The response of two half spaces to point dislocations at the material interface

Yehuda Ben-Zion
Department of Geological Sciences, University of Southern California, Los Angeles, CA 90089-0740, USA

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SUMMARY
The exact solution to the problem of a general point dislocation situated at an interface between two half spaces of different elastic moduli is derived using a vector function expansion and the Cagniard-de Hoop technique. For source functions that are nonsingular in either time or space the solution is given in terms of finite range integrals suitable for numerical integration. The integration ranges can be divided into two intervals: the first corresponds to a time-space response carried by head wave disturbances, the second to a time-space response carried by direct geometrical waves. Additional interface waves (Stoneley waves, leaky modes) enter the solution when the integration contours pass near their corresponding poles in the complex ray parameter domain. The head waves and other interface waves strongly distort the near field waveforms from corresponding waveforms in a homogeneous full space. It is shown that the distribution of travel times, amplitudes and motion polarities due to slip between two dissimilar media is different from what is predicted for slip in a homogeneous full space. It is therefore suggested that the interpretation of observed seismic data, especially those recorded in the near field of faults, be undertaken with consideration given to effects arising from material heterogeneity in the source region. The solution presented in this paper may be used as an integration kernel to give the response of two dissimilar half spaces to a finite propagating rupture at the material interface. Using the early parts of borehole seismograms to avoid free surface phases, such a response may be utilized to obtain source and fault parameters.

key words: heterogeneous source region, half spaces, head waves

INTRODUCTION
The sites of most earthquakes are along geological faults which are surfaces of material discontinuity in the earth. Yet earthquake source models traditionally assume that the dislocation plane lies in a homogeneous medium. To investigate the effects of faulting at a material discontinuity Ben-Zion (1989) calculated the response of two quarter spaces to SH line dislocation at the material interface and found that the resulting displacement waves differ from corresponding waves in a homogeneous half space in the distribution of the three most commonly measured seismic parameters, travel times, amplitudes and motion polarities. Similar differences were indicated by Cormier & Beroza (1987) who used ray theory to calculate synthetic seismograms due to sources in a low velocity zone embedded in a laterally homogeneous half space.

The response of a medium to a dislocation source can be derived by directly imposing the specified dislocation as a boundary condition requirement or by using body-force equivalents (e.g. Aki & Richards 1980, chapter 3) to simulate the effects of the dislocation. The first approach is easier to use since it only entails solving homogeneous wave equations while the second approach obtains the same response by summing derivatives of solutions (Green functions) to inhomogeneous wave equations, thus employing higher order tensors and additional algebra. Furthermore, the resulting scaling factor in the Green function approach is the seismic moment density tensor which is ambiguously defined when the source region is heterogeneous (Woodhouse 1981, Heaton & Heaton 1989, Ben-Zion 1989). The ambiguity stems from the appearance of material constants in the definition of the seismic moment density tensor. When a seismic source is moved an infinitesimal distance across a material discontinuity interface the elastic parameters and the seismic moment density tensor jump in magnitude while the response wavefield remains unchanged. Thus, significantly different seismic moments can produce the same displacement field. Heaton & Heaton (1989) concluded that the seismic moment is not a good measure of the size of an earthquake and suggested instead the use of the unambiguous parameter potency, defined by Ben-Menahem & Singh (1981) as the integral of the slip over the rupture area. To synthesize 3-D wavefields radiated by finite slip along an interface which separates different media Ben-Zion (1989) suggested a representation (eq. 3.1) which requires the response of the assumed structure to a general point dislocation at the interface between the different media. Such representation utilizes the easier first approach and the resulting size scaling parameter is the potency in accord with Ben-Menahem & Singh (1981) and Heaton & Heaton (1989).
In this paper I derive the kernel of the representation (3.1) of Ben-Zion (1989) for a medium consisting of two different half spaces joined along a common boundary. For realistic source functions the exact solution is given in terms of finite range integrals amenable for numerical evaluation. Using asymptotic expansion it is easy to obtain instead a high frequency approximate solution in algebraic form. Numerical calculations for a simple shear dislocation demonstrate that, as in the 2-D SH case, commonly observed contrasts in material properties across faults can significantly modify the response wavefield. Near the material interface the waveforms undergo radical changes from what is predicted for slip in a homogeneous medium. This is mainly due to the radiation (into regions on both sides of the fault) of head wave disturbances which propagate along the material interface with the velocities and motion polarities of the faster and slower P waves and the faster S wave. The importance of these head waves for seismic oil exploration was pointed out by Fertig (1982), Bortfeld & Fertig (1982) and Daley & Hron (1983). Gutowski et al. (1982) identified such head waves in seismic records of a field shooting experiment. McNally & McEvilly (1977), Ben-Zion et al. (1989) and others observed similar head waves in the near field of San Andreas fault earthquakes.

The results derived in this paper affirm what was found for the 2-D SH case. When the media on the opposite sides of a fault are assumed dissimilar, the distribution of travel times, amplitudes and motion polarities differ from what is calculated for a corresponding homogeneous medium. Thus, earthquake source parameters which are obtained using models that assume homogeneity across the fault may be erroneous. It is suggested that observed seismic data be interpreted in a framework which allows for material discontinuity to exist in the source region. The solution presented here may be superposed, using the representation (3.1) of Ben-Zion (1989), to calculate wavefields that are radiated by finite rupture surfaces propagating along interfaces which separate different half spaces. Fitting the synthetic waveforms to the early parts of observed borehole seismograms such fields may be used to obtain kinematical source parameters and impedance contrasts across faults that are approximately planar.

FORMULATION AND TRANSFORM DOMAIN SOLUTION

Consider the model shown in Figure 1 consisting of two elastic half spaces joined along the interface $z=0$. Both Cartesian $(x,y,z)$ and cylindrical $(r,\phi,z)$ coordinate systems are used. The top half space ($z<0$) is called medium 1 and the bottom one ($z>0$) medium 2. The rigidity and compressional and shear velocities of medium $j$, $j=1,2$, are denoted, respectively, by $\mu_j$, $\alpha_j$ and $\beta_j$. In formulating the problem I follow section 7.4.2 of Aki & Richards (1980). The elastic displacement field is expressed using scalar displacement potentials $\phi, \psi, \chi$ as

$$\ddot{u}_j(x,t) = \nabla \phi_j(x,t) + \nabla \times \nabla \times \{0,0,\psi_j(x,t)\} + \nabla \times \{0,0,\chi_j(x,t)\}$$

(1)

The potentials $\phi, \psi$ and $\chi$ represent, respectively, P, SV and SH waves satisfying the homogeneous scalar wave equations

$$\frac{\partial^2}{\partial t^2} - \alpha_j^2 \nabla^2 \phi_j(x,t) = 0, \quad \frac{\partial^2}{\partial t^2} - \beta_j^2 \nabla^2 \psi_j(x,t) = 0, \quad \frac{\partial^2}{\partial t^2} - \beta_j^2 \nabla^2 \chi_j(x,t) = 0$$

(2.1)

Transforming (2.1) to frequency domain via the Fourier operator $e^{i\omega t}dt$ we get

$$\nabla^2 + \omega^2 / \alpha_j^2 \phi_j(\vec{x},\omega) = 0, \quad \nabla^2 + \omega^2 / \beta_j^2 \psi_j(\vec{x},\omega) = 0, \quad \nabla^2 + \omega^2 / \beta_j^2 \chi_j(\vec{x},\omega) = 0$$

(2.2)

Using transform techniques (or separation of variables) to solve the Helmholtz equations (2.2), the solutions for the

\[ \begin{array}{c}
\mu_1\alpha_1^2 \beta_1 \\
\mu_2\alpha_2^2 \beta_2
\end{array} \]

\[ \begin{array}{c}
\phi \\
\psi
\end{array} \]

Figure 1. Cartesian $(x,y,z)$ and cylindrical $(r,\phi,z)$ coordinate systems. The interface $z=0$ separates two half spaces of different elastic moduli.
displacement potentials in frequency-space domain are

$$\phi_j(x, \omega) = \sum_{m=-\infty}^{\infty} \int_{k=0}^{\infty} A_j(k, m) Y_k^m(r, \theta) P_j(z) \, dk$$

$$\psi_j(x, \omega) = \sum_{m=-\infty}^{\infty} \int_{k=0}^{\infty} B_j(k, m) Y_k^m(r, \theta) S_j(z) \, dk$$

$$\chi_j(x, \omega) = \sum_{m=-\infty}^{\infty} \int_{k=0}^{\infty} C_j(k, m) Y_k^m(r, \theta) S_j(z) \, dk$$

where $Y_k^m(r, \theta) = J_m(kr)e^{im\theta}$ with $J_m(kr)$ being the $m$th order Bessel function of the first kind and $P_j = e^{-n(v_j)}$, $S_j = e^{-n(\beta_j)}$ with $n(v_j) = \sqrt{k^2 - \omega^2/v_j^2}$, $v_j = \alpha_j, \beta_j$. To satisfy the radiation condition we use the branches $\text{Re} \, n(v_j) \geq 0$ with $\text{Re}$ denoting the real part of a complex function.

The stresses across planes normal to the z-axis are

$$\tau_{zr} = \mu \left( \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} \right)$$

$$\tau_{z\theta} = \mu \left[ (1/r) \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right]$$

$$\tau_{zz} = \mu \left[ (\alpha^2/\beta^2 - 2) \nabla \cdot \mathbf{u} + 2\alpha \frac{\partial u_r}{\partial z} \right]$$

Operating (1) on the solution (3) and substituting the result into (4) the displacement vector $\mathbf{u}$ and traction vector $\mathbf{T}$ across planes parallel to the interface $z=0$ are found to be

$$\mathbf{u}_j(x, \omega) = \sum_{m=-\infty}^{\infty} \int_{k=0}^{\infty} \left\{ R_k^m \left( B_j S_j k^2 A_j P_j z \right) + S_k^m \left( B_j S_j k^2 A_j P_j z \right) + T_k^m C_j S_j z \right\} \, dk$$

$$\mathbf{T}_j(x, \omega) = \sum_{m=-\infty}^{\infty} \int_{k=0}^{\infty} \left\{ R_k^m \left[ \left( B_j S_j k^2 A_j P_j \right) 2k^2 + A_j \left( \alpha_j^2/\beta_j^2 \right) \left( P_j k^2 - P_j z^2 \right) \right] + S_k^m \left[ B_j S_j k^2 + 2A_j P_j z \right] \right\} \, dk$$

where subscript $z$ after a comma denotes a derivative with respect to the $z$ coordinate.

The set (5) makes use of three vector functions given by

$$R_k^m(r, \phi) = \frac{1}{k} \nabla \times (0, 0, Y_k^m) = \frac{1}{kr} \frac{\partial Y_k^m}{\partial \phi} \hat{\phi} - \frac{1}{k} \frac{\partial Y_k^m}{\partial r} \hat{r}$$

$$S_k^m(r, \phi) = \frac{1}{k} \nabla Y_k^m = \frac{1}{k} \frac{\partial Y_k^m}{\partial r} \hat{r} + \frac{1}{kr} \frac{\partial Y_k^m}{\partial \phi} \hat{\phi}$$

$$T_k^m(r, \phi) = -\mathbf{Y}_k^m \hat{z}$$

with $\hat{r}$, $\hat{\phi}$ and $\hat{z}$ being the unit vectors in the $r$, $\phi$ and $z$ directions, respectively.
These vector functions are orthogonal to each other, and each satisfies the orthogonality relation

$$\int_0^{2\pi} \int_0^\infty \mathbf{V}_k^m(r,\varphi) \cdot \mathbf{V}_{k'}^m(r,\varphi) \, rdrd\varphi = \frac{2\pi \delta_{m,m'} \delta(k-k')}{\sqrt{kk'}} \quad ; \quad \mathbf{V} = \mathbf{R}, \mathbf{S}, \mathbf{T}$$

where $\delta_{m,m'}$ and $\delta(k-k')$ are, respectively, the Kronecker and Dirac delta functions and $^*$ denotes complex conjugate. (7)

We now specify a source term of a general point dislocation at the material interface, $z=0$, as

$$\mathbf{u}_2(x,y,0^+,t) - \mathbf{u}_1(x,y,0^+,t) = \Delta \mathbf{u} h(t) \delta(x) \delta(y)$$

where $\Delta \mathbf{u}$ is the vector amplitude of the point dislocation and $h(\cdot)$ is the Heaviside unit step function. To obtain the response to a point dislocation at general $(x_0,y_0)$ coordinates along the interface $z=0$, one only needs to substitute $x$ with $(x-x_0)$ and $y$ with $(y-y_0)$ in the final solution. The response to a point dislocation with an arbitrary source time function can be obtained from the response to the source (8.1) by convolution.

Transforming (8.1) to frequency domain we get

$$\hat{\mathbf{u}}_2(x,y,0^+,\omega) - \hat{\mathbf{u}}_1(x,y,0^+,\omega) = \mathbf{\Delta} \hat{\mathbf{u}} \delta(x) \delta(y)$$

Expanding (8.2) in the space spanned by the vector functions $\mathbf{R, S, T}$ we write

$$\frac{\mathbf{\Delta} \hat{\mathbf{u}} \delta(x) \delta(y)}{-\text{i}\omega} = \frac{1}{2\pi} \sum_{m=0}^{\infty} \int_{k=0}^{\infty} \left[ f_R \mathbf{R}_k^m + f_S \mathbf{S}_k^m + f_T \mathbf{T}_k^m \right] dk$$

and using the orthogonality relations (7) (see Aki and Richards 1980, p 308-309) the coefficients $f_R$, $f_S$, $f_T$ are found to be

$$f_T(k,m\neq1) = 0$$
$$f_T(k,m=1) = \frac{(\Delta u_y + i\Delta u_x)}{(2i\omega)}$$
$$f_T(k,m=-1) = \frac{-(\Delta u_y + i\Delta u_x)}{(2i\omega)}$$

$$f_S(k,m\neq1) = 0$$
$$f_S(k,m=1) = \frac{-(\Delta u_x + i\Delta u_y)}{(2i\omega)}$$
$$f_S(k,m=-1) = \frac{(\Delta u_x + i\Delta u_y)}{(2i\omega)}$$

$$f_R(k,m\neq0) = 0$$
$$f_R(k,m=0) = \Delta u_z / (i\omega)$$

To the displacement boundary conditions (8.1)-(8.4), we add a requirement for stress continuity. The boundary conditions at the interface $z=0$, for each $(k,m)$ member, are then

$$[ \mathbf{u}_2(\hat{x},\omega) - \mathbf{u}_1(\hat{x},\omega) ]_{z=0} = \frac{1}{2\pi} \left[ f_R(k,m) \mathbf{R}_k^m + f_S(k,m) \mathbf{S}_k^m + f_T(k,m) \mathbf{T}_k^m \right] k$$
$$[ \mathbf{T}_2(\hat{x},\omega) - \mathbf{T}_1(\hat{x},\omega) ]_{z=0} = 0$$

Now from (5) it is seen that $A_j$ and $B_j$ appear only in the coefficients of the vector functions $\mathbf{R}$ and $\mathbf{S}$ while $C_j$ alone is attached to the vector $\mathbf{T}$. The problem is thus decoupled into P-SV waves, represented by the $\mathbf{R}$ and $\mathbf{S}$ terms, and SH waves,
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given by the term $T$. Applying (9) to the $T$ term of (5), the SH boundary conditions are

$$C_2 - C_1 = \frac{1}{2\pi} f_T$$

$$-C_2 \mu_2 n(\beta_2) = C_1 \mu_1 n(\beta_1)$$

from which we get

$$C_j(k,m) = \frac{1}{2\pi} c_j(k)f_T(m), \ j=1,2$$

with $c_1(k) = \begin{bmatrix} I_2 \\ I_1 + I_2 \end{bmatrix}$, $c_2(k) = \begin{bmatrix} I_1 \\ I_1 + I_2 \end{bmatrix}$, $I_j = \mu_j n(\beta_j)$ and $f_T(m)$ given in (8.4).

Similarly, applying (9) to the $R$ and $S$ terms of (5), the P-SV boundary conditions give

$$A_1 n(\alpha_1) - B_1 k^2 + A_2 n(\alpha_2) + B_2 k^2 = \frac{k}{2\pi} f_R$$

$$A_1 + B_1 n(\beta_1) - A_2 - B_2 n(\beta_2) = -\frac{1}{2\pi} f_S$$

$$[A_1(\omega^2/\beta_1^2) - 2k^2 + 2B_1 n(\beta_1)k^2]\mu_1 + [A_2(2k^2 - \omega^2/\beta_2^2) + 2B_2 n(\beta_2)k^2]\mu_2 = 0$$

$$[2A_1 n(\alpha_1) - B_1 \omega^2/\beta_1^2]\mu_1 + [2A_2 n(\alpha_2) + B_2 \omega^2/\beta_2^2]\mu_2 = 0$$

and after some algebra we find

$$A_1(k,m) = \frac{1}{2\pi} [a_{1R}(k)f_R(m) + a_{1S}(k)f_S(m)]$$

$$B_1(k,m) = \frac{1}{2\pi} [b_{1R}(k)f_R(m) + b_{1S}(k)f_S(m)]$$

$$A_2(k,m) = \frac{1}{2\pi} [a_{2R}(k)f_R(m) + a_{2S}(k)f_S(m)]$$

$$B_2(k,m) = \frac{1}{2\pi} [b_{2R}(k)f_R(m) + b_{2S}(k)f_S(m)]$$

where

$$a_{1R}(k) = \left\{ k^4 CH + k^2 GF - k^2 q_1 [BG + k^2 Hg + n(\alpha_2) n(\beta_2) (q_1 - 2q_2) + k^2 n(\alpha_1) n(\beta_1)] + k^2 [\mu_1 p^2 \beta_1^2 (BC-gF) + n(\alpha_1) n(\beta_1) (2\mu_1 q_1 - n(\alpha_2) q_2)] - (BE-AF) 2n(\alpha_2) - k^2 (EG-CA) \mu_2 p^2 \beta_2^2 \right\} k/[n(\alpha_1) D]$$

$$a_{1S}(k) = \left\{ k^2 CH + GF - n(\alpha_2) [EH-Fq_1] + k^2 q_1 C + EG \right\} k^2 / D$$

$$b_{1R}(k) = \left\{ k^2 CH + GF - q_1 [BG + k^2 Hg + 2\mu_1 (BC-Fg)] \right\} k / D$$

$$b_{1S}(k) = \left\{ k^2 CH + GF \right\} n(\alpha_1) / D$$

$$a_{2R}(k) = \left\{ k^2 Hn(\alpha_1) n(\beta_1) [2\mu_1 - q_1] + 2\mu_1 (BE-AF) - k^2 q_1 n(\alpha_1) n(\beta_2) [2\mu_2 - q_1] \right\} k / D$$

$$a_{2S}(k) = \left\{ EH-Fq_1 \right\} k^2 n(\alpha_1) / D$$

$$b_{2R}(k) = \left\{ 2\mu_1 [EG-CA] + n(\alpha_1) n(\beta_1) [q_1 - 2\mu_1] + k^2 q_1 n(\alpha_1) [q_1 - q_2] \right\} k / D$$

$$b_{2S}(k) = \left\{ k^2 q_1 C + EG \right\} n(\alpha_1) / D$$

with

$$p = k/\omega, \ n(v) = \sqrt{k^2 - \omega^2/\gamma^2}, \ v_j = \alpha_j, \beta_j, \ g = n(\alpha_1) + n(\alpha_2), \ I_j = \mu_j n(\beta_j), \ q_j = \mu_j (p^2 \beta_j^2 - 2), \ A = n(\alpha_1) n(\beta_1) + k^2$$

$$B = n(\alpha_1) n(\beta_2) - k^2, \ C = n(\alpha_1) q_2 + n(\alpha_2) q_1, \ E = 2I_1 n(\alpha_1) + k^2 q_1, \ F = 2I_2 n(\alpha_1) - k^2 q_1, \ G = 2n(\alpha_2)(\mu_1 - \mu_2)$$

$$H = 2\mu_1 - \mu_2 p^2 \beta_2^2, \ D = k^2 H(EG-AC) + G(BE-AF) + k^2 q_1 (BC-Fg), \ j=1,2$$

and $f_R(m), f_S(m)$ given in (8.4).
Now the source terms $f_R, f_S, f_T$ are nonzero only for $m=0, \pm 1$. Writing the sum over $m$ explicitly and splitting the displacement vector $\vec{u}(\vec{x}, \omega)$ of (5) into the $R$, $S$ and $T$ terms, and the $R$, $S$ terms into $P$ and $SV$ components, we have

$$\vec{u}_j(\vec{x}, \omega) = \frac{1}{2\pi} \int_{k=0}^{\infty} \left[ c_j S_j \left( f_T(-1) T_k^{-1} + f_T(+1) T_k^{+1} \right) \right] dk$$

(12.1)

$$\vec{u}_j^S(\vec{x}, \omega) = \frac{1}{2\pi} \int_{k=0}^{\infty} \left[ S_{jS} \left( b_R f_R(0) S_k^0 + b_S f_S(-1) S_k^{-1} + f_S(+1) S_k^{+1} \right) \right] dk$$

(12.2.1)

$$\vec{u}_j^P(\vec{x}, \omega) = \frac{1}{2\pi} \int_{k=0}^{\infty} \left[ P_{jP} \left( a_R f_R(0) P_k^0 + a_S f_S(-1) P_k^{-1} + f_S(+1) P_k^{+1} \right) \right] dk$$

(12.2.2)

$$\vec{u}_j^S,SV(\vec{x}, \omega) = \frac{1}{2\pi} \int_{k=0}^{\infty} \left[ S_{jS} \left( b_R f_R(0) S_k^0 + b_S f_S(-1) S_k^{-1} + f_S(+1) S_k^{+1} \right) \right] kdk$$

(12.2.3.1)

$$\vec{u}_j^P,SV(\vec{x}, \omega) = \frac{1}{2\pi} \int_{k=0}^{\infty} \left[ P_{jP} \left( a_R f_R(0) P_k^0 + a_S f_S(-1) P_k^{-1} + f_S(+1) P_k^{+1} \right) \right] kdk$$

(12.2.3.2)

Next we use the Cagniard-de Hoop method (Cagniard, 1962; de Hoop, 1960; Aki and Richards 1980, chapter 6) to invert the set (12) into time and space.

**INVERSION OF THE $T$ TERM - SH WAVES**

The first step in the Cagniard-de Hoop method is to write an integral solution in Laplace-space domain, the integration corresponding to an inverse Fourier (or Hankel) transform. The integration path is then deformed in a complex ray parameter domain until the solution resembles a forward Laplace transform, from which the time-space solution can be identified. The complex domain manipulation is, in effect, a cancellation process between the inverse Laplace and Fourier (or Hankel) transforms. In 2-D wave propagation problems the cancellation is exact and solutions can be obtained in algebraic form. In 3-D problems (as the case at hand) the cancellation process yields finite range integrals which are suitable for numerical integration.

We recall from (6) and (3) the definition of $T_k^m$

$$T_k^m(r, \phi) = \frac{\imath m}{k r} J_m(k r) \ e^{\imath m \phi} - J'_m(k r) \ e^{\imath m \phi} \ (\hat{r}, \hat{\phi})$$

where prime denotes a derivative with respect to the argument.

(Appendix A lists properties of Bessel functions which are invoked in this paper. Using (13) and properties (A.1),(A.2) we sum the bracket of (12.1) and get

$$\vec{u}_j^T(\vec{x}, \omega) = \frac{1}{2\pi} \int_{k=0}^{\infty} \left[ \frac{c_j S_j}{\omega} \left( \Delta u_r J'_0(\kappa r) + \Delta u_\phi \frac{\partial}{\partial r} J'_0(\kappa r) \right) \right] dk$$

(14)

where $\Delta u_r = \Delta u_x r \cos \phi + \Delta u_y \sin \phi$ and $\Delta u_\phi = \Delta u_y \cos \phi - \Delta u_x \sin \phi$.

From relation (A.3) and the evenness in $k$ of $c_j$ and $S_j$ we rewrite (14) as

$$\vec{u}_j^T(\vec{x}, \omega) = \frac{1}{4\pi} \int_{k=-\infty}^{\infty} \left[ \frac{c_j S_j}{\omega} \left( \Delta u_r H_0^{(1)}(kr) + \Delta u_\phi \frac{\partial}{\partial r} H_0^{(1)}(kr) \right) \right] dk$$

(15)

where $H_0^{(1)}$ is the zero order Hankel function of the first kind.
We now set \( \omega = is \) and \( k = isp \), where \( s \) is a real positive (Laplace) parameter and \( p \) is the ray parameter. We then obtain from (15)

\[
\tilde{u}_j^T(x,s) = \frac{1}{4\pi} \int_{p=-\infty}^{\infty} c_j S_j \left[ \frac{\Delta u_r}{r} H_0^{(1)*}(isp) + \hat{\phi} \Delta u_\phi \frac{\partial}{\partial r} H_0^{(1)*}(isp) \right] dp
\]

where \( n(\beta_j) \) in \( c_j \) and \( S_j \) is now written as \( n(\beta_j) = s\eta(\beta_j) \), \( \eta(\beta_j) = s\beta_j^2 - p^2 \) and \( \text{Re} \eta(\beta_j) \geq 0 \).

Substituting in (16) equality (A.4) we get

\[
\tilde{u}_j^T(x,s) = \frac{-i}{2\pi} \int_{p=-\infty}^{\infty} c_j S_j \left[ \frac{\Delta u_r}{r} K_0^{*}(spr) + \hat{\phi} \Delta u_\phi \frac{\partial}{\partial r} K_0^{*}(spr) \right] dp
\]

where \( K_0 \) is the modified Bessel function of the second kind.

Noting again that \( c_j \) and \( S_j \) are even functions of \( p \), and invoking relation (A.5), it is seen that the integrand along the negative imaginary axis is the negative complex conjugate of the integrand at corresponding points along the positive imaginary axis (Schwartz's reflection principle). We thus have

\[
\tilde{u}_j^T(x,s) = \frac{1}{\pi r^2} \text{Im} \int_{p=0}^{\infty} c_j S_j \left[ \frac{\Delta u_r}{r} K_0^{*}(spr) + \hat{\phi} \Delta u_\phi \frac{\partial}{\partial s} K_0^{*}(spr) \right] dp
\]

where \( \text{Im} \) denotes the imaginary part of a complex function.

We now change the integration variable from the ray parameter \( p \) to the travel time along the ray \( \tau \), given by

\[
\tau(v_j) = \rho + \eta(v_j) \rho
\]

where \( v_j = \alpha_j \), \( \beta_j \) is used in (19) in anticipation of future need.

Inverting (19) for \( p \) we obtain the parameterization

\[
p(\tau) = \begin{cases} 
\frac{\tau \sin \theta - \cos \theta \sqrt{R^2 v_j^2 - \tau^2}}{R} & \text{if } \tau > R/v_j \\
\frac{\tau \sin \theta + \cos \theta \sqrt{R^2 v_j^2 - \tau^2}}{R} & \text{if } \tau < R/v_j
\end{cases}
\]

where \( R = \sqrt{r^2 + z^2} \) is the distance along the ray from the source to the observation point and \( \theta = \tan^{-1}(r/\rho) \) is the incident angle of the ray (Figure 2a).

When \( \tau \) varies from \( d/v_j \) to \( \infty \), \( p(\tau) \) traces in the complex \( p \) plane a path called Cagniard path. Figure 2b shows the Riemann sheet \( \text{Re} \eta(\beta_1) \geq 0 \), \( \text{Re} \eta(\beta_2) \geq 0 \) (assuming for concreteness \( \beta_1 > \beta_2 \)). Indicated in the figure are the pole at the origin, branch cuts given by \( |\text{Re} (p)| \geq 1/\beta_j \), \( j=1,2 \) and various integration paths. To avoid the pole at \( p=0 \) the integration path of (18) \( J \) is slightly deflected from the origin. Two possible Cagniard paths, \( C_1 \) and \( C_2 \), are shown. \( C_1 \) corresponds to the case \( \theta < \theta_c \beta_2 \beta_3 = \sin^{-1}(\beta_2/\beta_1) \) and \( C_2 \) to \( \theta > \theta_c \beta_2 \beta_3 \). \( \ell_\epsilon \) is a small arc with radius \( \epsilon \) around the pole at the origin. \( I_R \) is an upper connecting arc with radius \( R \). Now the integrand of (18) is analytic in the regions bounded by \( I + I_\epsilon + I_{C_1,2} + I_R \). Since it goes to zero on \( I_R \) when \( R \) tends to \( \infty \) (Jordan's lemma) we write symbolically (using Cauchy's theorem)

\[
I = \int_{I_\epsilon} + \int_{I_{C_1,2}}
\]

The first term on the right side of (21) is a Residue contribution. To evaluate it we set \( p = \epsilon e^{i\lambda} \), with \( \epsilon \) tending to 0 and \( \lambda \) varying from \( \pi/2 \) to 0, and substitute for \( K_0'(spr) \) and \( K_0'(spr) \) the leading term of their asymptotic expansions for small
(a) The source-receiver geometry. \( R = \sqrt{r^2 + z^2} \) is the distance along the ray from the source to the observation point and \( \theta = \tan^{-1}(\tau/|z|) \) is the incident angle of the ray. (b) A schematic diagram of the Riemann sheet \( \text{Re} \left( \eta(\beta_i) \right) \geq 0, \ j = 1, 2 \), for the case \( \beta_1 > \beta_2 \) and an observation point in medium 2. Heavy lines indicate various integration paths: \( I \) is the integration path of (18), \( I, \) is a small semicircular detour around the pole at the origin, \( C_{1,2} \) are two possible Cagniard paths corresponding, respectively, to \( \delta(\cdot) \in \text{ep}, \) \( \text{Re}(p) = 1/\beta_j \) argument \( (A.6, A.7) \). We then get

\[
\vec{u}_j(x, s)_{\text{Res}} = \frac{1}{\pi^2 r^2} \int_{\lambda = \pi/2}^{0} c_j s \left[ \hat{\Delta} u_r \frac{1}{s \text{e}^{i\lambda}} + \hat{\phi} u_{\phi} s \text{e}^{i\lambda} \right] \text{e}^{i\lambda} d\lambda
\]

\[
= \frac{e^{-i\lambda \beta_j}}{s} c_j(p=0) \left[ \hat{\Delta} u_r \hat{\phi} u_{\phi} \right] 2\pi r^2
\]

where \( \vec{u}_j(\cdot)_{\text{Res}} \) denotes the Residue contribution to \( \vec{u}_j \).

Appendix B gives equivalence relations between the Laplace and time domains needed to complete the inversion process. From (B.1), the time-space Residue contribution of the \( \Upsilon \) term is

\[
\vec{u}_j(x,t)_{\text{Res}} = \frac{c_j(p=0)}{2\pi r^2} \left[ \hat{\Delta} u_r \hat{\phi} u_{\phi} \right] h(t-|z|/\beta_j)
\]

The displacement field (23) is noncausal in that it propagates in the \((r, \phi)\) plane with infinite speed. Harkrider (1976) and Harkrider & Helmberger (1978) discussed these phases. They showed that when the various terms that make up the total displacement field are added, the noncausal terms are eliminated. In the following we shall not be concerned with these Residue contributions. We assume that they are artifacts of the representation and method of solution and that they cancel out when the total displacement field is synthesized from the P and S waves of the \( \mathbf{R}, \mathbf{S} \) and \( \Upsilon \) vector functions.

We now turn to the second term on the right side of (21), the Cagniard integral. Using in (18) the parameterization
Two half spaces response to point dislocations

The response to point dislocations is given by

\( \mathbf{u}_j(x, t) = \frac{1}{\pi \tau^2} \text{Im} \int_{\tau - \beta_j}^{\infty} \left\{ \frac{c_i}{p} e^{-\eta \beta_j \tau} [ \hat{\Delta} u_r + \hat{\phi} \Delta u_\phi - \frac{\partial}{\partial s} K_0(s) ] \right\} \frac{dp}{d\tau} d\tau \)

where \( \frac{dp}{d\tau} \), from (19) and (20), is

\[
\frac{dp}{d\tau} = \begin{cases}
\frac{\eta(\nu_j)}{\sqrt{R^2 \nu_j^2 - \tau^2}} & R/\nu_j > \tau \\
\frac{R/\nu_j}{\sqrt{\tau^2 - R^2 \nu_j^2}} & \tau > R/\nu_j
\end{cases}
\]

with \( \nu_j = \alpha_j, \beta_j, j=1,2 \).

Using the Laplace transform properties (B.2)-(B.5), we find the time-space response of the \( \mathbf{I} \) term to be

\[
\mathbf{u}_j(x, t) = \frac{1}{\pi \tau^2} \text{Im} \int_{t+\beta_j \tau}^{\infty} \left\{ \frac{c_i}{p} [ -\hat{\Delta} u_r + \hat{\phi} \Delta u_\phi - \frac{\partial}{\partial t} (f^2 Q) ] \right\} \frac{dp}{d\tau} d\tau
\]

where \( f = t - \tau + \rho \), \( Q = g^{-1/2} \) and \( g = (t-\tau)(t-\tau+2\tau) \).

The step function in \( Q \) results in a cutoff of the upper integration limit for the \( \mathbf{I} \) term. A similar upper limit cutoff applies to the \( \mathbf{I} \) term when the response to a realistic source time function is synthesized from (26) by convolution. The integration range can be further restricted by choosing the lower limit to be the value for which the integrand first becomes complex. Considering for concreteness \( \mathbf{I}_2 \), it is seen from (10.2), (20) and (25) that if \( \beta_2 > \beta_1 \) the integrand is complex only for \( \tau > R/\beta_2 \), when the Cagniard path is off the \( \text{Re}(p) \) axis. Here the first motion corresponds to the arrival time of a geometrical wave, traveling along the direct ray path from the source to the observation point with \( \beta_2 \) velocity. On the other hand, if \( \beta_1 > \beta_2 \) (see Figure 2) the above situation holds only for \( \theta < \theta_c \), where the integrand of (26) is still real for \( t < R/\beta_2 \). For \( \theta > \theta_c \), however, \( C_2(p) \) becomes complex when \( p > 1/\beta_1 \) since then \( \eta_1 \) is pure imaginary and given by \( \eta(\beta_1) = -i \eta(\beta_1) \), \( \eta(\beta_1) = \sqrt{p^2 - \beta_1^2} \). Here the time of the first arrival, \( t_0 \), is found by setting \( p = 1/\beta_1 \) in (19). This corresponds to the arrival of a head wave that travels from the source along the interface with the faster \( \beta_1 \) velocity and then along a ray path with the critical angle to the observation point with \( \beta_2 \) velocity. From the above considerations (26) is rewritten as

\[
\mathbf{u}_j(x, t) = \frac{1}{\pi \tau^2} \text{Im} \left\{ \frac{c_i}{p} \int_{t+\beta_j \tau}^{\infty} \left[ \frac{\hat{\Delta} u_r}{t+\beta_j \tau} + \hat{\phi} \Delta u_\phi \right] \frac{dp}{d\tau} d\tau \right\}
\]

with \( t_{\beta_2} = t/\beta_2 + \tilde{d}(\beta_{\theta}^2 - \beta_{\beta}^2)^{1/2} \) where \( \beta_{\theta}, \beta_{\beta} \) denote, respectively, the bigger and lesser of \( \beta_1, \beta_2 \).

With (27) we have an integral representation of the complete SH displacement in time and space. Using in (24) asymptotic expansion for \( K_0 \), valid for large values of its argument, we can alternatively obtain (e.g. Heaton 1978, Helmberger 1983) a high frequency approximate solution in algebraic form.

**INVERSION OF THE R AND S TERMS - P,SV WAVES**

The P and SV waves separate in the inversion since they follow different Cagniard paths. We thus have to invert here four different waves. We sketch the inversion of the SV wave of the S term, following the inversion of the I term. The other three waves are inverted in parallel ways.
Using the definition of $S_k^m$ (equations 6 and 3) and the Bessel function properties (A.1), (A.2) we get from (12.2.1)

$$\mathbf{u}_j^S \cdot \mathbf{V} (\hat{x}, \omega) = \frac{1}{2\pi} \int_{k=0}^{\infty} \frac{-\text{sgn}(z) \eta(\beta_j) \zeta_j}{i \omega} \left\{ b_{jR}(k) r \Delta u_k J_0'(kr) + b_{jS}(k) r \Delta u_k \frac{\partial}{\partial r} H_0^{(1)}(kr) + \frac{\Delta u_k \phi}{r} J_0'(kr) \right\} \, dk$$

where \(\text{sgn}(z)\) denotes the sign of \(z\) and \(b_{jR}(k), b_{jS}(k)\) are given in (11.2).

Using (A.3) and the respective oddness, evenness in \(k\) of \(b_{jR}(k), b_{jS}(k)\) we write

$$\mathbf{u}_j^S \cdot \mathbf{V} (\hat{x}, \omega) = \frac{i}{4\pi} \int_{k=\infty}^{\infty} \frac{\text{sgn}(z) s \eta(\beta_j) \zeta_j}{\omega} \left\{ b_{jR}(p) r \Delta u_k J_0^{(1)}(p) + b_{jS}(p) r \Delta u_k \frac{\partial}{\partial r} H_0^{(1)}(p) + \frac{\Delta u_k \phi}{r} H_0^{(1)}(p) \right\} \, dp$$

Setting \(\omega = is\), \(k = isp\) we get

$$\mathbf{u}_j^S \cdot \mathbf{V} (\hat{x}, s) = \frac{i}{4\pi} \int_{p=-\infty}^{\infty} \text{sgn}(z) s \eta(\beta_j) \zeta_j \left\{ b_{jR}(p) r \Delta u_k r \phi s J_0^{(1)}(p) + b_{jS}(p) r \Delta u_k r \phi s H_0^{(1)}(p) + \frac{\Delta u_k \phi}{r} r \phi s H_0^{(1)}(p) \right\} \, dp$$

where \(\bar{b}_{jR}(p), \bar{b}_{jS}(p)\), from (11.2) and the substitution \(k = isp\), are

$$\bar{b}_{jR} = (GF-CH+q_1 p^{-2}[HY-BG]+2 \mu_1 [BC-P\gamma]) / p s D, \bar{b}_{jS} = (GF-CH) \eta(\alpha_1) / s D$$

$$\bar{b}_{2R} = (2 \mu_1 [GY-CA]+G \eta(\alpha_1) \eta(\beta_j) q_1 p^{-2}[2 \mu_1]+q_1 p^{-2} \eta(\alpha_1) q_2-q_1)) / p s D, \bar{b}_{2S} = (q_1 C-EG) \eta(\alpha_1) / s D$$

The terms \(A, B, C, D, E, F, G, H, q_j\) and \(g\) are all even(p). They are written below the set (33). Using (A.4), Schwartz's reflection principle and the respective oddness, evenness in \(p\) of \(\bar{b}_{jR}(p), \bar{b}_{jS}(p)\) we find

$$\mathbf{u}_j^S \cdot \mathbf{V} (\hat{x}, s) = \frac{1}{\pi^2} \int_{p=0}^{\infty} \text{sgn}(z) s \eta(\beta_j) \zeta_j \left\{ \frac{b_{jR}(p) r \Delta u_s r \phi s}{r} \frac{\partial}{\partial s} K_0(s r) - \frac{b_{jS}(p) r \Delta u_s r \phi s}{r^2} \frac{\partial^2}{\partial s^2} K_0(s r) + \frac{\Delta u_s \phi}{r} \frac{\partial}{\partial s} K_0(s r) \right\} \, dp$$

where in (31) \(b_{jR}(p) = (s-i) \bar{b}_{jR}(p)\) and \(b_{jS}(p) = s \bar{b}_{jS}(p)\)

Figure 3a shows the first quadrant of the Riemann sheet \(\Re \eta(\alpha_1) \geq 0, \Re \eta(\alpha_2) \geq 0, \Re \eta(\beta_1) \geq 0, \Re \eta(\beta_2) \geq 0\) assuming \(\alpha_1 > \alpha_2 > \beta_1 > \beta_2\). Indicated in the figure are Cagniard path, four branch cuts given by \(|\Re(p)| \geq 1/v_j, v_j = \alpha_j, \beta_j, j = 1, 2\), and a singular point at \(1/V_S\), a possible real root of \(D(p)\), with \(V_S\) being the Stoneley wave velocity. Such real cut exists only for a limited range of values of the elastic parameters and densities of the solid pair (e. g. Cagniard 1962, Gilbert & Laster 1962). The Cagniard path of Figure 3a corresponds to an observation point with \(\theta > \theta_{cB1} = \sin^{-1}\beta_2/\beta_1\) in medium 2. In this case the displacement field has three different head wave contributions before the arrival of the geometrical wave, due to waves that travel part of the way along the interface with the faster \(\alpha_1, \alpha_2\) and \(\beta_1\) velocities. Figure 3b shows corresponding direct and head waves paths for such source-receiver configuration. When the source-receiver angle is in the zone \(\theta \geq \theta_{cB2a_2} > \theta > \theta_{cB2a_1} = \sin^{-1}\beta_2/\alpha_1\) there is only one head wave contribution before the geometrical arrival, corresponding to the disturbance that propagates along the interface with \(\alpha_1\) velocity. In the zone \(\theta < \theta_{cB2a_1}\) the first motion is that of the regular geometrical wave at \(R/\beta_2\).
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Using the Laplace transform properties (B.2)-(B.5), we find from (31) the SV wave of the $S$ term in time and space

\[
\hat{u}_j^{SV}(x,t) = \frac{1}{\pi^2} \text{Im} \left\{ \frac{\Delta u_0}{r^2} \right\} - \frac{b_{S} \eta(\beta)}{p} \int_{\pi\beta_j = \pi \alpha \beta_j}^{\pi \beta_j = \pi \alpha \beta_j} \frac{d}{dt} \left[ \frac{1}{r} \right] \frac{dQ}{dt} dt - \frac{b_{S} \eta(\beta)}{p} \int_{\pi \beta_j = \pi \alpha \beta_j}^{\pi \beta_j = \pi \alpha \beta_j} \frac{d}{dt} \left[ \frac{1}{r} \right] \frac{dQ}{dt} dt
\]

where $t_{\alpha \beta}(v_j) = \frac{v_j}{c_{\alpha \beta}} + \frac{v_j^2 - \alpha^2}{2 v_j^2}$, $v_j = \alpha_j \beta_j$, $j = 1, 2$ and $\alpha$ is the greater of $\alpha_1, \alpha_2$.

The inversions of the $(S,P)$, $(R,SV)$, and $(R,P)$ terms are done in analogous ways. The response of two half spaces to a general point dislocation with step source time function at the material interface, denoted by $\hat{u}_j(x,t;\text{step})$, is summarized below

\[
\begin{align*}
\hat{u}_j^{SV}(x,t) &= \hat{u}_j^T(x,t;\text{step}) + \hat{u}_j^{SV}(x,t;\text{step}) + \hat{u}_j^{P}(x,t;\text{step}) + \hat{u}_j^{PV}(x,t;\text{step}) + \hat{u}_j^{P}(x,t;\text{step}) \\
\hat{u}_j^{T}(x,t) &= \frac{1}{\pi^2 \alpha^2} \text{Im} \left\{ \frac{\Delta u_0}{r^2} \right\} - \frac{c_i}{p} \int_{\alpha \beta_j = \pi \alpha \beta_j}^{\pi \alpha \beta_j} \frac{d}{dt} \left[ \frac{1}{r} \right] \frac{dQ}{dt} dt - \frac{c_i}{p} \int_{\alpha \beta_j = \pi \alpha \beta_j}^{\pi \alpha \beta_j} \frac{d}{dt} \left[ \frac{1}{r} \right] \frac{dQ}{dt} dt
\end{align*}
\]
\[ u_j^{SV}(\overrightarrow{X}, t; \text{step}) = \frac{1}{\pi^2} \text{sgn}(z) \text{ Im} \left\{ \begin{array}{l}
\frac{\Delta u_{\phi}}{r} \int_0^t f_{\phi} \left( \frac{\Delta u_{\phi}}{r^2} \right) dt + \frac{b_{\phi}}{r} \int_0^t f_{\phi} \left( \frac{\Delta u_{\phi}}{r^2} \right) dt - \frac{\Delta u_{\phi}}{r} \int_0^t \frac{b_{\phi}}{r} \frac{\partial f_{\phi}}{\partial t} \left( \frac{n}{r^2} \right) dt \\
\frac{\Delta u_{\phi}}{r^2} \int_0^t f_{\phi} \left( \frac{\Delta u_{\phi}}{r^2} \right) dt + \frac{b_{\phi}}{r} \int_0^t f_{\phi} \left( \frac{\Delta u_{\phi}}{r^2} \right) dt
\end{array} \right\} \]
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with

\[ p = k/\omega, \quad \eta(\nu_j) = \sqrt{v_j^2 - p^2}, \quad \gamma = \eta(\alpha_1) + \eta(\alpha_2), \quad I_j = \mu_1 \eta(\beta_j), \quad q_j = \mu_1 (\beta_j^2 - p^2), \quad A = \eta(\alpha_1) \eta(\beta_1) - p^2 \]

\[ B = \eta(\alpha_1) \eta(\beta_2) + p^2, \quad C = [\eta(\alpha_1) q_2 + \eta(\alpha_2) q_1] p - p, \quad E = 2 I_1 \eta(\alpha_1) q_1, \quad F = 2 I_2 \eta(\alpha_1) q_1, \quad G = 2 \eta(\alpha_2)(\mu_1 - \mu_2) \]

\[ H = 2 \mu_1 p^2 - \mu_2 \beta_2^2 \quad \text{and} \quad D = H(AC-E\gamma) + G(BE-AF) + q_1(F\gamma - BC). \]

The displacement field (33) is not suitable for a direct numerical evaluation since it contains singularities and, in the terms with time derivatives, unbounded integration limits. When the response field to a source with realistic time function is synthesized from the set (33) by time convolution the derivatives in the integrals are removed. Alternatively, the time derivatives can be substituted with space derivatives which are removed when the response to a source with finite spatial dimension is considered. After the derivatives in the integrands are removed, the infinite integration ranges are reduced to finite ones by the cutoff effect of the step function in Q. This is demonstrated in the next part where we write the response of two half spaces to a general point shear dislocation with ramp source time function.

The response of two half spaces to a point shear dislocation with ramp source time function at the material interface

For a shear crack we have \( \Delta u_x = 0 \) and the \( a_{jR}, b_{jR} \) terms of (33) drop.

A general time function \( \xi(t) \) can be represented in terms of the unit step function as

\[ \xi(t) = \int_0^t \xi(\xi) h(t-\xi) \, d\xi \tag{34.1} \]

The response of a linear system to a source with the time function \( \xi(t) \), denoted by \( R(\xi(t)) \), can then be written as

\[ R(\xi(t)) = \int_0^t R(h(\xi)) R(h(t-\xi)) \, d\xi \tag{34.2} \]

with \( R(h(t-\xi)) \) being the response of the system to a step source time function.

We consider a ramp source time function given by

\[ \xi(t) = (t h(t) - h(t-t_r)) + t_r h(t-t_r) \tag{34.3} \]

where \( t_r \) is the rise time interval.

Putting the derivative of (34.3) into (34.2) and integrating we obtain the representation

\[ R(\xi(t) = \text{ramp}) = \int_{t-t_r}^t R(h(\xi)) \, d\xi \tag{34.4} \]

Using \( \vec{u}_j(x,t;\text{step}) \) of (33) with \( \Delta u_x = 0 \) for \( R(h(\xi)) \) of (34.4), and changing the order of integration, we find the response of two half spaces to a point shear dislocation with ramp source time function at the material interface as

\[ \vec{u}_j(x,t;\text{ramp}) = \vec{u}_j(x,t;\text{ramp}) + \vec{u}_j^{\text{SV}}(x,t;\text{ramp}) + \vec{u}_j^{\text{SP}}(x,t;\text{ramp}) + \vec{u}_j^{\text{R},\text{SV}}(x,t;\text{ramp}) + \vec{u}_j^{\text{R},\text{SP}}(x,t;\text{ramp}) \]

with

\[ \vec{u}_j^{\text{ramp}}(x,t;\text{ramp}) = -\frac{1}{\pi^2 t^2} \text{Im} \left\{ \frac{\partial^2 u_{z\phi}}{\partial \beta_j \partial \beta_j} \left[ \int_0^t \frac{c_i}{p} g(t)^{1/2} \frac{dp}{dt} \, dt \right] \frac{c_i}{p} g(T)^{1/2} \frac{dp}{dt} \right\} \]
\[ u_j^{SV}(x,t;ramp) = \frac{\text{sgn}(z)}{\pi^2T^2} \text{Im} \left\{ \frac{\Gamma}{\pi^2T^2} \int_0^T \frac{b_\beta \eta(\beta_j) g(t)}{p} \frac{\Phi(t)}{\tau} dt + \int_0^{\tau} \frac{b_\beta \eta(\beta_j) g(t) g(T)}{p} \frac{\Phi(t)}{\tau} dt \right\} \]

\[ u_j^{P}(x,t;ramp) = \frac{1}{\pi^2T^2} \text{Im} \left\{ \int_0^T \frac{a_\Gamma g(t)}{p} \frac{\Phi(t)}{\tau} dt + \int_0^{\tau} \frac{a_\Gamma g(t) g(T)}{p} \frac{\Phi(t)}{\tau} dt \right\} \]

\[ u_j^{SV}(x,t;ramp) = -\frac{\hat{z}}{\pi T} \text{Im} \left\{ \int_0^T \frac{b_\beta p f(t) g(t)}{p} \frac{\Phi(t)}{\tau} dt + \int_0^{\tau} \frac{b_\beta p f(t) g(T)}{p} \frac{\Phi(t)}{\tau} dt \right\} \]

\[ u_j^{P}(x,t;ramp) = \frac{\hat{z} \text{sgn}(z)}{\pi^2T} \text{Im} \left\{ \int_0^T \frac{a_\Gamma \eta(\alpha_j) f(t)}{p} \frac{\Phi(t)}{\tau} dt + \int_0^{\tau} \frac{a_\Gamma \eta(\alpha_j) f(T)}{p} \frac{\Phi(t)}{\tau} dt \right\} \]

where \( f(t) = (t-\tau + pr), g(t) = (t-\tau)(t-\tau + 2pr), t = \tau, T \) and \( T = t - \tau \). The other terms are defined in (33).

Using transformation of variables as described in Appendix C, the set (35) can be evaluated numerically.

**NUMERICAL RESULTS**

Proceeding with the variable substitutions of Appendix C, the set (35) is put in a form suitable for numerical integration. As an example we assume \( \Delta u_0 = 0, \Delta u_y = \sqrt{2} \). In the geometry of Figure 4 this corresponds to a strike slip

![Figure 4](image-url)  
Figure 4. A rotation of the two different half spaces so that the interface \( z = 0 \) represents a vertical fault plane. A point strike slip dislocation is operating at the origin. The observation points are along the line \( r = \sqrt{2}, \phi = 45^\circ \).
The observation points are taken along the line \( x=-\ell \text{km}, y=\ell \text{km} \). Thus, in this example \( \Delta u_x=\Delta u_y \sin \varphi =1 \) and \( \Delta u_\varphi =\Delta u_y \cos \varphi =1 \). Figures 5-7 illustrate some features of the wave field in this example. The solutions are calculated, using simple trapezoidal integration, for time points separated by \( 10^{-3} \text{ sec} \) and linear interpolations are used to plot the results. The rise time interval is set at \( t_r=0.1 \text{ sec} \). The parameters of the two half spaces are taken to be \( \alpha_1=5.4 \text{ km/sec}, \beta_1=3.3 \text{ km/sec}, \mu_1=0.276 \times 10^{12} \text{ dyne/cm}^2; \alpha_2=5.0 \text{ km/sec}, \beta_2=3.1 \text{ km/sec} \) and \( \mu_2=0.225 \times 10^{12} \text{ dyne/cm}^2 \). These numbers characterize the different sides of the San Andreas fault in central California as given by laboratory measurements (Carmichael 1982) and field velocity studies (McEvilly 1966; Stewart 1968; Boore & Hill 1973; Walter & Mooney 1982).

Figure 5a shows the \( r \) component of motion near field observation points along the line \( x=-\ell \text{km}, y=\ell \text{km} \), at \( \theta=\pm 90^\circ, \pm 89^\circ, \pm 88^\circ \) and \( \pm 87^\circ \). The solid lines show displacement seismograms in medium 1 (\( \theta<0 \)) and the dashed lines corresponding seismograms in medium 2 (\( \theta>0 \)). Arrows pointing to the top traces (\( \theta=\pm 90^\circ \)) indicate the arrivals of body waves traveling with the faster P (\( \gamma_{p_1} \)), slower P (\( \gamma_{p_2} \)), faster S (\( \gamma_{s_1} \)) and slower S (\( \gamma_{s_2} \)) velocities. The superscripts i,s indicate, respectively, waves that are radiated from the initial and stopping phases of the rupture. As expected, corresponding waves due to the initial and stopping phases of the rupture are separated by the 0.1 sec of the rise time interval and have reversed polarities with respect to each other. The large peaks between the P and S arrivals are interface waves (leaky modes) traveling with a velocity of about 4.6 km/sec. These phases, which can be designated as \( \text{P} \) pulses using the notation of Gilbert & Laster (1962), are associated with poles of the reflection/transmission coefficients which lie on Riemann sheets where the radiation condition is not satisfied. Successive traces of decreasing \( |\theta| \) show the evolution of the various phases in the wavefield with increasing normal distance from the fault. Figure 5b shows the corresponding displacement wavefield in a full space characterized by the parameters of medium 1. It is seen that in the limit where the two media on the different sides of the fault are taken to be the same, the complicated wavefield of Figure 5a deteriorates to an elemental field composed of simple P and S wavelets. The complexity of the wavefield when the fault is a material interface is especially impressive in view of the moderate (7\%) material contrast used to calculate the field of Figure 5a. Comparing Figures 5a and 5b it is important to note that in the case of two dissimilar media, the first motion polarity in the zone \( \theta>\theta_{\text{cross}}=\sin^{-1}(\alpha_2/\alpha_1) \) in the slower medium 2 is opposite from what is expected in the framework of a homogeneous medium. This is because in this zone the first motion is that of a head wave which is controlled by the propagation of the disturbance in the faster medium on the opposite side of the fault. For the parameters of the two half spaces chosen above, the critical angle \( \theta_{\text{cross}} \) (the maximum angle for which the early part of the P wave in the slower medium 2 is a head wave contribution) is about 68°.

Figure 5c shows the \( r \) component of motion in the two half spaces at the observation coordinates \( x=-\ell \text{km}, y=\ell \text{km} \) and \( \theta=\pm 69^\circ, \pm 68^\circ, \pm 67^\circ \) and \( \pm 66^\circ \). For \( \theta<\theta_{\text{cross}} \), the P waveforms are similar to corresponding waveforms in a full space. The S waveforms, however, continue to be significantly different from corresponding full space pulses since in this zone \( \theta_{\text{cross}} \) the early parts of the S waves, in both media 1 and 2, are still the contributions of head wave disturbances which propagate along the material interface with the P waves, \( \alpha_1 \) and \( \alpha_2 \) velocities. From Figures 5a-c it is clear that when the fault is modeled as a material interface, the resulting distribution of travel times, amplitudes and motion polarities is different from what might be predicted for slip in a homogeneous medium. Figures 6a-c, 7a-c show the same as Figures 5a-c for the \( \varphi \) and \( z \) components of motions, respectively (the small differences between the traces of media 1 and 2 at the interface \( z=0 \) (\( \theta=\pm 90^\circ \)) are due to computational errors). As with the \( r \) component of motion, the \( \varphi \) and \( z \) components clearly demonstrate the prominent effects that a material discontinuity at the fault has on the resulting wavefield. If we consider again the observation line \( x=-\ell \text{km}, y=\ell \text{km} \), and assume the dip slip dislocation \( \Delta u_x=\sqrt{2}, \Delta u_y=0 \), we have \( \Delta u_x=\Delta u_y \sin \varphi =1 \) and \( \Delta u_\varphi =\Delta u_y \cos \varphi =-1 \). In this case the \( r \) and \( z \) components of motion are the same as those of Figures 5a-c and 7a-c, respectively, while the \( \varphi \) component is reversed from what is shown in Figures 6a-c.

**DISCUSSION**

I have derived the response of two different half spaces to a general point dislocation at the material discontinuity interface. For realistic source functions the exact solution is given as finite range integrals suitable for numerical integration. Using asymptotic expansion of the modified Bessel function, expressions for an approximate high frequency solution can be readily obtained in algebraic form. Numerical examples demonstrate that a moderate contrast in the elastic properties of the media on the different sides of a fault results in a significant alteration of the response field, especially near the fault. When
Figure 5. (a) The r component of the displacement field (35) in media 1 (solid lines) and 2 (dashed lines) at observation points $x = -1\ km$, $y = 1\ km$ and $\theta = \pm 90, \pm 89, \pm 88$ and $\pm 87^\circ$. The rise time interval is $t_r = 0.1\ sec$. The media parameters are $\alpha_1 = 5.4\ km/sec$, $\beta_1 = 3.3\ km/sec$, $\mu_1 = 0.276 \times 10^{12}\ dyne/cm^2$, $\alpha_2 = 5.0\ km/sec$, $\beta_2 = 3.1\ km/sec$ and $\mu_2 = 0.225 \times 10^{12}\ dyne/cm^2$. Arrows pointing to the top traces ($\theta = \pm 90^\circ$) indicate the arrivals of the faster P ($t_{p1}$), slower P ($t_{p2}$), faster S ($t_{s1}$) and slower S ($t_{s2}$) waves, radiated from the initial, stopping (superscripts i, s) phases of the rupture. The large peaks between the P and S arrivals are interface waves (leaky modes) traveling with a velocity of about 4.6 km/sec. (b) Same as (a) in a full space characterized by the parameters of medium 1. (c) Same as (a) for $\theta = \pm 69, \pm 68, \pm 67$ and $\pm 66^\circ$ ($\theta_{max} = \sin^{-1}(\alpha_2/\alpha_1) = 68^\circ$).
Figure 6. (a), (b), (c) Same as Figures 5 (a), (b), (c), respectively, for the $\varphi$ component of motion.
Figure 7. (a), (b), (c) Same as Figures 5 (a), (b), (c), respectively, for the z component of motion.
the media across the fault are dissimilar there are interface waves and P and S head waves which are radiated from the interface into certain regions on both sides of the fault. In the faster medium the head waves influence only the S pulse while in the slower medium both the P and S pulses are affected. The zones in which the head waves are the first arrivals increase with the distance down the fault of the dislocation and the amount of the material contrast across the fault. Using, for example, the material contrast of Figures 5-7 and a 10km focal depth of a typical San Andreas fault earthquake, I compute 4 and 14 km as the respective normal distances from the fault in which the early parts of the P and S pulses in the slower medium are head waves contributions. Considering a dislocation at a distance of 100km down a subducting slab, in a zone characterized by \(\alpha_1=8.6\text{ km/sec}, \beta_1=5.0\text{ km/sec}, \alpha_2=8.0\text{ km/sec and } \beta_2=4.7\text{ km/sec}\) (Hauksson 1985), the corresponding respective normal distances from the fault are 39 and 153 km. Although these numbers seem small the wavefields near faults are of a particular importance for understanding the details of rupture processes.

As in the 2-D SH case reported earlier (Ben-Zion 1989) I find here that when the fault is modeled as a material interface, the distribution of the three important seismic parameters, travel times, amplitudes and motion polarities, differ from what is predicted for slip in a homogeneous medium. As discussed in McNally & McEvilly (1977), Cormier & Beroza (1987) and Ben-Zion (1989), observations of these effects are reported in the literature. It follows that models of earthquake source regions which do not include material discontinuities may lead to the wrong interpretation of observed seismic data. Interpreting the observed data in a framework which allows for material discontinuities across faults should improve our knowledge of earth structures and earthquake processes. An example is the determination of fault planes using the seismic waves radiated from earthquake sources. It is well known that the response of a homogeneous medium to slip does not contain information that can be used to distinguish the fault plane from its auxiliary plane. This result might be expected since in a homogeneous medium the fault plane itself does not carry any special physical property. In contrast, in models such as considered here the fault plane is a unique physical interface which separates different materials. Sato & Matsuzaki (1974) and N. Biswas (unpublished manuscript) calculated the radiation pattern due to a point shear dislocation at the boundary between two different half spaces and found that in the case of slip between dissimilar media the radiation pattern along the fault plane is different from the radiation pattern along the auxiliary plane. Similarly, the response field (35) contains a certain radiation pattern along the x-y fault plane in terms of the angle \(\varphi\) (included in \(\Delta x, \Delta y, \Delta z\)) and a different radiation pattern along the auxiliary x-z plane in terms of the angles \(\theta\) (included in \(\rho, \psi\)) and \(\varphi\).

The response field derived in this paper can be used as a kernel field to synthesize, via the representation (3.1) of Ben-Zion (1989), wavefields that are radiated from finite rupture surfaces moving along interfaces which separate dissimilar half spaces. In the absence of free surface complications (i.e. in conjunction with borehole seismograms) such synthetics can be used to study earthquake source parameters and impedance contrasts across approximately planar faults such as the San Andreas fault.

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REFERENCES


APPENDIX A. PROPERTIES OF BESSEL FUNCTIONS
(Abramowitz & Stegun 1972, chapter 9)

\[ J_m(kr) = (-1)^m J_m(kr) \]  \hspace{1cm} (A.1)

\[ J_1(kr) = -J_0'(kr) \]  \hspace{1cm} (A.2)

\[ J_0'(kr) = \frac{1}{2} \left[ H_0^{(1)}(kr) + H_0^{(1)}(-kr) \right] \]  \hspace{1cm} (A.3)

\[ \frac{1}{2} H_0^{(1)}(ix) = -\frac{i}{\pi} K_0'(x) \]  \hspace{1cm} (A.4)

\[ K_0(Z^*) = K_0(Z) \]  \hspace{1cm} (A.5)

\[ K_0'(x) = -\frac{1}{x} \quad x << 1 \]  \hspace{1cm} (A.6)

\[ K_0''(x) = \frac{1}{x^2} \quad x << 1 \]  \hspace{1cm} (A.7)
APPENDIX B. EQUIVALENCE RELATIONS BETWEEN LAPLACE AND TIME DOMAINS
(Abramowitz & Stegun 1972, chapter 29)

<table>
<thead>
<tr>
<th>Laplace Domain - $F(s)$</th>
<th>Time Domain - $f(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{-sR/v_j}/s$</td>
<td>$h(t-R/v_j)$</td>
</tr>
<tr>
<td>$K_o(spr)$</td>
<td>$h(t-pr)/\sqrt{t^2-p^2r^2}$</td>
</tr>
<tr>
<td>$(\partial/\partial s)^n F(s)$</td>
<td>$(-t)^n f(t)$</td>
</tr>
<tr>
<td>$sF(s)$</td>
<td>$(\partial/\partial t)f(t), \ f(0)=0$</td>
</tr>
<tr>
<td>$e^{-snRd} F(s)$</td>
<td>$f(t-nRj)d\ h(t-nRj)d$</td>
</tr>
</tbody>
</table>

(B.1)  
(B.2)  
(B.3)  
(B.4)  
(B.5)

APPENDIX C. REMOVAL OF THE SQUARE ROOT SINGULARITIES FROM THE INTEGRATION RANGE

The integrals in the displacement field (35) contain square root singularities which, although being integrable, present difficulties for numerical integration schemes. This problem can be avoided by substitution of variables which remove the singularities from the integration range. We consider separately the integrals with the function $g^{-1/2}$ and the integrals with the function $g^{+1/2}$.

The integrals with $g^{-1/2}$ can be written as

$$I(t) = \text{Im} \int_{h\alpha}^{t} f(\tau) \ (t-\tau)^{-1/2} \frac{dp}{dt} \ d\tau$$  
(C.1.1)

If $t < R/v_j$, (C.1.1) has one singular point at the upper integration limit. The substitution $\tau = t \sin^2(\lambda)$ transforms (C.1.1) into

$$I(t) = 2\sqrt{t} \ \text{Im} \int_{\lambda = \sin^{-1}\sqrt{h\alpha/t}}^{\pi/2} f(\tau(\lambda)) \frac{dp}{dt}(\lambda) \sin(\lambda) \ d\lambda$$  
(C.1.2)

If $t > R/v_j$, we divide the integration range into two parts and write (C.1.1) (using (25) for $dp/d\tau$) as

$$I(t) = I_1(t) + I_2(t)$$  
(C.1.3)

$$I_1(t) = \text{Im} \int_{h\alpha}^{R/v_j} F(\tau)(t-\tau)^{-1/2}(R^2v_j^2-\tau^2)^{-1/2} \ d\tau$$

$$I_2(t) = \text{Re} \int_{R/v_j}^{t} F(\tau)(t-\tau)^{-1/2}(\tau^2-R^2v_j^2)^{-1/2} \ d\tau$$

where $F(\tau) = f(\tau)\eta(v_j)$
\( I_1 \) contains a singularity at the upper integration limit. With the substitution \( \tau = R \sin(\lambda)/v_j \) \( I_1 \) becomes

\[
I_1(t) = \text{Im} \int_{\lambda = \sin^{-1} \sqrt{\frac{v_j}{\tau}}}^{\pi/2} F[\tau(\lambda)][\tau(\lambda)]^{-1/2} d\lambda
\]  
(C.1.4)

\( I_2 \) contains two singularities at both integration limits. Setting (Heaton 1978, Appendix D) \( \tau = R \sin(\lambda)/v_j + t \cos^2(\lambda) \) puts \( I_2 \) in the form

\[
I_2(t) = 2 \text{Re} \int_{\lambda = 0}^{\pi/2} F[\tau(\lambda)][\tau(\lambda) + R/v_j]^{-1/2} d\lambda
\]  
(C.1.5)

The integrals with \( g^{1/2} \) can be written as

\[
J(t) = \text{Im} \int_{\text{Re}} f(\tau) \frac{d\tau}{d\xi}
\]  
(C.2.1)

If \( t < R/v_j \) there is no singularity in (C.2.1). If \( t > R/v_j \) we divide the integration range and write (C.2.1) (using equation 25) as

\[
J(t) = J_1(t) + J_2(t)
\]  
(C.2.2)

\[
J_1(t) = \text{Im} \int_{\text{Re}} F(\tau)(R^2v_j^2 - \tau^2)^{-1/2} d\tau
\]

\[
J_2(t) = \text{Re} \int_{R/v_j}^{t} F(\tau)(\tau^2 - R^2v_j^2)^{-1/2} d\tau \quad \text{with} \quad F(\tau) = f(\tau)\eta(v_j)
\]

\( J_1 \) is similar to \( I_1 \) and the same substitution, \( \tau = R \sin(\lambda)/v_j \), gives

\[
J_1(t) = \text{Im} \int_{\lambda = \sin^{-1} \sqrt{\frac{v_j}{\tau}}}^{\pi/2} F[\tau(\lambda)] d\lambda
\]  
(C.2.3)

\( J_2 \) has one singularity at the lower integration limit. Here we set \( \tau = R \cosh(\lambda)/v_j \) and get

\[
J_2(t) = \text{Re} \int_{\lambda = 0}^{\cosh^{-1} \sqrt{v_j/R}} F[\tau(\lambda)] d\lambda
\]  
(C.2.4)

The integrals (C.1.2), (C.1.4), (C.1.5), (C.2.3) and (C.2.4) no longer contain singularities and are amenable for numerical evaluation.