest of the plate-like range (see Figure 5.4e,f). The models form stronger plates and sharper deformation zones, but the convective planform becomes very long wavelength and some rotational motion is lost. The back-arc
spreading centers also disappear, as the surface is too strong to deform there with the higher yield stress. The system now consistently forms one linear, sheet-like downwellings that is approximately half an Earth diameter in length, and stays roughly stationary through time. Spreading ridges form easily about a third of the way around the Earth from the subduction zone, sometimes merging into as single spreading ridge. There are usually 2 or 3 “plates,” converging at the single downwelling, and usually diverging at two separate spreading ridges. In between the two ridges is a virtually stagnant plate, as it is not connected to any downwellings.

The heat flux and RMS velocity time series plots mainly show what regime of surface mobility the model falls into, but the general numbers and trends also can be compared to the Earth. The heat flux trend makes sense, with increased flow during overturns and decreased heat loss during periods of stagnation. Models all show oscillations around a non-dimensional value of 20, which scales to ~15 mW/m². This is lower than the ~100 mW/m² average for oceanic crust for the Earth (since continents are not present in this model) [Fowler, 2005]. The RMS velocity trends also make sense physically, with spikes at overturns and lulls during stagnant periods or periods with less vigorous subduction. However, the typical velocities observed both as the RMS for the whole system scale to ~1 mm/yr. This is much lower than the 4-9 cm/yr estimated for the Earth’s RMS velocity [Lithgow-Bertelloni et al, 1993]. The fact that both values are lower than observed indicates that the system as a whole is not convecting as vigorously as Earth. To see if this is true, higher Rayleigh numbers were tested.

5.2 Increasing Rayleigh Number

Unlike in Paul Tackley’s experiments, models with varying Rayleigh number were run. Since 10⁶ (based on mantle thickness) is on the low end of estimates for the Earth, this was used as the starting point and higher Rayleigh numbers were tested up to 10⁷. Past this unfeasably
a), b): Yield Stress = 100 MPa
The lowest end of the yield stress range; the surface is very weak and there is significant intraplate deformation. With the higher Rayleigh number, there are now more downwellings.

c), d): Yield Stress = 125 MPa
With lower Rayleigh number, this was the most plate-like case. Now it shows a weak surface, though back-arc spreading is present.

e), f): Yield Stress = 150 MPa
Shows similar features to 6c and 6d. Shows some strike-slip style motion, with a diffuse weak zone.

g), h): Yield Stress = 250 MPa
At lower Rayleigh number, this yield stress produces episodic lid behavior, and here shows strongly plate-like motion. There are two downwellings instead of one for low Rayleigh numbers.
Figure 5.7: RMS Velocity and Surface Heat Flux Plots Corresponding to Models in Figure 5.6
Note: These models used previously run, steady-state results as their initial condition.
Figure 5.8: Surface Viscosity and Velocity Vectors With Cold Temperature Isosurfaces for $\text{Ra} = 30^\circ$

a), b): Yield Stress = 125 MPa
The surface is weak, and shows many areas of intraplate deformation. Downwellings form a ring around a great circle of the model due to the higher Rayleigh number.

c), d): Yield Stress = 250 MPa
The surface is stronger, $\log(\text{viscosity}) \sim 2.5$. There are still approximately two major downwellings, and many detailed plate-like features form, such as back-arc spreading, and strike-slip deformation. The higher Rayleigh number is forming $\sim 10$ plates.

e), f): Yield Stress = 275 MPa
Surface now doesn’t break into as many plates, but there are still two downwellings present. These results look more like higher yield stress results at lower Rayleigh numbers, but with more downwellings present.

g), h): Yield Stress = 750 MPa
This falls into the episodic lid mode of convection. Now the system prefers to form only one downwell ing, as it does for lower Rayleigh number models.
Figure 5.9: RMS Velocity and Surface Heat Flux Plots Corresponding to Models in Figure 5.8
Note: Models that start from 0 were run from full start-up conditions, models that start at a time later than 0 were run using other models steady-state condition as an initial condition
high resolution is required \((128 \times 128 \times 128)\) and the number of CPUs and amount of time runs would take is too long for this project. The general trends in yield stress and Rayleigh number have already been discussed, but there is also a strong effect on the wavelength pattern of convection.

Cartesian models have generally used a constant Rayleigh number \([\text{Tackley, 2000a,b; Stein et al, 2004}]\) and my results show no strong dependence of convective wavelength on Rayleigh number. However, the spherical models show a jump from degree-1 convection to degree-2 convection as Rayleigh number goes from \(10^6\) to \(30^6\), with a transition region at \(20^6\) (see Figures 5.6, 5.7, 5.8, and 5.9). At the higher Rayleigh number, the convective planform changes dramatically. Instead of one large downwelling that spans half the Earth, there are two such downwellings, and they form in various orientations. Some models show the subduction zones link up to form a stable ring roughly around the equator, while others show the downwellings clustering on one side of the Earth. Either way there is a clear shorter wavelength pattern to the motion, and more plates form; on the order of 10. During the transition region, the single subduction zone breaks into two or three smaller downwellings, each sheet-like and causing plate-like behavior in the model. Models at this Rayleigh number do not produce the additional large downwelling, so the pattern is more of a transition between degree-1 and degree-2 convection.

The degree of convection is also analyzed using plots of the temperature power spectrum (see Figure 5.10), which is plotted as a function of depth, showing the intensity of the spherical harmonic degrees. All images shown are taken from the last timestep recorded by the model run. Temperature power spectra are computed using a spherical harmonic expansion of the temperature field, that then determines how strong each degree is shown in the measured
temperature field. The low Rayleigh number models show very high intensity at the lowest spherical harmonic degrees and very low intensities elsewhere. Degrees of 5 or lower show log(intensity) values of between -0.5 and 0, with the intensity dropping sharply to -1.5 to -2 at degree 10 and less than -2.5 by degree 15. Higher Rayleigh number models show the intensity distributed more evenly through the low degrees, and a more gradual drop off through the higher degrees. The highest intensities, 0 to -1, show up at degree 8 and below, with the only intensity bands approaching 0 showing up in the degree 2 to 4 range. In higher degrees, there are intensities of -1.5 out to about degree 15, and -2 at degree 20, before dropping to -1.5 at the highest degrees. The higher Rayleigh number models agree with the tomographic power spectrum shown in Figure 5.11 [Becker and Boschi, 2002].

In addition to being an interesting aspect of the dynamics of this system, it also makes the model more Earthlike. The Earth has an approximately degree 2 convective pattern, and degree 1 is certainly too long wavelength. Though some models form an ring of subduction circling the globe along a rough great circle, which could be appropriate for Earth, most results show a ring-shaped cluster of subduction on one side of the Earth, reminiscent of the circum-Pacific region today, which contains most of the world’s subduction. In the models with this set up, the opposite side of the sphere does not break into as many plates as one Earth, but additional complications such as continents could be the reason for this discrepancy.

Not only is the wavelength of motion more Earthlike, the surface viscosity and velocity show more of the detailed features of plate tectonics. Most of the downwellings show back-arc spreading, sharp spreading ridges form, often arc-shaped, and some strike-slip style motion. This last point is especially important, as Cartesian models do not show any strike-slip fault formation in the absence of lateral viscosity variations [Tackley, 1999; Bercovici, 2000]. These
Figure 5.10: Temperature Power Spectra for Three Yield Stress Values With Increasing Ra

Left: Power spectra for model with yield stress $= 125$ MPa and Ra as indicated.

Left: Power spectra for model with yield stress $= 250$ MPa and Ra as indicated.

Left: Power spectra for model with yield stress $= 275$ MPa and Ra as indicated.
Figure 5.11: Tomographic Power Spectrum and Associated Tomographic Model (after Becker and Boschi, 2002) Showing Degree-2 Convection Both in the Power Spectrum and in the Tomographic Model
models do not produce sharp strike-slip faults, more diffuse zones of distributed shear
deformation across a lateral velocity gradient. The best example is shown in Figure 5.12. Here
two plates can be seen moving past each other in opposite directions, with a broad (~1000 km)
zone of deformation connecting them. Strike-slip faults as sharp as the subduction zones and
spreading ridges are never observed. The last important difference is in the surface velocity,
which has a higher overall than in the degree 1 case. The RMS numbers show average motion of
~2 mm/yr, or double the average RMS velocity of low Rayleigh number models. Toroidal
motion is approximately unchanged, with typical toroidal to poloidal ratios of 0.4 observed.

5.3 Conclusions

Using a visco-plastic rheology in a mantle convection model in spherical geometry shifts
the range of yield stress that produces plate like behavior higher than for models with the same
rheology using Cartesian geometry. For Cartesian models the yield stress range that produces
plate-like behavior is from approximately 25 MPa to 150 MPa, and in spherical geometry the
range is 60 MPa to 200 MPa. The reason for this must be related to the change in geometry, as
this is the only aspect altered from the Cartesian models. The change in geometry changes the
flow patterns, and allows stress to focus at fundamental, preferred patterns for the system,
namely the degree 1 pattern. Episodic models have degree 1 patterns for all Rayleigh numbers
(see Figures 5.8 and 5.4) because they need stress to focus in one area to break the stagnant lid.
This shows how the stress can focus more easily for spherical models, and allows the higher
yield stress to be used.

This is encouraging because it makes the model more realistic; the yield stress is still
lower than the ~700-1000 MPa range measured in laboratory experiments, but any physical
aspect that increases the yield stress is an improvement. This problem can then be interpreted in
two ways: that adding Earth like complexities to the models, such as spherical geometry, continents, etc. will eventually bring the usable yield stress range up to match experimental results, or that an entirely different physical process is causing shear localization. My results do not directly support or refute either hypothesis. They do indirectly support the former interpretation, because the addition of spherical geometry has allowed for higher yield stresses to be used. Other studies have shown that adding continents in Cartesian geometry increases the usable yield stress range, so a model with continents in spherical geometry could boost plate-like yield stress ranges even closer to experimental values. On the other hand, just adding complexity to boost the yield stress is somewhat artificial, and doesn’t explain why yield stress must be so low for the more simple models. This research makes a contribution here, but the yield stress problem is still very much unsolved.

The surface viscosity shows some of the more detailed features of plate tectonics that Cartesian models lack, back-arc spreading and some strike-slip style motion (see Figure 5.12). Models on the low end of plate-like yield stress range show significant weakening, approximately one order of magnitude lower than the average surface, in the back-arc region of subduction zones. This feature was described in the results section, and the mechanism is weakening caused by small scale convection. The end result is a rapid increase in velocity across the weak zone, causing a net spreading across the back-arc region. Back-arc spreading ridges tend to form on both sides of active subduction zones, because subduction is double sided in the models instead of single sided as on Earth.

Strike-slip faults have always been a huge problem for convection models, because all of the motion in isoviscous convection is divergent and convergent. Rotational motion only forms for lateral viscosity variations, which a visco-plastic rheology creates, but this rotational motion
Figure 5.12: Surface Viscosity and Velocity, in map view, for Plate-like Models at low Ra and high Ra

Left side (from top to bottom): yield stress = 100 MPa, yield stress = 125 MPa, yield stress = 200 MPa
Right side (from top to bottom): yield stress = 125 MPa, yield stress = 250 MPa, yield stress = 275 MPa

My models show no real strike-slip faulting, but do show strike-slip motion across a large, diffuse weak zone. They also form for unique geometries, such as when the end of a spreading
ridge and beginning of a subduction zone line up. This then causes a segment of strike-slip motion linking the two. Also, oblique spreading ridges sometimes form at very sharp angles, making faults that are mostly transform but partly divergent.

Some have argued that strike slip faults on earth are just that, special cases where geometry forces the system into strike-slip motion [Tackley, 2000a]. However, toroidal motion over the past 120 Ma makes up ~55% of all surface motion on Earth, so strike-slip faulting must be important [Lithgow-Bertelloni et al, 1993]. The increased toroidal to poloidal ratios I show are an improvement on Cartesian models, which show average ratios of ~0.3 for most plate-like models. However, the planform of convection is still problematic because the diffuse nature of the strike-slip “faults” is not Earthlike, and shows that an additional mechanism is necessary to channel the toroidal motion into sharp weak zones. In these models the weak zones are not only diffuse, but they tend to fizzle out and the two plates join together into one. The system seems to favor this configuration over strike-slip fault formation, so perhaps a damage rheology would fix this problem. If the strike-slip shear caused damage, shear would localize and perhaps cause the strike-slip fault to persist as well. Furthermore, if previously existing weak zones exist the shear would tend to localize on this zone and form a sharper, more persistent strike-slip fault.

The other major problem with convection models is that subduction is double sided instead of single sided as it is on Earth. My models also do not show single sided subduction, though they occasionally show some asymmetry, with one of the plates moving at a faster velocity than the other. This aspect of plate tectonics is not well understood in terms of fitting into mantle convection. It’s probable that continents play a major role here, since they’re buoyancy will cause an inherent asymmetry. Models with continents show one sided, asymmetric subduction imposed by the continents [Tackley 1999].
The most important development with spherical geometry is a Rayleigh number dependence on wavelength pattern of convection (ie the size of convection cells that form). For low Rayleigh numbers, $10^6$, the largest possible wavelength, two hemispherical convection cells forms. This degree-1 pattern is often found in spherical convection models, and is possibly the style of convection on other planets such as Venus and Mars. However, Earth appears to have a shorter wavelength, higher spherical harmonic degree pattern. Some propose that the Earth favors degree-1 convection, but that continents force the pattern into a degree-2 style convection [Zhong et al, 2007]. The results presented here provide another explanation for the observed planform of convection on Earth; that it is Rayleigh number controlled. Degree-2 models form around the order of 10 plates, most can be clearly outlined by sharp low viscosity zones. In addition, the higher Rayleigh number models also showed the detailed features described above, such as back-arc spreading, without the problems of the weaker surface found in low Rayleigh number models. These models show that not only does the appropriate yield stress range but the appropriate Rayleigh number to produce Earth like wavelengths. Furthermore, it is an interesting empirical development of the dynamics of the visco-plastic convecting system.

Overall, I have shown that using spherical geometry in a visco-plastic mantle convection model forms a top thermal boundary layer that breaks into strong plates moving at a uniform velocity for certain yield stress values. Spherical geometry shows many improvements over Cartesian geometry: the yield stress range that produces plate-like behavior is higher (60-250 MPa), though still lower than experimental values (700-1000 MPa); toroidal motion is increased and strike-slip motion is observed, but sharp faults do not form; and back-arc spreading occurs in most models. Most importantly, the wavelength is found to be controlled by the Rayleigh number (a previously undiscovered dependence). This implies that a plate tectonics model needs
to find the appropriate Rayleigh number range as well as yield stress range to produce plate-like surface motions on an Earthlike wavelength.

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