HETEROGENEITY OF EARTHQUAKE STRESS DROPS, FOCAL MECHANISMS AND
ACTIVE FAULT ZONES

by

Iain W. Bailey

A Dissertation Presented to the
FACULTY OF THE GRADUATE SCHOOL
UNIVERSITY OF SOUTHERN CALIFORNIA
In Partial Fulfillment of the
Requirements for the Degree
DOCTOR OF PHILOSOPHY
(EARTH SCIENCES)

August 2009

Copyright 2009

Iain W. Bailey
Acknowledgements

I am grateful to my principal advisor Yehuda Ben-Zion and co-advisor Thorsten Becker for all that they have taught me and their assistance throughout my Ph.D. studies at USC. I also thank my other co-author Matthias Holschneider for his help and for hosting me in Potsdam in July, 2007, as well as my other committee members Aiichiro Nakano, John Platt and Charlie Sammis, for their patience and advice.

The work presented in this thesis has benefited strongly from discussions with my teachers, fellow students at USC and those I have met at numerous meetings and conferences over the last four and a half years. In addition to my committee members, these include but are not limited to Jeanne Hardebeck, Yan Kagan, Thomas Heaton, Gert Zöller, Zhegang Peng, James Dolan, Thomas Jordan, Thomas Henyey, Wenzheng Yang, Michael Lewis, Jeremy Zechar, Peter Powers, Neta Wechsler, Zheqiang Shi, Adam Fischer, Clare Steedman, Kurt Frankel, Plamen Ganev, Erik Frost, Po Chen, Lisa Alpert, Boris Kaus, Attreyee Ghosh, Katrin Plenkers, Hubert Saleur, Bart Kosko and Craig Taylor.

The focal mechanism data used in Chapter 2 and Chapter 3 were all generated by Jeanne Hardebeck and sent to me upon request. I am grateful for her patience in answering my questions and for sending the various catalogs available.

The three studies in this thesis were supported by the Southern California Earthquake Center. I have also been funded via four Teaching Assistant fellowships, a final year dissertation fellowship and a summer dissertation fellowship from the University of Southern California, as well as several departmental summer funding awards. I am grateful to Yehuda Ben-Zion, Thorsten Becker and John McRaney for their help in generating and organizing the funding for me throughout my Ph.D, and to Cindy Waite for her help with all of the bureaucracy. I thank John Yu for his fantastic computer support, as well as the other office staff in the department and SCEC for all things administrative: Barbara Grubb, Vardui Ter-Simonian, Dana Coyle, Shelly Werner, Sally Henyey and Tran Hyunh,
For the study in Chapter 2, I thank Fred Pollitz, Olaf Zielke and Tom Hanks for constructive comments that helped to improve the manuscript. For the study in Chapter 3, I am grateful to the editor, Frank Krueger, and two anonymous reviewers for their comments on the original manuscript.

For various meetings and work, I have been fortunate to travel all over the US, to Israel, to Australia and to Germany. I am very grateful for these opportunities to see new places and meet new people, which wouldn’t have been possible without the enthusiasm and support of Yehuda. I have been lucky to receive funding for travel to meetings via Department of Earth Sciences travel awards.

Finally, I wish to thank my family and friends for all of their encouragement throughout my years in southern California.
Table of Contents

Acknowledgements ii
List of Figures vi
List of Tables ix
Abstract x

Chapter 1 Introduction 1

Chapter 2 Statistics of Earthquake Stress Drops on a Heterogeneous Fault in an Elastic Half-Space 6
  2.1 Introduction 6
  2.2 Method of Investigation 9
    2.2.1 Description of Fault Model 9
    2.2.2 Fault Model Heterogeneity 11
    2.2.3 Quantities Computed 13
  2.3 Results 16
    2.3.1 Different Types of Heterogeneity Distribution 16
    2.3.2 Different Heterogeneity Levels 20
    2.3.3 Depth Dependency of $\langle \tau_s - \tau_d \rangle$ 23
  2.4 Discussion 25

Chapter 3 Patterns of Co-seismic Strain Computed from Southern California Focal Mechanisms 31
  3.1 Introduction 32
  3.2 Theoretical Background and Data 35
    3.2.1 The Seismic Potency Tensor 35
    3.2.2 DC Potency Tensor Catalog 37
  3.3 Analysis 38
    3.3.1 Summed Potency Tensors 38
    3.3.2 Summations Over Different Magnitude Bins 45
    3.3.3 Summations Over Different Tectonic Regions 46
    3.3.4 Magnitude Bins for Tectonic Regions 49
    3.3.5 Quality-Based Analysis 51
    3.3.6 Gridded Tensor Summations for 1 km-Scale Spatial Analysis 54
List of Figures

2.1  Schematic diagram of fault model. ......................................................... 11
2.2  Strength profiles for different model setups. ........................................... 12
2.3  Imposed distributions of $\tau_s - \tau_a$ ..................................................... 14
2.4  Cumulative frequency-size statistics for 150 years of simulated events ........ 17
2.5  Scalar potency as a function of rupture area ........................................... 18
2.6  Earthquake stress drops vs. magnitudes for 3 different heterogeneity models . 21
2.7  Earthquake stress drops vs. magnitudes for uniform distributions of $\tau_s - \tau_a$ with different ranges ................................................................. 22
2.8  The per-cent of events and total potency release for which $T_F \geq 0.8$ in models of uniformly distributed $\tau_s - \tau_a$. ..................................................... 23
2.9  Cumulative frequency-size statistics of earthquakes for three model realizations with depth-dependence ......................................................... 25
2.10 Earthquake stress drops vs. magnitudes for depth-dependent models ........... 26
2.11 Hypocenter depths vs. earthquake stress drops and number of hypocenters .... 27
3.1  Location map. ....................................................................................... 39
3.1  Summation results for the entire southern California catalog ..................... 42
3.1  Histograms of (a) the summed scalar potency and (b) the number of events for each magnitude bin using the entire data set. ................................. 44
3.1  Summations for magnitude subsets. ....................................................... 47
3.1  Summations for regional subsets ............................................................ 50
3.2 Summation results for magnitude bins of regional subsets displayed in terms of the principle strain axes orientations ................................................................. 52
3.3 Summations for magnitude subsets of catalogs with different quality restrictions . 55
3.4 CLVD component of normalized summations over magnitude subsets of catalogs with different quality restrictions ................................................................. 56
3.1 Results of gridded tensor summations for region TRIF ............................... 59
3.2 Gridded summations for magnitude subsets of region TRIF ....................... 61
3.3 Gridded summation results in terms of \( r_{CLVD} \) for magnitude bins in the region TRIF 62
3.4 Gridded summation results in terms of dominant fault style for magnitude bins of the region SBM ................................................................. 63
3.5 Gridded summation results in terms of \( r_{CLVD} \) for magnitude subsets of region SBM 64
4.1 Location map showing focal mechanism data, fault zones and source mechanism tensor summation results. ................................................................. 74
4.1 Cumulative length of the San Jacinto fault trace segments as a function of azimuth. 77
4.1 Summed source mechanism tensor results and angular histograms of the fault traces for each of the fault zones. ................................................................. 82
4.1 Classification of focal mechanism heterogeneity by \( \Delta r_{NORM} \) and \( r_{CLVD} \) for each of the nine fault zones. ................................................................. 83
4.2 Focal mechanism heterogeneity measures as a function of fault trace orientation properties. ................................................................. 85
4.3 Partitioning of focal mechanism styles for four fault zones. ................. 86
4.4 Smoothed density of epicenter locations in the San Jacinto fault zone for three different faulting styles. ................................................................. 88
4.5 Smoothed density of hypocenter strike-depth coordinates in the San Jacinto fault zone for three different faulting styles. ................................................................. 89
4.6 Time vs. number of earthquakes in the San Jacinto fault zone for three different faulting styles. ................................................................. 90
4.7 Frequency-size distributions of earthquakes in the San Jacinto fault zone for three different faulting styles. ................................................................. 91
4.1 Total cumulative slip estimates for each of the fault zones as a function of two heterogeneity measures ................................................. 94
4.2 Illustrative examples showing how misalignment of faults with relative plate motions can lead to heterogeneous faulting. ............................ 95
C1 Alternative measures of the CLVD component compared to $r_{CLVD}$. ........ 113
D1 Distribution of $r_{CLVD}$ for summations of simulated random double-couple orientations ......................................................... 114
F1 Differences between summation results for different magnitude bins ........ 115
F2 Differences between summation results for regional subsets .................. 116
G1 (a) Summed scalar potency and (b) total number of events in each of the seven regions shown in Figure 3.1. ................................. 118
G2 Cumulative potency release over the catalog time interval for $0 < M_L \leq 5$ earthquakes in each subregion ................................. 119
I1 Histograms of fault plane uncertainty for different quality restrictions on the focal mechanism catalog. ........................................ 121
I2 Comparison of heterogeneity results for two different quality levels in the catalog. ......................................................... 122
I3 Relation between fault plane uncertainty and the concentration of the von Mises-Fisher distribution. ........................................ 123
I4 Comparison of heterogeneity for data that sample three kernel densities representing different quality levels in the catalog. .................. 125
J1 Angular variation of the P and T axes and values of $\Delta r_{NORM}$ for a single dominant faulting type with different values of $\kappa$. ........ 127
J2 Compiled results for simulated DC variation based on mixing of faulting styles and Gaussian-like variations. .................. 128
K1 Average minimum rotation angle between all DC pairs in the fault zones as a function of $\Delta r_{NORM}$. ......................................................... 129
K2 Density of rotation axes orientations for minimum rotations between all DC pairs in the fault zones. ........................................ 130
List of Tables

3.1 Summary of the earthquake data set ........................................... 40

4.1 Examples showing how a CLVD component can be generated from mixing focal mechanism orientations. ........................................... 80

H1 Summary of data for the fault zones used in this study. .................. 120

L2 Compiled results for total cumulative offset of the faults relevant to this study. ........ 131
Abstract

Faults and fault networks are heterogeneous structures where elastic strain from ongoing tectonic motions is released in earthquakes. Three studies presented in this thesis aim to illustrate the importance of fault heterogeneity using numerical models and quantify fault heterogeneity using earthquake data. In the first part, a model fault consisting of a grid of discrete slip patches surrounded by a 3-D elastic half-space is used to investigate the statistics of earthquake stress drops. Predictions for stress drops on a homogeneous fault with properties constrained by experiments are an order of magnitude larger than typical observations from seismic data. In the model, heterogeneous frictional properties result in an evolving distribution of spatially heterogeneous stress. Since stress must exceed the fault strength only in the hypocenter region, lower average stress at times of rupture lead to stress drops consistently lower than predictions for a homogeneous fault. The second part of the thesis concerns quantifying the geometry of earthquake faulting for different spatial scales and magnitude ranges in a plate boundary region. We compute and compare potency tensor summations of subsets of a catalog containing \( \sim 170,000 \) focal mechanisms for \( 0 < M_L \leq 5 \) earthquakes from southern California. Results indicate that earthquake deformation is influenced by scales related to plate tectonic motions (\( \sim 700 \) km), large scale fault zones (\( \sim 50–250 \) km) and smaller geometric complexities (\( \sim 5–50 \) km). However, individual fault zone length-scales have distinct characteristics, indicating departure from self-similarity despite the lack of a single characteristic length-scale. In the third part of the thesis higher quality focal mechanisms are used to compare levels of heterogeneity between nine strike-slip fault zones in California. Focal mechanism heterogeneity is quantified using two measures that represent the scatter and asymmetry in faulting orientations, both computed from normalized potency tensor summations. The scatter correlates with complexity of the fault surface traces, and asymmetry correlates with the dominant fault orientation relative to plate motion directions. These observations indicate that focal mechanism heterogeneity relates
both the long term evolutionary properties of the fault zone and the ability of the fault to efficiently release plate tectonic strains.
Chapter 1

Introduction

The relationship between active faulting of the Earth’s crust and earthquakes was originally suggested by Gilbert (1884), McKay (1890) and Koto (1893) following independent observations after earthquakes in California, New Zealand and Japan, respectively. In 1910 H. F. Reid published the theory of elastic rebound based on observations of the 1910 San Francisco earthquake (Reid, 1910), which became the foundation for the field of earthquake physics. The theory suggested that elastic strain energy is built up in the crust, which we now relate to the constant motions of tectonic plates, and is released on localized fault surfaces by rapid brittle fracture. We have seen great advances in our understanding of the mechanics of earthquakes in relation to faulting (e.g., Anderson, 1951; Rice, 1980; Scholz, 2002). However, much of the physics is based on models of faults as planar surfaces separating homogeneous elastic material. It is clear from any geological map of a plate boundary region that elastic strain energy is released on a network of faults with non-planar geometries and varying material properties resulting from the complex distribution of rock types. This dissertation focuses on effects of fault heterogeneity; using a numerical approach to investigate earthquake stress drops on heterogeneous faults, and using observational approaches to quantify heterogeneity in active fault zones.

Several studies have indicated that departures from planar geometries or changes in material properties can affect many features of the earthquakes associated with a given fault. For example, Segall & Pollard (1980) and Harris & Day (1991) showed that breaks and steps in a fault can suppress the propagation of ruptures, Dieterich & Smith (2009) showed that sliding on a fault with a rough surface leads to strong stress concentrations that explain background seismicity and aftershock patterns, and Zöller et al. (2005) showed that the range of length scales related to frictional properties of a fault can affect the distributions of earthquake magnitudes. These studies are supported by observational works which indicate that start and end points of large earthquakes are correlated with geometrical complexities in the fault (e.g., King & Nábělek, 1985; Wesnousky, 2006).
In Chapter 2 the numerical model of Ben-Zion & Rice (1993) is used to investigate the impact of fault heterogeneity on the size of earthquake stress drops. This is a problem of relevance to seismic hazard assessment since the amplitude of peak ground velocities is proportional to the stress drop. Estimates of potential stress drop on a homogeneous fault constrained by experiment data gives results an order of magnitude higher than typically observed. On a heterogeneous fault stress drops can be lower, since the stress at the time of rupture initiation only needs to exceed the failure strength at the hypocenter. The model fault plane consists of a grid of discrete slip surfaces governed by friction and creep parameters embedded in a 3-D elastic halfspace. The frictional properties of the slip surfaces provide a simple way of incorporating heterogeneity onto the fault which approximate the various heterogeneities associated with real faults. Approximations made to the mechanics isolate key aspects of elastic stress transfer as well as brittle and ductile deformation, which allow for the generation of hundreds of years of seismicity on the fault. The model is well suited to the investigation of stress drop potential on a fault because we can analyze the statistical behavior from a range of initial stress drop distributions. The heterogeneous fault properties lead to an evolving stress field that never reaches a level where the fault is uniformly close to failure. As a result, the stress drops of the largest earthquakes generated on a heterogeneous fault are consistently ∼ 30% of those predicted for a homogeneous fault. This relative reduction of stress drops is found for three different spatial distributions of fault heterogeneity investigated, including random roughness, fractal roughness and a model with several near-vertical barriers at different points on the fault. Results that are most consistent with observations correspond to the barrier model, where heterogeneities impose distinct length scales upon the fault. As suggested in other works summarized by Ben-Zion (2008), quantifying the range of length scales is important to the characterization of fault heterogeneity.

A system for which the range of length scales can be quantified is distinct from systems described as scale-invariant for which there is similarity at all scales and there are effectively no length scales of importance. Scale invariance has been proposed to apply to fault and earthquake systems in several studies (e.g., King, 1983; Kagan, 1982; Amelung & King, 1997; Turcotte, 1997).
In contrast, Ben-Zion & Sammis (2003) summarize evidence that faults evolve towards less complex structures over time, implying that differences exist in the heterogeneity of different faults, hence indicating that the length scales of individual faults are important. Furthermore, inversion of earthquake and geodetic data for tectonic stress fields indicate smooth $\sim 50$ km scale variations (e.g. Becker et al., 2005; Hardebeck & Michael, 2006), implying that length scales are relevant with respect to the plate tectonic driving forces. Chapter 3 addresses this issue by comparing the patterns of earthquake related deformation for different spatial scales and magnitude ranges.

Both Chapter 3 and Chapter 4 are concerned with the analysis of small earthquake focal mechanism data from southern and central California. Focal mechanisms describe the orientation of coseismic strain associated with radiation of seismic energy, from which we can infer the direction of faulting. A population of focal mechanisms can therefore be used to investigate heterogeneity in faulting orientations throughout the top $\sim 15$ km of the Earth’s crust. Following the method of Kostrov (1974), summations of potency tensors are used to describe geometrical properties of a population of focal mechanisms. The method is useful for investigating characteristics of subsurface deformation by seismic processes, and notable uses have been to study deformation in the Tonga slab (Fischer & Jordan, 1991), the deformation patterns associated with earthquakes of different sizes (Amelung & King, 1997), coseismic strain release around the San Jacinto fault (Sheridan, 1997) and temporal changes in the earthquake strain release across California (Sipkin & Silver, 2003). In Chapter 3 we concentrate on differences between summation results for spatial bins and earthquake magnitude ranges with the aim of understanding the role of different length scales in earthquake deformation. We find that earthquake characteristics are affected by a wide range of length scales from those of the plate tectonic boundary ($\sim 700$ km) to those of $\sim 5$ km geometric complexities within fault zones. This hierarchical nature in the scales influencing earthquake deformation is suggestive of self-similar processes. However, structures of consistent deformation style that we can relate to faults or fault complexities have distinct characteristics and persist through multiple magnitude ranges, indicating that specific length scales are important. Differences between results for tectonic regions of $\sim 50 – 250$ km size indicate that it is appropriate to treat large fault zones as distinct length scales within a plate boundary system.
In Chapter 4, it is assumed that individual fault zones are distinct length scales and we aim to quantify more specifically how those fault zones are different. This study defines nine different fault zones in southern and central California based on data from geologic mapping. Variations in both focal mechanism and fault trace orientations are used to quantify relative differences in fault zone heterogeneity. In order to minimize the effects of focal mechanism uncertainties, the analysis uses only data constrained by higher quality restrictions. The analysis builds upon the methods introduced in Chapter 3, concentrating on the aspects of summed tensors that illustrate heterogeneity in the population. Specifically, two measures which quantify the degree of scatter and asymmetry in focal mechanism orientations are used. The degree of scatter correlates with the amount of complexity in the surface trace of faults within the fault zone. This indicates that orientations of faulting by earthquakes over the \( \sim 20 \) year period is representative of the long term evolution of the fault zone. The asymmetry in focal mechanism orientations correlates with the dominant direction of faulting of the fault zone, implying that misalignment of a fault zone with the direction of relative plate motion can be a source of heterogeneity. Investigation of the partitioning into different faulting styles within the San Jacinto fault zone indicates that the spatial sampling of the focal mechanism data exerts the strongest control on the heterogeneity results. Most of the data are confined to clusters of seismicity which themselves are heterogeneous and possibly relate to geometrical heterogeneities of faults that cause strong stress concentrations. The use of statistics of focal mechanism data to investigate fault properties is therefore limited by how representative the clusters of seismicity are with respect to the entire fault.

All three studies illustrate the importance of understanding heterogeneity of specific fault zones. The methods presented in Chapter 3 and Chapter 4 provide a useful tool for the quantification of earthquake properties at seismogenic depths. These are used to show in Chapter 3 that there are distinct differences in properties that relate to the overall maturity of fault zones. Differences between fault zones are consistent with fault geometries becoming smoother and a reduction in the range of length scales with overall slip. The results of Chapter 2 indicate that we can expect lower stress drops from small amounts of heterogeneity on a fault, since failure initiates in small critically stressed regions, and stress transfer leads to propagation onto parts of the fault that are
not critically loaded. If mature faults are smoother, the heterogeneities will be reduced and it will be easier for rupture to propagate. Results in Chapter 4 indicate that some heterogeneity persists in all fault zones, and it is thus realistic to expect lower stress drops of the largest events in all cases. We should, however, expect more variation in the stress drops of smaller events in more complex fault zones (e.g., San Jacinto Fault and Eastern California Shear Zone) than in more homogeneous regions such as the Parkfield section of the San Andreas.

References for related publications:


References for submissions:

Chapter 2

Statistics of Earthquake Stress Drops on a Heterogeneous Fault in an Elastic Half-Space

SUMMARY
We investigate properties of earthquake stress drops in simulations of evolving seismicity and stress field on a heterogeneous fault. The model consists of an inherently-discrete strike-slip fault surrounded by a 3-D elastic half-space. We consider various spatial distributions of frictional properties and analyze results generated by 150–300 model years. In all cases, the self-organized heterogeneous initial stress distributions at the times of earthquake failure lead to stress drops that are systematically lower than those predicted for a homogeneous process. In particular, the large system-sized events have stress drops that are consistently $\sim 25\%$ of predictions based on the average fault strength. The type and amount of assumed spatial heterogeneity on the fault affect the stress drop statistics of small earthquakes ($M_L < 5$) more than those of the larger events. This produces a decrease in the range of stress drops as the earthquake magnitudes increase. The results can resolve the discrepancy between traditional estimates of stress drops and seismological observations. The general tendency for low stress drops of large events provides a rationale for reducing the statistical estimates of potential ground motion associated with large earthquakes.

2.1 Introduction
The rapid drop of shear stress on a fault during an earthquake rupture $\Delta \tau_{eq}$ controls the seismic radiation from the source region, and hence the generated ground motion (e.g. Hanks & McGuire,
1981; Aki & Richards, 2002; Ben-Zion, 2003). It is thus important to have realistic estimates of earthquake stress drops that can be used to calculate the associated seismic shaking hazard.

The maximum potential stress drop for a given fault can be estimated by assuming an instantaneous reduction from a static frictional strength level to zero dynamic strength. The static strength of crustal faults is well described by the Coulomb-Byerlee friction,

\[ \tau_s = C + f_s \sigma'_n, \quad (2.1) \]

where \( C \) is a cohesion term, \( f_s \) is the coefficient of static friction and \( \sigma'_n \) is the effective normal stress. Byerlee (1978) found that for most rocks \( C = 0 \) and \( f_s \approx 0.85 \) when \( \sigma'_n < 200 \) MPa, while \( C \approx 50 \) MPa and \( f_s \approx 0.6 \) when \( \sigma'_n \geq 200 \) MPa. To estimate the static fault strength we set \( C = 0 \), \( f_s = 0.75 \), and \( \sigma'_n = (\rho - \rho_w)gz = 18z \) MPa/km, where \( \rho = 2.84 \) kg/m\(^3\) and \( \rho_w = 1.00 \) kg/m\(^3\) are typical densities of rock and water, respectively, \( g = 9.8 \) m/s\(^2\) is the gravitational acceleration, and \( z \) is the depth in km. For a depth range of 3.0–12.5 km, which approximately spans the seismogenic zone of crustal faults in California (Marone & Scholz, 1988), this gives an average static shear strength (and maximum \( \Delta \tau_{eq} \)) of \( \sim 150 \) MPa. This value is compatible with estimates based on measurements in geophysical boreholes (e.g., McGarr & Gay, 1978; Zoback et al., 1993).

Stress drops that are determined empirically from measurements of the radiated seismic waves are typically in the range 0.2–20 MPa (see Shearer et al., 2006, and references therein), significantly lower than the estimated available stress drop. These lower values may be partially explained by a non-zero dynamical frictional strength \( \tau_d \). However, the high values of \( \tau_d \) that are required to resolve the discrepancy would produce significant frictional heating, leading to heat flow anomalies around faults which are not observed (Brune et al., 1969; Lachenbruch & Sass, 1992). Moreover, if seismic slip occurs within a narrow zone of a few cm thick or less, as indicated by various geological studies (e.g., Chester & Chester, 1998; Sibson, 2003; Wibberley & Shimamoto, 2003; Heermance et al., 2003; Rockwell & Ben-Zion, 2007), high values of \( \tau_d \) would lead to widespread melting of fault zones which is again not observed.

The discrepancy between the measured values of \( \Delta \tau_{eq} \) and the potential given by \( \tau_s - \tau_d \) can be explained by considering a heterogeneous distribution of initial shear stress on the fault at the times
of earthquake failure. A homogeneous shear stress on a fault implies that \( \tau = \tau_s \) across the entire rupture, but in a heterogeneous process \( \tau = \tau_s \) is only required at the hypocenter. Since stress is transferred as the rupture propagates, parts of the fault where \( \tau < \tau_s \) may sustain dynamic increase of stress and fail as part of the same earthquake. The stress drop during an earthquake is the spatial average of the initial minus final stress distributions over the rupture area and is therefore reduced if the initial stress is lower. A statistical tendency to have a relatively low initial stress over most of the eventual rupture area can lead to a substantial reduction of the potential \( \Delta \tau_{eq} \).

In the present paper we investigate statistical properties of \( \Delta \tau_{eq} \) using model calculations of long deformation histories on a heterogeneous strike-slip fault with evolving stress fields and stress drop distributions. The fault is defined by a grid of discrete slip patches that incorporate static and dynamic frictional levels, as well as the potential for stable sliding, and is embedded in an elastic half-space (Ben-Zion & Rice, 1993; Ben-Zion, 1996). The discrete patches provide a simple representation of heterogeneity that may be associated with changes in fault geometry or material properties, while the elastic half-space accounts for realistic long range interactions between slipping regions.

The model simulations use quasi-static calculations that allow us to investigate the statistics of \( \Delta \tau_{eq} \) from large earthquake populations generated using different distributions of frictional fault properties. Dynamical effects are incorporated approximately through the use of a dynamic over-shoot coefficient (see section 2.1). The property distributions are based on previous works that studied the resulting seismicity patterns in detail (Ben-Zion, 1996; Zöller et al., 2007). These studies and others have shown that the model produces many statistical features of seismicity compatible with observations, including frequency-size and temporal event statistics, hypocenter distributions, and scaling of source-time functions (Ben-Zion, 1996; Ben-Zion et al., 2003; Zöller et al., 2009; Dahmen & Ben-Zion, 2009).

We find that in nearly all of the investigated model realizations, stress drops for large earthquakes which rupture the entire seismogenic zone are about 25% of the predictions based on the average static strength of the fault. For smaller earthquakes, there is a much greater range in \( \Delta \tau_{eq} \) values that reflects the assumed local fault properties. The simulated stress drop distributions are
compatible with the seismological observations. The results provide a theoretical basis for reduced estimates of the ground motion that is likely to be generated by large earthquakes.

2.2 Method of Investigation

2.2.1 Description of Fault Model

A complete description of the model can be found in Ben-Zion (1996). Here we only outline the main properties of the model and simulation algorithm. The model (Figure 2.1) consists of a vertical strike-slip fault embedded in a 3D elastic half-space. The fault contains a computational grid (region II of Figure 2.1) where evolving stress and displacement fields are generated in response to ongoing loading imposed as slip boundary conditions on the other fault regions (chosen to represent deformation along the central San Andreas fault). Regions III and V of the fault creep at constant plate velocity $V_{pl} = 35$ mm/yr, while regions I and IV follow staircase slip histories with recurrence times of 150 yr. This loading style is the same as in Ben-Zion (1996) but the specific loading details do not significantly affect the behavior of this model (e.g., Zöller et al., 2007; Ben-Zion, 2008). In the following we use $i = 1, \ldots, M$ and $j = 1, \ldots, N$ to represent, respectively, the along-strike and depth coordinates of cells belonging to the computational grid.

The stress within an individual cell of the computational grid, $\tau(i,j,t)$, is computed at time $t$ using a discretized form of a boundary integral equation,

$$
\tau(i,j,t) = \sum_{k,l} K(i,j,k,l)[V_{pl}t - u(k,l,t)] ,
$$

where $u(k,l,t)$ is the total slip in cell $(k,l)$ at time $t$, and $K(i,j,k,l)$ defines the stress at the center of cell $(i,j)$ due to uniform unit slip in the $x$ direction of cell $(k,l)$. The stiffness matrix $K(i,j,k,l)$ is computed from the static solution of Chinnery (1963) for dislocations in a 3D elastic half-space, and is associated with long-range stress transfer that falls with distance $r$ from the source as $1/r^3$. 
The total slip at each cell, quantified by \( u(i,j,t) \), is the sum of aseismic and seismic slip contributions which vary in their relative importance from cell to cell. The aseismic slip-rate \( V_c \) is governed by power-law dislocation creep of the form

\[
V_c(i,j,t) = c(i,j)\tau(i,j,t)^3,
\]
where \( c(i,j) \) is a set of space dependent creep coefficients that increase exponentially with depth and with distance from the southern edge of the computational grid. This slip rate is computed at each time step and partially or completely reduces the slip deficit due to plate loading, depending on the values of \( c(i,j) \) and \( \tau(i,j,t) \). The seismic slip is governed by frictional properties of the cells, defined by spatial distributions of the static friction \( \tau_s(i,j) \), the dynamic friction \( \tau_d(i,j) \) and an arrest stress level \( \tau_a(i,j) \). The distribution of \( \tau_s(i,j) \) is given by Eq. 2.1, with \( f_s = 0.75 \) and \( \sigma_n' = 18 \text{ MPa/km} \), such that the static strength increases linearly with depth. The cohesion term \( C \) has little effect on the model behavior, but is always greater than the maximum possible stress drop. In Figure 2.2a we display the strength profile for an arbitrarily selected column of cells in the middle of the computational grid by combining Eq. 2.1 with Eq. 2.3 where \( V_c = V_{pl} \). This illustrates that the fault is dominated by brittle behavior above 10 km and creep behavior below.

Initial failure of a cell occurs when \( \tau(i,j,t) \geq \tau_s(i,j) \), at which point the strength of that cell changes abruptly to its dynamic strength \( \tau_d(i,j) \) and remains there for the duration of the event. The stress drops locally to its arrest level \( \tau_a(i,j) \), which is lower than the dynamic friction to accommodate approximately inertial effects. The static strength, dynamic strength and arrest stress in each grid cell are related by a constant dynamic overshoot coefficient,

\[
D = \frac{\tau_s - \tau_a}{\tau_s - \tau_d},
\]
which is set to 1.25 throughout this study. The brittle stress drop at a cell \((i,j)\) leads to local slip given by,

\[
\Delta u(i,j) = \frac{\tau(i,j,t) - \tau_a(i,j)}{K(i,j,i,j)},
\]
where the “self-stiffness” of the cell $K(i, j, i, j)$ is proportional to the rigidity of the surrounding half-space divided by the length of the slipping patch, consistent with the stress-strain relation for an isotropic elastic solid. The associated stress transfer computed via Eq. 2.2 may lead to further failures in other cells, in which case slip, stress drop and stress transfer are computed again until the stress levels of all cells are below their brittle failure threshold ($\tau_d$ if the cell has failed, $\tau_s$ if it has not). At the end of each seismic event the brittle strength is reset everywhere to the static level $\tau_s(i, j)$.

As mentioned, the computations associated with the evolution of stress and slip on the fault are done quasi-statically (i.e., at zero effective time). More elaborate quasi-dynamic calculations incorporating finite communication speed for stress transfer, and related kinematic rupture propagation over finite earthquake times, were shown to have little effects on the statistical properties of the seismicity and stress drops (Zöller et al., 2004, 2009). In the interseismic periods, time progresses via variable time steps given by the minimum of 3 days and the time required to initiate a brittle slip instability (hypocenter) in a single cell.

### 2.2.2 Fault Model Heterogeneity

The spatial distribution of $\tau_s(i, j) - \tau_d(i, j)$ represents time-independent heterogeneity of the fault and is used as a tuning parameter for the simulated seismicity patterns (Ben-Zion, 1996; Zöller et al.,...
Figure 2.2: Static strength (black), creep strength (gray) and arrest stress (dashed) for a single column of the computational grid under three heterogeneity models. (a) Model U of Ben-Zion (1996) where \( \tau_a \) is very close to \( \tau_s \), (b) the depth dependent model where \( f_d = 0.5 \pm 0.1 \), and (c) the depth dependent model where \( f_d = 0.2 \pm 0.1 \). Differences only exist in the depth profile of \( \tau_a \) indicating the potential stress drop for the fault under the different heterogeneity models. The creep strength is defined as the stress required for zero accumulation of slip deficit. The lower creep strength for a cell close to 6 km depth shows one of the few cells where random fluctuations have altered the behavior. In (a) the cohesion is 0.6 MPa, corresponding to models U and F of Ben-Zion (1996). In (b) and (c), the cohesion is 1.0 MPa.

2005; Ben-Zion, 2008). Although we specify heterogeneity in terms of frictional properties, this may be seen as a representation of several types of fault heterogeneity such as geometrical disorder, spatial variations in the normal stress, fluid pressure and elastic properties. Since the true nature of fault heterogeneity is not well understood, we investigate several different model setups and their effects on the earthquake stress drop characteristics. The model setups fall into three approaches associated with the type of heterogeneity, the amount of heterogeneity and the depth-dependence of the heterogeneity.

The effect of different types of heterogeneity are investigated by using three different stochastic models for the spatial distribution of \( \tau_s - \tau_a \). The distributions and parameters are the same as cases 1, 3 and 4 of Ben-Zion (1996), and are referred to as models U, F and M, respectively. In
model U, values of \( \tau_s(i,j) - \tau_a(i,j) \) are chosen from a uniform distribution of uncorrelated random numbers in the range 0.6–1.8 MPa. In model F, values of \( \tau_s(i,j) - \tau_a(i,j) \) are taken from a fractal distribution generated by the algorithm of Brown (1995) with fractal dimension 2.3, mean value 3 MPa and standard deviation 1.2 MPa. In model M, low stress drop segments with \( \tau_s - \tau_a = 1 \) MPa are separated by high stress drop barriers where \( \tau_s - \tau_a = 9 \) MPa. The barriers are imposed by selecting cells at the free surface with a probability of 0.2 and then propagating down using a random walk. Figure 2.3 shows the distribution of \( \tau_s - \tau_a \) across the computational grid in each case.

In a second approach we vary the range of \( \tau_s - \tau_a \) in model U to investigate effects due to the amount of heterogeneity. In order to investigate a wide range of heterogeneity, we increase the mean of the uniform distributions from 1.2 MPa to 10 MPa, as used by Zöller et al. (2007) for a study of earthquake recurrence times with the same model. In total, we investigate five uniform distribution cases for \( \tau_s - \tau_a \): 10 ± 0.5 MPa, 10 ± 1 MPa, 10 ± 2.5 MPa, 10 ± 5 MPa and 10 ± 7.5 MPa. In all cases, the same spatial distribution of selected random numbers are multiplied by the appropriate factor to obtain the desired distribution.

Thirdly, we investigate the effects of depth-dependence in the difference between static and dynamic friction by introducing a depth-dependent dynamic strength, \( \tau_d = C + f_d \sigma'_n \), where \( C \) and \( \sigma'_n \) are the same as for \( \tau_s \) and \( f_d \) is the dynamic coefficient of friction. We investigate two values of \( f_d \): 0.5 (cases 1 and 2) and 0.2 (case 3), and include variations of \( f_d \) based on uncorrelated uniform distributions such that \( f_d \) varies from 0.4 to 0.6 in case 1, from 0.49 to 0.51 in case 2, and from 0.1 to 0.3 in case 3. Combining Eq. 2.4 with Eq. 2.1, \( \tau_s - \tau_a = 22.5(0.75 - f_d)z \) describes the range of available stress drops at different depths. The range of potential stress drops on the fault as a function of depth for cases 1 and 3 is illustrated in the depth profiles shown in Figures 2.2b and 2.2c.

### 2.2.3 Quantities Computed

For each model earthquake, the stress drop is computed as

\[
\Delta \tau_{eq} = \frac{1}{N_f} \sum_{i,j \in failed} \tau_0(i,j) - \tau_1(i,j),
\]
Figure 2.3: Three imposed distributions of \( \tau_s - \tau_a \) used earlier by Ben-Zion (1996). (a) Values for each grid cell are selected from a uniform random distribution with range 0.6–1.2 MPa (model U). (b) A fractal distribution of local stress drops with fractal dimension 2.3 (model F). (c) A distribution where segments with \( \tau_s - \tau_a = 1 \) MPa are separated by quasi-vertical barriers with \( \tau_s - \tau_a = 9 \) MPa (model M). Although the distributions over the entire fault are shown, aseismic slip dominates below 10 km (Figure 2.2) and these parameters have little influence on those regions.

where \( N_f \) is the number of failed cells, and the subscripts 0 and 1 denote the stress immediately before and immediately after the earthquake for the failed cells.

The range of values of \( \Delta \tau_{eq} \) produced by the model are largely determined by our choices of \( \tau_s - \tau_a \) in the initial setup. In order to compare absolute values to the average potential stress drop and assess the reduction caused by evolving stress heterogeneities, we compute the following non-dimensional numbers for each earthquake:

\[
T_F = \frac{\Delta \tau_{eq}}{\langle \tau_s - \tau_a \rangle_{fault}},
\]

(2.7)
\[
T_R = \frac{\Delta \tau_{eq}}{\langle \tau_s - \tau_a \rangle_{\text{failed}}},
\tag{2.8}
\]

where \( \langle \tau_s - \tau_a \rangle_{\text{fault}} \) is the average available stress drop over the entire fault and \( \langle \tau_s - \tau_a \rangle_{\text{failed}} \) is the average available stress drop over the rupture area (i.e. failed cells). The parameter \( T_R \) has a maximum of 1 and measures the proportion of the potential stress drop that is actually released by the earthquake, while \( T_F \) may be larger than 1 and measures the stress drop relative to an estimate based on a uniform process. In any given simulation, \( T_F \) is proportional to \( \Delta \tau_{eq} \) since \( \langle \tau_s - \tau_a \rangle_{\text{fault}} \) is the same for all ruptures. For faults with a homogeneous stress field, \( \tau_s - \tau_d \leq \Delta \tau_{eq} \leq \tau_s - \tau_a \). With \( D = 1.25 \), values of \( T_F \) and \( T_R \) less than 0.8 cannot occur on a fault with homogeneous stress field and reflect the effects of stress heterogeneities on the fault. The use of both the average properties for the entire fault (\( T_F \)) and properties for only the rupture area (\( T_R \)) helps to assess the stress drops relative to estimates for a homogenous fault, as well as to assess the influence of lower initial stress on the eventual rupture area. For the case where \( \tau_s - \tau_d \) is depth dependent, the fault average in Eq. 2.7 is replaced by the average of the dominantly brittle depth range of 0–10 km (28.1 MPa when \( f_d = 0.5 \) and 61.9 MPa when \( f_d = 0.2 \)).

In most of our results, we display stress drop as a function of earthquake magnitude since the stress drop characteristics are largely controlled by how far the earthquake has propagated. We compute the magnitude from the scalar seismic potency, which is given by

\[
P_0 = A_{\text{cell}} \sum_{i,j \in \text{failed}} u_1(i,j) - u_0(i,j),
\tag{2.9}
\]

where \( A_{\text{cell}} \) is the area of a single slip patch (\( \sim 0.3 \text{ km}^2 \)) and subscripts 0 and 1 denote the displacement before and after the earthquake in each of the failed cells. The local magnitude \( M_L \) is calculated using the empirical scaling relation of Ben-Zion & Zhu (2002),

\[
\log_{10} P_0 = 0.06M_L^2 + 0.98M_L - 4.87
\tag{2.10}
\]
where $P_0$ is in km$^2$cm. We note that the differences between $M_L$ computed via Eq. 2.10, and $M_W$ computed via the scaling relation of Hanks & Kanamori (1979) are 0.3 and 0.1 at the respective lower and upper magnitude limits considered in this study.

2.3 Results

Before examining results in any simulation, we condition the stress-state on the fault by imposing 125 years of plate motion and running the model for another 25 years. This reduces the effect of the initial conditions on the analyzed results. As described in Section 2.2, the locked sections to the north and south of the computational grid (Regions I and IV in Figure 2.1) follow a staircase slip history, such that two large earthquakes are imposed during the 150 years of simulated data generation. A large earthquake with 5.25 m of slip is imposed on Region I at $t = 150$ years. We then record earthquakes for the following 150 years of simulated seismicity, within which a large earthquake that has the same dimensions is imposed on Region IV at $t = 200$ years. The earthquake on Region IV has little effect on the simulated results because of its distance from the computational grid. However, the parameters of the earthquake on Region I strongly influence the characteristics of the first event that occurs on the grid at $t = 150$, so that data point is removed from our analysis.

In the case where $\tau_s - \tau_a$ is depth-dependent, the potential stress drops for deeper parts of the grid may be over five times higher than in the depth-independent cases. This results in greater time intervals between earthquakes as the plate motion repeatedly builds up stress to $\tau_s$, leading to comparatively fewer earthquakes over 150 years of simulation. We therefore double the length of simulation over which data is generated to 300 years for the results presented in Section 2.3.3.

2.3.1 Different Types of Heterogeneity Distribution

Each of the three heterogeneity models (U, F and M) generates $\sim 10,000$–$50,000$ events with magnitudes in the range 3.2–6.7 (Figure 2.4). The size of the smallest event is governed by the cell size and the lowest value of $\tau_s - \tau_a$, while the size of the largest event is limited by the dimensions of the brittle part (i.e., $z < 10$ km and $x < 60$ km) of the computational grid and $\langle \tau_s - \tau_a \rangle_{\text{fault}}$. 

16
Figure 2.4: Cumulative frequency-size statistics for 150 years of simulated events in three types of heterogeneous distribution of brittle properties.

Figure 2.4 (a reproduction of Figure 15a of Ben-Zion, 1996) shows that all three heterogeneity models result in a power-law distribution of event sizes between the minimum magnitude and $M_L \approx 5.0$. For $M_L > 5.0$, the curves are dominated by a characteristic earthquake size corresponding to rupture over the entire seismogenic part of model. This characteristic event occurs quasi-periodically with recurrence time of $\sim 10$–20 years. In model M with a wide range of length scales, the frequency-size statistics are closest to the Gutenberg-Richter distribution. Further details of the seismicity characteristics in these three model setups are discussed by Ben-Zion (1996).

Figure 2.5 shows $P_0$ and $\Delta \tau_{eq}$ as a function of rupture area for earthquakes generated by model U. The dashed lines with a slope of $3/2$ correspond to constant values of stress drop over a circular rupture geometry in a homogeneous elastic medium with $\mu = 30$ GPa (Aki, 1967; Brune, 1968). This approach is typically used to summarize the stress drops from seismic data. Our computed stress drops (shadings) generally agree with values that would have been inferred from the position
Figure 2.5: Scalar potency as a function of rupture area for earthquakes produced by 150 years of seismicity on a fault with a uniform distribution of heterogeneity. The diagonal dashed lines show the stress drop that would be predicted based on a circular rupture geometry, while shading of points shows the simulated stress drops.

of points with respect to the constant stress drop lines. As noted by Ben-Zion & Rice (1993), the scaling of $P_0$ with $A$ for earthquakes in this model exhibits a transition around $A \sim 30 \text{ km}^2$ corresponding to a circular rupture of radius $r \sim 3 \text{ km}$. The initial potency-area slope of $\sim 1$ for events with $M_L < 5.0$ is indicative of fractal-like failure in a rough stress field (Fisher et al., 1997), while the change to a steeper slope of $\sim 3/2$ for larger events indicates a transition toward the scaling of a classical crack in a homogeneous stress field (e.g., Ben-Zion, 2008). The transition to a classical crack growth leads to fewer events in the magnitude range $5.5 \leq M_L \leq 6.1$, giving rise to the characteristic earthquake distribution observed in Figure 2.4.

Figure 2.6 shows the distributions of $\Delta \tau_{eq}$ and $T_R$ as a function of magnitude for models U, F and M with key values of $T_F$ marked as dashed lines. The results indicate that for all three cases of the assumed property distribution, (1) the stress drops are generally lower for larger earthquakes.
and (2) there is less variation of stress drop values for larger earthquakes. In models U and F, about half of the events (53% and 63%, respectively) have stress drops consistent with estimates for a homogeneous fault (i.e. $T_F \geq 0.8$, since $\Delta \tau_{eq} \geq \langle \tau_s - \tau_a \rangle_{\text{fault}}$). However, these events account, respectively, for only 9% and 12% of the total potency release on the fault because they are all small earthquakes. In the results for case M, the events with $T_F \geq 0.8$ account for only 2% of both total earthquake number and total potency release since most of the seismicity is confined to the low stress drop patches. The almost linear decrease in the median value of $T_R$ with increasing magnitude for all models shows that the average initial stresses are lower for larger earthquakes. The progressive reduction of $T_R$ with magnitude stems from the scaling of stress concentration in elastic solid with rupture size, which increases the ability of larger events to propagate through areas of low initial stress (Ben-Zion & Rice, 1993; Ben-Zion, 2008). In all three cases $T_F \approx 0.3$ and $T_R \approx 0.25 - 0.4$ for the large characteristic events, implying that a maximum of 40% of the stress drop potential for a fault is released in a big earthquake. The results for $\Delta \tau_{eq}$ are in the range 0–10 MPa, consistent with observations, but this is largely controlled by the imposed values of $\tau_s - \tau_a$. The difference between the median values of $T_F$ for large and small earthquakes is smallest for model M where there is a wide range of size-scales.

For earthquakes where only one or two cells fail, the magnitude and stress drop are largely pre-defined by $\tau_s - \tau_a$ for those specific cells. This produces the distinct discretization of magnitude results on the left side of Figures 2.6a, 2.6c, and 2.6e. In those earthquakes only the hypocentral region fails so the initial stress is always close to $\tau_s$. As a result, the distribution of $\Delta \tau_{eq}$ is similar to the imposed heterogeneity in $\tau_s - \tau_a$ and $T_R$ is always close to 1. As the rupture area increases, the elastic stress transfer exerts stronger and stronger influence on the final earthquake properties. The stress drop characteristics of larger events are therefore less sensitive to small-scale spatial variations in properties, and appear similar regardless of the underlying property distribution.

A secondary effect that influences the stress drops for larger earthquakes is the brittle failure of relatively deep grid cells that are typically dominated by creep deformation. Since creep does not occur over short time scales, stress may build up on these cells during rupture propagation to the static strength levels that are not attainable during the interseismic periods. This is similar to
brittle failure of a normally viscous material under high strain rates. If the stress in these cells at the
time of hypocenter failure is lower than the arrest stress, there is a locally negative stress drop. For
system-sized earthquakes, this has a notable effect on the values of $\Delta \tau_{eq}$.

### 2.3.2 Different Heterogeneity Levels

Figure 2.7 shows distributions of $T_F$ and $T_R$ as a function of magnitude for model U using three
different ranges of $\tau_s - \tau_a$: (a) & (b) $10 \pm 0.5$ MPa, (c) & (d) $10 \pm 5$ MPa and (e) & (f) $10 \pm 7.5$ MPa.
As in Figure 2.6, all cases show a decrease in both the median values of $\Delta \tau_{eq}$ and the variation of
$\Delta \tau_{eq}$ as the event magnitudes increase. The case here where $\tau_s - \tau_a = 10 \pm 5$ MPa (Figure 2.7c)
is analogous to the case in the previous section where $\tau_s - \tau_a = 1.2 \pm 0.6$ MPa (Figures 2.6a),
since the range of the uniform distribution is equal to the mean in both model realizations. A
comparison of Figures 2.7c and 2.6a shows that the resulting distribution shapes for $T_F$ and $T_R$ are
similar despite changes in the absolute values of $\Delta \tau_{eq}$. In the case of $\tau_s - \tau_a = 10 \pm 5$ MPa there are
fewer earthquakes and generally larger magnitudes because of the increased available stress.

A comparison of the results in Figure 2.7 indicates that a wider range of values in the imposed
uniform distribution has little effect on the $\Delta \tau_{eq}$ characteristics of larger magnitude earthquakes,
but clearly changes the amount of variation in smaller earthquakes. Again, this shows that the
characteristics of small events are much more sensitive to the local fault properties than the larger
earthquakes.

Figure 2.8 illustrates the reduction in the number and combined potency release of events with
$T_F \geq 0.8$ as the range of $\tau_s - \tau_a$ is increased. Interpreting the range in the uniform distribution as
a degree of heterogeneity, we find that the more heterogeneous the fault, the greater the proportion
of earthquakes for which $\Delta \tau_{eq}$ is smaller than predicted for a homogeneous fault. A comparison of
the reduction in event number with the reduction in total potency release illustrates again that the
effect of heterogeneity changes are largely limited to the small earthquakes that are an unsubstantial
component of the total potency release. The reduction in stress drops for the largest earthquakes,
which contribute most to the total potency release, requires only a small amount of heterogeneity
along the fault.
Figure 2.6: Earthquake stress drops vs. magnitudes for 3 different heterogeneity models: (a) and (b) the uniform distribution model U, (c) and (d) the fractal model F, and (e) and (f) the barrier model M. The simulated $\Delta \tau_{eq}$ for individual earthquakes are represented by gray circles and median values for magnitude bins of 0.2 width are shown by a black line. The left column of plots shows results for the absolute stress drops and $T_F$, while the right column shows results in terms of $T_R$. 
Figure 2.7: Earthquake stress drops vs. magnitudes for uniform distributions of $\tau_s - \tau_a$ in the grid cells with ranges (a) & (b) 9.5–10.5 MPa, (c) & (d) 5–15 MPa, and (e) & (f) 2.5–17.5 MPa. The $\Delta \tau_{eq}$ values of individual earthquakes are represented by gray circles and the median values for magnitude bins of 0.2 width are shown by a black line. The left column of plots shows results for the absolute stress drops and $T_F$, while the right column shows results in terms of $T_R$. 
Figure 2.8: The per-cent of events and total potency release for which $T_F \geq 0.8$ in models of uniformly distributed $\tau_s - \tau_a$ with ranges 9.5–10.5 MPa, 9–11 MPa, 7.5–12.5 MPa, 5–15 MPa and 2.5–17.5 MPa.

2.3.3 Depth Dependency of $\langle \tau_s - \tau_d \rangle$

Figure 2.9 shows the frequency-size distributions for the three cases where there is a difference between the static and dynamic friction coefficients: (1) high dynamic strength with moderate heterogeneity ($f_d = 0.5 \pm 0.1$), (2) high dynamic strength with low heterogeneity ($f_d = 0.5 \pm 0.01$), and (3) low dynamic strength with moderate heterogeneity ($f_d = 0.2 \pm 0.1$). In all three situations there are fewer earthquakes generated than in previous simulations (Figure 2.4), despite the longer time period. The slope of the distributions is shallower than without the depth dependence in all three cases, which indicates an increased ratio of large to small events. A maximum likelihood estimate of the $b$-value for the case with $f_d = 0.5 \pm 0.01$ lies in the range 0.8–1.3 depending upon the cutoff magnitude used, while the value is closer to $\sim 1.5$ for the Gutenberg-Richter-like events from model U in Section 2.3.1. A comparison of the distributions for the three cases considered in this section indicates a similar $b$-value in all cases, but the $a$-value of the seismicity is greater when $f_d$ is higher. By changing the range of $f_d$ from $\pm 0.1$ to $\pm 0.01$, we find little difference in the frequency-size distributions except for a small increase in the size of the largest magnitude event. Ben-Zion (2008) summarizes the characteristics of the frequency-size statistics generated by this model for different values of key parameters representing the fault heterogeneity and dynamic weakening $\langle \tau_s - \tau_d \rangle$ with
no depth dependence on the fault. The results of Figure 2.4 and 2.9 show that depth-dependent variations of $\tau_s - \tau_a$ provide a simple way for obtaining Gutenberg-Richter distributions with a $b$-value near 1 for realistic values of $\tau_s - \tau_d$ and $D \approx 1.25$. This may be related to an increase in the effective heterogeneity of the stress on the fault because of depth variations of the potential stress drops.

The growing difference between $\tau_s$ and $\tau_a$ with increasing depth leads to the largest potential stress drops in considered thus far, as illustrated by the depth profiles in Figure 2.2. Figure 2.10 shows that there is a much greater range in values of $\Delta\tau_{eq}$ as a result. In the case where $f_d = 0.2 \pm 0.1$ (Figure 2.10c) we find stress drops that approach the $\sim 100$ MPa predicted by laboratory experiments. However, such high stress drops are confined to small earthquakes and the large earthquakes show consistently low values of $T_F$ and $T_R$. The non-dimensional number $T_F$ is sensitive to the choice of denominator in Eq. 2.7, so $T_F$ would be larger if we divide by the average available stress drop at a shallow depth where most seismicity occurs (Figure 2.11) rather than the average over the top 10 km. The results for $T_R$ are more meaningful and they show that cascading ruptures lead, as in the previous results (Figures 2.6 and 2.7), to stress drops smaller than the available stress drop. A minor difference is that for the results here there is a plateau in the values of $T_R$ for smaller earthquakes where stress drops are close to 90% of the available stress drop for the rupture area. This corresponds to higher variation in the values of $\Delta\tau_{eq}$ when only one or two cells fail due to the increase in $\tau_s - \tau_a$ with depth.

In these simulations the median stress drop increases slightly as a function of magnitude due to the tendency of larger earthquakes to rupture more of the deeper parts of the fault than smaller events. However, the stress drops of largest events are still only about $\sim 25\%$ of the available stress drop and the results again show decreasing variations with increasing size. A comparison of the three cases shows that the variation of $T_F$ and $T_R$, but not $\Delta\tau_{eq}$, is higher for the two cases where $f_d$ is larger. Comparing Figures 2.10b and 2.10d, we find that increasing the amount of heterogeneity in $f_d$ increases the variation of $T_R$ in the small events, which are also more numerous, but has little effect on the larger earthquakes.
Figure 2.11 compares the stress drops of earthquakes with different hypocenter depths for a depth-independent heterogeneity model with $\tau_s - \tau_a = 10 \pm 5$ MPa (Figure 2.11a–c), and a depth-dependent model with $f_d = 0.5 \pm 0.1$ (Figure 2.11d–f). The lines representing the median values of $\Delta \tau_{eq}$ show that the general trends of $\Delta \tau_{eq}$ follow the trend of $\tau_s - \tau_a$. This is because the smallest events are more numerous and are also the most sensitive to the imposed values of $\tau_s - \tau_a$. Figure 2.11f shows that in the depth-dependent case nearly half of the seismicity occurs in the upper 2 km. This is due to the smaller values of $\tau_s - \tau_a$ there. We find that the variation of $\Delta \tau_{eq}$ for individual hypocenters, shown by the scatter of points, is greatest for depths around 5 km which is in the middle of the seismogenic zone. The variation of $T_R$ (Figure 2.11e) is greatest at 1.5 km, implying that most larger events with small $T_R$ initiate at the top of the fault.

2.4 Discussion

The simulations in this study show that for a heterogeneous fault with a self-organizing stress field, the overall stress for the entire fault never reaches a level that is comparable to the average fault strength. As a result, the stress drops of the largest earthquakes on the fault are always smaller.
Figure 2.10: Earthquake stress drops vs. magnitudes for depth-dependent difference between static and dynamic friction, with a uniform random distribution for variations in $f_d$. Individual earthquakes are represented by gray circles and the median values for magnitude bins of 0.2 width are shown by a black line. The left column of plots shows results for the absolute stress drops and $T_F$, while the right column shows results in terms of $T_R$. 
Figure 2.11: Hypocenter depths vs. earthquake stress drops and number of hypocenters for cases with depth-independent and depth-dependent potential stress drops. (a) Stress drops along with (b) $T_R$ and (c) number of hypocenters for the uniform distribution $\tau_s - \tau_a = 10 \pm 5$ MPa. (d) Stress drop along with (e) $T_R$ and (f) number of hypocenters for the depth-dependent $f_d$ in the range 0.4–0.6. In (a), (b), (d) and (e), individual earthquake hypocenters are represented by gray circles and the median for each depth level is shown by a solid line. In (a) and (d), the $\Delta \tau_{eq}$ value of $T_F = 1$ is shown by a vertical dashed line.
than estimates based on the experimental results outlined in the introduction. The simulated results indicate that the stress drops of moderate to large earthquakes on a heterogeneous fault are only \( \sim 25\% \) of the values predicted for a corresponding homogeneous fault. The underlying physical reason is that earthquake ruptures have an effective inertia that leads to failure of parts of the fault with low initial stress. The generality of this process in all the examined models indicates that the key requirements are some heterogeneity of the fault properties and elastic stress transfer during ruptures. We find that increased fault heterogeneity and the inclusion of a creep process that reduces the level of stress on some parts of the fault enhance the reduction of the earthquake stress drops. The results are likely to apply to natural fault processes since faults are generally heterogeneous and they reside in a surrounding elastic solid. We note that the relatively low evolving average initial stress on the fault in our simulations is consistent with rupture propagation at sub-shear velocities (e.g., Andrews, 1976; Zheng & Rice, 1998; Shi et al., 2008), as observed for most large earthquakes (e.g., Mai, 2004; Somerville et al., 1999).

As mentioned in the introduction, observed stress drops are typically an order of magnitude less than estimates based on a homogeneous fault. The median stress drops in our results are generally larger than 20\% of the average available stress, so we do not resolve the discrepancy fully. Previous results from this model have shown that heterogeneities are necessary to reproduce many statistical features of seismicity on a fault, and this study demonstrates that similar heterogeneities can account for a substantial reduction in expected stress drops. A further reduction may be generated by simulations that incorporate a wider range of heterogeneities, as well as elastodynamic calculations that increase the effective inertia of earthquake ruptures.

Nearly all of our results for depth-independent variation of \( \tau_s - \tau_a \) suggest that the median stress drop decreases as a function of earthquake magnitude. Analysis of microearthquakes in the Parkfield section of the San Andreas fault show reduction of stress drops with event size (Nadeau & Johnson, 1998), but most observations using standard techniques tend to imply either approximately constant or slightly increasing stress drops (Aki, 1972; Hanks, 1977; Abercrombie, 1995; Hardebeck & Hauksson, 1997). The closest of our results to magnitude independent stress drops are generated by the strongly heterogeneous model M, which has a wide range of size scales and
depth-independent difference between the static and dynamic friction levels. This implies that localized patches of high stress drop and a wide range of length scales, as in a heterogeneous fault zone, are important for maintaining constant levels of stress drop over a range of earthquake magnitudes. The wide range of length scales leads to an effective self-similarity of rupture processes, consistent with the argument first put forward by Aki (1972). Incorporating an increase in the average offset between $\tau_s$ and $\tau_a$ with depth leads to a slightly increasing median stress drop as a function of magnitude. The increasing median stress drop in this case is explained by the deeper penetration of larger magnitude earthquakes.

For the cases where potential stress drop is depth-independent, model M produces the most realistic earthquake size and hypocenter depth distributions. The simulations with depth-increasing potential stress drop produce realistic distributions of $\Delta \tau_{eq}$ and frequency-size statistics, but the simulated distribution of hypocenter depths is unrealistically weighted towards shallow events. The abundant shallow seismicity can be suppressed by increasing the efficiency of the creep process at shallow depth where there is currently no creep. However, this is not central to the main focus of our study and has therefore not been implemented. The general trend of the simulated stress drops vs. depth corresponds to the imposed distribution of $\tau_s - \tau_a$. Thatcher & Hanks (1973) did not find a correlation with depth for their study of 138 earthquakes in southern California but found evidence for regional variations. Fletcher (1980) found a moderate correlation between stress drop and depth of $\sim 0.1$ MPa/km within considerable scatter in the 1975 Oroville aftershock sequence. A similar correlation of $\sim 0.2$ MPa/km was obtained for depths less than 8 km by Shearer et al. (2006) for earthquakes across southern California, where the hypocenter depths are better resolved than by Thatcher & Hanks (1973). At Parkfield, depth-dependence is apparent in the top ten kilometers (Allmann & Shearer, 2007), but the authors note a strong sensitivity to the shear wave velocity model which may bias the results. Yang et al. (2009) obtained results that are consistent with mild increase of stress drops with depth along the Kardere-Düzce branches of the North Anatolian fault, though the along strike variations are also of a similar size. These results may be simulated by considering a combination of the depth-dependent models and model M.
Our general finding of reduced variations in stress drops with increasing magnitude is intuitive since large earthquakes are associated with larger spatial averages of local stress drops. However, this is difficult to compare with observational studies. There is a general lack of data where $\Delta \tau_{eq}$ has been computed by consistent methods over a broad magnitude range from a similar region, and scatter in the results tends to be dominated by uncertainty involved in the computational method. The results of Shearer et al. (2006) for over 60,000 southern California earthquakes suggest very little reduction in stress drop variance as a function of magnitude within the narrow size-range of $1.5 \leq M_L \leq 3.1$ considered. However, results from 10,000 earthquakes associated with a single fault zone at Parkfield for the same magnitude range (Allmann & Shearer, 2007) show a noticeable reduction of scatter with increasing magnitude. This may be explained by a decrease in uncertainty as earthquakes become better recorded, but may also relate to the explanations provided by our model since the analysis is for a single fault structure rather than the southern California fault network. Our results are also supported by detailed inversions of rupture histories for earthquakes which show comparable internal variations of slip and stress drop along portions of the rupture (e.g. Wald et al., 1996; Somerville et al., 1999; Delouis et al., 2002; Dreger et al., 2007).

Andrews et al. (2007) estimated the maximum ground motion generated by a system-size earthquake on the Solitario Canyon fault, close to the proposed Yucca Mountain nuclear waste repository, by simulating a dynamic rupture that sustains a uniform reduction of shear stress that is roughly 20 MPa (based on their Figure 5). The simulations produced peak ground velocities of 3.6 m/s and 5.7 m/s in the horizontal and vertical directions, respectively. Since ground motion is proportional to the stress drop, our study indicates that more typical values for large earthquakes should be about 0.3 times those numbers, i.e., 1.08 m/s and 1.71 m/s, respectively. More realistic simulations of self-evolving heterogeneous stress field on a fault that incorporates effects ignored in the present study (e.g., details of the friction law, elastodynamic calculations, etc.) may provide better estimates of expected stress drops. However, the key general tendencies for relatively low stress drops of large events and reduced variability with increasing size that are simulated in this work are likely to remain unchanged.
Chapter 3

Patterns of Co-seismic Strain Computed from Southern California Focal Mechanisms

SUMMARY

Geometrical properties of an earthquake population can be described by summation of seismic potency tensors that provide a strain-based description of earthquake focal mechanisms. We apply this method to \( \sim 170,000 \) potency tensors for \( 0 < M_L \leq 5 \) southern California earthquakes recorded between January 1984 and June 2003. We compare summed tensors for populations defined by faulting region and earthquake magnitude in order to investigate the relation between earthquake characteristics, tectonic domains and fault-related length scales. We investigate spatial scales ranging from \( \sim 1–700 \) km and use the results to identify systematic differences between seismic behavior for different faults and different regions. Our results show features that are indicative of both scale-invariant and scale-dependent processes. On the largest scale the overall potency tensor summation for southern California \( 0 < M_L \leq 5 \) earthquakes over \( \sim 20 \) years corresponds closely to a double-couple mechanism with slip direction parallel to relative plate motion. The summed tensors and derived quantities for the different regions show clear persistent variations that are related to the dominant tectonic regime of each region. Significant differences between the non-double-couple components of the summed tensors, which we relate to fault heterogeneity, indicate systematic differences in deformation associated with earthquake populations from different fault zones or different magnitude ranges. We find an increase of heterogeneity for populations of smaller earthquakes and for regions where faulting deviates strongly from the overall sense of deformation, even
when corrected for quality. The results imply an overall organization of earthquake characteristics into domains that are controlled to first order by geometrical properties of the largest faults and the plate motion. Smaller scale characteristics are related to local variations in the orientation, complexity and size of faults.

3.1 Introduction

This study is concerned with characterizing the geometrical properties of earthquake populations over multiple scales of magnitude and fault length based on focal mechanism observations. Theoretical studies of earthquakes and fault mechanics are typically based upon one of two end-member descriptions: homogeneous smooth faults in an elastic solid (e.g., Reid, 1910; Rice, 1980) or scale-independent faults with a fractal geometry (e.g., Kagan, 1982; King, 1983; Turcotte, 1997). As summarized by Ben-Zion (2008), each case has very different implications for the mechanics of earthquakes, but both are simplistic in the sense that all faults are treated as belonging to a single dynamic regime. A wide variety of multi-disciplinary observations indicate differences between fault structures and suggest an evolution with ongoing deformation, from highly disordered networks that have band-limited fractal properties toward dominant connected structures that have relatively simple tabular geometries (Ben-Zion & Sammis, 2003, and references therein). The limited amount and resolution of the available data present difficulties for studies aiming to clarify the geometrical properties of fault populations. In the present paper we attempt to overcome these difficulties by performing multi-scale and multi-signal calculations, focusing on results that emerge from several different types of analysis.

Differentiating between classes of fault geometries can be approached by analysis of (i) surface fault traces (e.g., Wesnousky, 1988; Stirling et al., 1996; Sagy et al., 2007), (ii) locations of earthquake hypocenters (e.g., Kagan & Knopoff, 1980; Fehler et al., 1987; Nicholson et al., 2000; Holschneider & Ben-Zion, 2006), and (iii) earthquake focal mechanisms (e.g., Kagan, 1990; Amelung & King, 1997; Hardebeck, 2006; Twiss & Unruh, 2007). While fault traces are more representative of the long-term faulting processes than ~ 30 years of earthquake catalog data, observations at the surface may not correspond to processes at seismogenic depths. Locations of earthquake
hypocenters sample faulting throughout the entire brittle crust, and recent advances in the computation of these locations (e.g., Waldhauser & Ellsworth, 2000) have led to better resolution. The instrumental record is too short to provide a complete sampling of a given fault network, but data sets are now large enough to compare populations associated with different fault zones or tectonic domains. Focal mechanism catalogs contain fewer data than hypocenter catalogs due to limitations on the minimum number of instruments needed to compute a focal mechanism. However, representing earthquakes as point hypocenters neglects important geometrical information about the associated deformation (e.g., Libicki & Ben-Zion, 2005). This study utilizes the information provided by focal mechanisms while maximizing the number of data, and in the following sections we present analyzes of a catalog based on \(\sim 170,000\) earthquakes. The data consist of \(0 < M_L \leq 5\) southern California earthquakes from the period January 1984 – June 2003, and the large number of events allows us to investigate geometrical properties at multiple spatial scales in the range \(\sim 1–700\) km.

By representing individual focal mechanisms as seismic potency tensors (the strain-based equivalent of the moment tensor), we are able to describe geometrical properties of earthquake populations by summations of potency tensors (Kostrov, 1974).

Potency tensor summations or analogous moment tensor summations have been used in previous studies by e.g., Fischer & Jordan (1991) to study the subducting Tonga slab, Amelung & King (1997) for northern California, Sheridan (1997) for the San Jacinto fault zone and Sipkin & Silver (2003) for aftershock sequences in southern California. Amelung & King (1997) and Sheridan (1997) illustrated the dominant effect of large (\(\sim 50–100\) km) faults in a given region with respect to the summed tensor orientations, which have principle strain axes consistent with the fault orientations regardless of the magnitude range used for summation. Large faults of similar scales have also been shown to be important to stress inversion results by Hardebeck & Michael (2004) and Becker et al. (2005). When the summation region was expanded to a plate boundary scale, Amelung & King (1997) observed further that principle strain axes orientations are consistent with plate tectonic motions for nearly all magnitude ranges, implying that small earthquakes collectively deform the crust in the same way that large earthquakes do. Plate tectonic scales were also shown to be important to the properties of summed moment tensors by Fischer & Jordan (1991) and Sipkin
Due to the persistent orientations of summed tensors for earthquakes in different magnitude ranges, Amelung & King (1997) interpreted their results as self-similar behavior of all earthquake populations implying scale-invariant deformation processes. However, consistent orientations may also be explained by sets of relatively homogeneous deformation within finite scales associated with the large faults. Such length-scale dependence indicates a deviation from self-similar behavior, which can be illustrated by showing that there are differences between the deformational patterns of large individual fault zones or tectonic domains.

As in previous studies, we find that the principle strain axes of summed tensors are aligned consistently with faulting directions for a range of different magnitude earthquakes. However, examining the non-double-couple (non-DC) component of the summed tensors indicates that the population geometries cannot be considered to be similar for different regions and magnitude bins. The non-DC component of the summed tensors can be interpreted as a measure of fault heterogeneity reflecting mixing of faulting styles within the sample, and we find that some regions show greater mixing of faulting styles than others. Furthermore, there is a tendency for populations of larger magnitude earthquakes to behave more homogeneously than smaller magnitude populations in the same region, which can be related to the existence of faulting structures with varying levels of geometrical heterogeneity. Treating earthquake and fault-related systems as self-similar processes is therefore an inadequate description, since an earthquake population associated with a particular fault-related length-scale can have distinct characteristics. The overall patterns that we observe can be explained by the interaction of processes with a wide range of spatial scales, from those of plate tectonic motions (∼ 700 km) to large fault zones (∼ 50–250 km) to geometrical complexities such as fault step-overs and kinks (∼ 5–50 km).

To perform a thorough analysis of earthquake potency tensors in southern California we develop and implement several analysis techniques. In Section 3.2 we describe potency tensor quantities and details of the southern California data used. In Section 3.3 we use six sub-sections to illustrate our analysis methods and associated results. Concentrating on the largest spatial scale and using the entire southern California dataset as an example, we explain our two methods of tensor
summation and how we interpret the results of these summations in Section 3.3.1. We then examine characteristics of deformation for different sized earthquakes by comparing summed tensors for different magnitude subsets in Section 3.3.2. A second spatial scale related to large fault zones is examined using regional subsets of the data in Section 3.3.3, and we partition these datasets by magnitude in Section 3.3.4. In Section 3.3.5 we investigate the robustness of our results and effects of data uncertainties by analysis of higher quality catalogs. Our smallest spatial scale is investigated in Section 3.3.6, where we outline additional methods for investigating spatial patterns at a 1 km scale for two regions, defining two new metrics and a method for assessing robustness to aid the interpretation of results. A discussion of our results is presented in Section 3.4.

3.2 Theoretical Background and Data

3.2.1 The Seismic Potency Tensor

The properties of the inelastic deformation in the earthquake source region can be described by the seismic potency tensor, \( P_{ij} \), which is formally defined as

\[
P_{ij} = \int_{V_s} \varepsilon_{ij}^P dV^s,
\]

where \( \varepsilon_{ij}^P \) is the transformational strain tensor and \( V^s \) is the source volume (e.g., Ben-Zion, 2003). The subscripts \( i \) and \( j \) denote the three Cartesian axis directions, and we take the convention that \( x_1 \) points east, \( x_2 \) points north, and \( x_3 \) points up. The transformational strain refers to the irreversible deformation that resets the zero reference level of the elastic stress during a given earthquake failure (Eshelby, 1957). The seismic moment tensor, \( M_{ij} \), can be computed from \( P_{ij} \) by

\[
M_{ij} = \sum_{k,l} c_{ijkl}^s P_{ij},
\]

where \( c_{ijkl}^s \) is the tensor of elastic moduli for the source region. However, \( c_{ijkl}^s \) is poorly constrained for the space-time domains associated with earthquakes (e.g., Ben-Zion, 2001, 2003), and it does not affect the seismic radiation in the surrounding elastic solid (Woodhouse, 1981; Ben-Zion, 1989; Heaton & Heaton, 1989; Ampuero & Dahlen, 2005). We therefore prefer to use \( P_{ij} \) and related strain
quantities, which make no assumptions about material properties at the source, rather than the more commonly used $M_{ij}$.

Assuming zero net torque and zero net rotation, both $P_{ij}$ and $M_{ij}$ are symmetric ($P_{ij} = P_{ji}$) and have six independent components (Aki & Richards, 2002). Changes in volume due to regular tectonic earthquakes are typically considered to be negligible and we make this assumption in our study, constraining all potency tensors to be deviatoric such that $P_{11} + P_{22} + P_{33} = 0$. A deviatoric $P_{ij}$ can be decomposed into a summation of double-couple (DC) and compensated linear vector dipole (CLVD) parts (Knopoff & Randall, 1970; Jost & Hermann, 1989). While the DC component can be associated with slip on a planar surface, the CLVD component corresponds to compensated uniaxial compression or extension of a volume which requires a more complicated faulting geometry. Although it is generally accepted that many tectonic earthquakes contain a non-zero CLVD component (Julian et al., 1998), this is considered to be a second-order feature and not well constrained by inversion (Frohlich & Davis, 1999). We therefore assume that it is appropriate to use potency tensors with zero CLVD component to represent the individual small ($M_L \leq 5$) earthquakes used in this study. Potency tensors used to represent populations of earthquakes are computed by summation of DC tensors (Section 3.3.1) and will not necessarily have a zero CLVD component. In such a case, the relative size of the CLVD component indicates a degree of fault heterogeneity that implies a mixing of different faulting regimes. We describe this heterogeneity more specifically in Section 3.3.1.

The size of a potency tensor is given by the scalar potency,

$$P_0 = \sqrt{2P_{ij}P_{ij}} = \|P_{ij}\| ,$$

which is related to slip, $\Delta u$, on a surface, $A$, by

$$P_0 = \int_A \Delta u \, dA .$$

(3.2)
The scalar potency can be used to compute the source mechanism tensor (Riedesel & Jordan, 1989),

\[ \hat{P}_{ij} = \sqrt{2} \frac{P_{ij}}{P_0}, \]  

(3.4)

which has unit Euclidean norm and is used in this study to describe the orientation part of the potency tensor.

### 3.2.2 DC Potency Tensor Catalog

Our data set is based on a catalog of focal mechanisms generated using the HASH algorithm (Hardebeck & Shearer, 2002) for southern California earthquakes with \( M_L > 0 \) in the period January 1984 to June 2003. This is an extended version of a catalog available via the Southern California Earthquake Data Center (Hardebeck et al., 2005). In our catalog, initial quality restrictions for the required station distribution are relaxed, so as to increase the number of data (J. Hardebeck, private communication, 2006). Maximization of the number of data results in an increased number of less well constrained focal mechanisms, but we assume that there will be no significant bias in average focal mechanism properties. The catalog generated by HASH contains mechanism quality estimates that take into account the sensitivity of the inversion to both the station distribution and velocity model.

Rearranging eq. 3.4, the potency tensor is given by \( P_{ij} = P_0 \hat{P}_{ij} / \sqrt{2} \). We compute \( \hat{P}_{ij} \) from the fault plane solutions using geometrical transformations described in Appendix A, and \( P_0 \) using the magnitude-potency scaling relation of Ben-Zion & Zhu (2002),

\[ \log_{10} P_0 = 0.0612 M_L^2 + 0.988 M_L - 4.87, \]  

(3.1)

where \( P_0 \) has units of km\(^2\) cm, and \( M_L \) is the local magnitude obtained from the catalog.

The inversion of seismic data for a DC mechanism assumes that the seismic radiation originates from a single point source. For a circular crack of radius \( r \), the scalar potency scales as \( P_0 = c \Delta \varepsilon_s r^3 \) (Eshelby, 1957; Ben-Zion, 2003), where \( \Delta \varepsilon_s \) is the static strain drop and \( c = 16/7 \) for an infinite Poissonian solid. Combining this with eq. (3.1) and assuming that \( \Delta \varepsilon_s = 1 \times 10^{-4} \), we estimate the
rupture dimension of the largest employed earthquakes with $M_L = 5.0$ as $r \sim 1.2$ km. Thus, at the scales used in this study the point source approximation is generally appropriate for earthquakes with $M_L \leq 5$ ($P_0 \leq 1.6 \text{km}^2 \text{cm}$), and larger events are removed from the catalog.

Errors associated with calculation of the orientation part of the focal mechanism introduce the largest uncertainty into the potency tensor parameters. We therefore interpret the focal mechanism inversion stability as a measure of potency tensor quality and consider errors related to the measurement of $M_L$ and computation of $P_0$ as negligible. By applying the \textit{HASH} inversion to first motion data in a region where the faulting structures are well studied, Kilb & Hardebeck (2006) inferred that the most reliable estimates of quality are the focal mechanism probability (PROB), which records the proportion of acceptable solutions within $30^\circ$ of the preferred solution, and fault plane uncertainty (FPU), which records the RMS angular difference of acceptable solutions from the preferred solution. Both PROB and FPU are used in this study. According to the \textit{HASH} output, location errors are such that 98.5% of horizontal errors and 92% of vertical errors are smaller than 1 km. Since we do not perform spatial analyzes at scales smaller than 1 km, we do not relocate events. Further details about our potency tensor catalog, which contains 169,866 events, are provided in Table 3.1. Locations of the earthquakes are shown in Figure 3.1.

3.3 Analysis

3.3.1 Summed Potency Tensors

Quantities computed and physical interpretations

For a population of $N$ earthquakes, the combined potency tensor can be calculated by a potency tensor summation,

$$P_{ij}^{TOT} = \sum_{k=1}^{N} P_{ij}^{(k)}.$$  \hspace{1cm} (3.2)

The associated inelastic strain for a volume including the earthquakes, $V$, is given by $P_{ij}^{TOT}/(V)$, and the mean rate of deformation due to the earthquakes over a time period, $\Delta t$, is given by $P_{ij}^{TOT}/(V \Delta t)$. Kostrov (1974) defines these quantities in terms of $M_{ij}$, and uses a factor of $1/(2\mu)$, where $\mu$ is
Figure 3.1: Red dots show the locations of earthquake focal mechanisms used in this study. Major Faults are shown by black lines based on Jennings (1975). Blue rectangles denote sub-regions outlining distinct regions of seismicity which are used for spatial analysis of potency tensor summations: Owens Valley (OWV), Kern County (KERN), Los Angeles Basin (LA), Landers and Hector Mine rupture areas (LAN), San Andreas and San Jacinto junction in the San Bernadino Mountains (SBM), San Jacinto and Elsinore faults (SJ), and Salton Sea area of the San Andreas (SSAF). The 75 × 75 km region TRIF, marked by a black outline, is used for detailed spatial analyzes. Though not included in the analysis, hypocenters of notable large earthquakes are marked for reference.
Number of Events 169,866  
Source Data SCSN polarity data (available at http://www.scecdc.scec.org/STP/stp.html)  
Location Method SIMULPS (Thurber, 1983)  
Minimum No. Stations for inversion 5  
No. Inversions using S-wave arrivals 21,700  
No. Multiple Solutions Removed 30,646 (based on value of PROB)  
No. large events ($M_L > 5$) removed 56  
Time Period covered 01/01/1984 – 2/6/2003  
Longitude Range 113.96 – 121.66°W (∼ 690 km)  
Latitude Range 31.63 – 37.89°N (∼ 720 km)  
Depth Range 0 – 15 km  
$M_L$ Range 0.01 – 5.00 ($1.34 \times 10^{-5} < P_0 \leq 3.98 \times 10^1$ km² cm⁻¹)  
FPU Range 11 – 64°  
PROB Range 0.09 – 1.00  

Table 3.1: Summary of the earthquake data set used in this study, which contains 169,866 events. FPU and PROB correspond to measures of focal mechanism quality (Hardebeck & Shearer, 2002).

rigidity, to account for elastic properties of $V$. In a population with a wide range of earthquake sizes, $P_{ij}^{TOT}$ is likely to be dominated by the largest earthquakes, so advantages provided by a large number of small events, such as the lesser influence of outliers and greater spatial sampling of a region, are somewhat diminished. It is therefore also useful to examine summations of source mechanism tensors. The source mechanism summation is given by

$$P_{ij}^{SM} = \sum_{k=1}^{N} \hat{P}_{ij}^{(k)}.$$  \hspace{1cm} (3.3)

The summation in eq. 3.3 is not influenced by individual earthquake sizes, so an analysis of the two different normalized tensors, $\hat{P}_{ij}^{TOT} = P_{ij}^{TOT} / \| P_{ij}^{TOT} \|$ and $\hat{P}_{ij}^{SM} = P_{ij}^{SM} / \| P_{ij}^{SM} \|$, allows us to compare the orientation of the total earthquake potency with the average orientation properties of the population. Standard stress inversions (e.g., Angelier et al., 1982; Michael, 1987) weight earthquake focal mechanism data equally regardless of magnitude, and so are more analogous to $\hat{P}_{ij}^{SM}$ than $\hat{P}_{ij}^{TOT}$, though we emphasize that we are calculating the strain rather than inverting for the most likely stress.
The interpretation of a normalized summed potency tensor can be aided by separating its four remaining degrees of freedom into three describing the orientation of the principle strain axes and one describing the size of the CLVD component. The principle strain axes are given by the eigenvectors, \( e_1, e_2 \) and \( e_3 \), of \( P_{ij} \), which correspond to the maximum compressive (P) axis, the intermediate (B) axis and the maximum extensive (T) axis, respectively. Taking a convention of compression being negative, the corresponding eigenvalues are given by \( \lambda_1 \leq \lambda_2 \leq \lambda_3 \). For a DC potency tensor, \( \lambda_2 = 0 \), and as such the value of \( \lambda_2 \) relates directly to the size of the CLVD component (further details are given in Appendix B). In this study, we quantify the CLVD component by the ratio (Julian et al., 1998),

\[
r_{CLVD} = \frac{\sqrt{6}}{2} \lambda_2,
\]

which ranges between the two pure CLVD cases of \( r_{CLVD} = -0.5 \) and 0.5, via the pure DC case of \( r_{CLVD} = 0 \). The factor 2 is a matter of convention, since a pure CLVD mechanism may also be described by the summation of two separate DC components with orientations differing by 90°. Alternative quantification of the CLVD size for a normalized deviatoric tensor is given by the Gamma-index (Kagan & Knopoff, 1985),

\[
\Gamma = -(3/2) \sqrt{6} \lambda_1 \lambda_2 \lambda_3, \quad \text{or} \quad f_{CLVD} = -\lambda_2 / \max(\lambda_1, \lambda_3),
\]

(Giardini, 1984), which also range between \(-0.5\) and 0.5 and are compared to \( r_{CLVD} \) in Appendix C.

The orientation of the strain axes relates to the dominant direction of faulting, while the value of \( r_{CLVD} \) describes the nature of deformation that cannot be explained by the dominant fault orientation, thus indicating a degree of heterogeneity in DC orientations. Since the CLVD component will be zero for summations of DCs where (a) all B-axes are parallel, (b) all fault planes are parallel or (c) all slip vectors are parallel (Julian et al., 1998; Kagan, 2009), its existence indicates a departure from simple fault geometries. Furthermore, the CLVD component will tend to be zero for summations of random fluctuations about a dominant DC orientation, since variations in one direction will generally be canceled out by variations in the opposite sense (this is illustrated by simulations of uniformly random DC orientations in Appendix D). For example, if the dominant deformation is consistent with pure strike-slip faulting, equal second-order contributions of reverse and normal faulting will result in \( r_{CLVD} = 0 \). If the second-order contributions are not equal such that there is more potency
Figure 3.1: Results of (a) a normalized potency tensor and (b) a normalized source mechanism tensor based on summation for the entire southern California data set of $0 < M_L \leq 5$ earthquakes. These plots show lower hemisphere, equal area projections, such that orientations where the strain is extensive are colored and orientations where the strain is compressive are white. The orientations of the three principal strain axes are overlain as open symbols (P-axis = circle, B-axis = diamond and T-axis = square). Labels for the axes are shown next to each symbol. Beneath each plot, we display the value of $r_{CLVD}$ which quantifies the size of the CLVD component in each of the summed tensors.

released by normal faulting mechanisms than reverse faulting mechanisms $r_{CLVD} < 0$, while the opposite case leads to $r_{CLVD} > 0$. A CLVD component therefore requires the existence of fault heterogeneity as well as asymmetry within that heterogeneity. The specific fault heterogeneity that may lead to a given CLVD component is not uniquely defined by focal mechanism data since the slip direction also depends upon the details of the fault loading.

**Southern California summation results**

Strain axes orientations and the size of the CLVD component can be displayed by a “beachball plot”, as in Figure 3.1 for the two summations over the entire $0 < M_L \leq 5$ southern California data set. The downward pointing orientations of the principal strain axes are shown using open symbols, while the CLVD size is indicated by the departure of the plot from a pure DC, for which case the two white and non-white regions would intersect in the direction of the B axis.

In Figure 3.1, both plots show near vertical B axes close to the middle of the circle, while the horizontal P- and T-axes plot close to the edge. This implies a dominance of strike-slip faulting in both number of earthquakes ($\hat{P}_{ij}^{SM}$) and the combined potency release ($\hat{P}_{ij}^{TOT}$). From a given set of principle strain axes, an associated fault plane and auxiliary plane for the DC component are
described by normal vectors that are 45° between the P- and T-axes, which can be converted to fault strike and dip angles via simple geometrical relations (e.g., Stein & Wysession, 2003, p. 218). The corresponding slip direction can then be found by resolving the T-axis onto each of the possible fault planes. For \( \hat{P}_{ij}^{TOT} \) in Figure 3.1, the P- and T-axis orientations would be consistent with left-lateral slip on a fault with strike 56° and dip 87°, or right-lateral slip on a fault with strike −34° and dip 86°. Neither are consistent with the section of the right-lateral San Andreas Fault shown in Figure 3.1, which has a strike largely in the range ∼−67° to −61° before bending to ∼−45° in the northwest of the region. The left-lateral Garlock Fault has a more consistent strike in the range ∼57° to 90°. The axis orientations in Figure 3.1 are consistent with relative plate motion directions given by the NNR-NUVEL-1A model (DeMets et al., 1994), where in the southern California region the Pacific plate moves relative to North America with an azimuth in the range ∼−35° to −41°.

**Comparison of summed tensors**

To quantify the difference in orientation of the strain axes, we consider the minimum angle, \( \Omega \), required to rotate about any pole from one orientation of P-, B- and T-axes to another. Given axial symmetries, the maximum value of \( \Omega \) is 120°. We calculate \( \Omega \) using an algorithm outlined by Kagan (1991) and the relevant equations are given in Appendix E. This angle is called \( \Omega_{3d} \) by Frohlich and Davis (1999). For the two summation types shown in Figure 3.1, \( \Omega = 7.6° \), reflecting the small difference between the dominant faulting styles. The significance of this similarity, despite the difference in summation type, is highlighted in Figure 3.1, which shows that the varying contribution of earthquakes from different magnitude bins is related to a typical Gutenberg-Richter distribution. It is clear that the potency tensor summation, \( \hat{P}_{ij}^{TOT} \) is dominated by events in the largest (4 < \( M_L \) ≤ 5) magnitude bin. The source mechanism summation is based on normalized tensors, and hence dominated by more numerous events in the 1 < \( M_L \) ≤ 2 range. Based on inspection of the frequency-magnitude conformance to a Gutenberg-Richter distribution, the magnitude of completeness is close to 1 in the focal mechanism catalog, and earthquakes in the range 0 < \( M_L \) ≤ 1 are therefore under-represented.
Figure 3.1: Histograms of (a) the summed scalar potency and (b) the number of events for each magnitude bin using the entire data set.

While the strain axes orientations are similar for the two summation types in Figure 3.1, the CLVD components are different. For \( \hat{P}_{ij}^{SM} \), \( r_{CLVD} = -0.225 \) and the negative sign indicates compression in the direction of the B-axis. Since the B-axis is vertical, this implies that normal faulting mechanisms mix with the dominating strike-slip mechanisms in the earthquake population. For \( \hat{P}_{ij}^{TOT} \), \( r_{CLVD} = 0.050 \), indicating a slightly larger contribution of reverse faulting than normal faulting in the background of the overall strike-slip deformation. We compute uncertainties for the values of \( r_{CLVD} \) by bootstrap re-sampling of the summed tensor, using the method described by Press et al. (1992), p. 691. We resample each summed tensor and compute \( r_{CLVD} \) 10,000 times to produce a density distribution for \( r_{CLVD} \) and from this calculate the 95% confidence interval for the quantity. In the case of \( \hat{P}_{ij}^{SM} \), the confidence limits are \([-0.230, -0.220]\), implying a robust CLVD component. For \( \hat{P}_{ij}^{TOT} \), the 95% confidence limits are \([-0.028, 0.128]\), indicating that there is no strong evidence that \( \hat{P}_{ij}^{TOT} \) is different from a pure DC mechanism. With regard to the result for \( \hat{P}_{ij}^{SM} \), summations of simulated random DC orientations in Appendix D indicate that \( r_{CLVD} > |0.225| \) in \( \sim 34\% \) of cases, but random fluctuations cannot explain the consistent strain axes orientations for the two summations. If we consider both CLVD results to be non-random, the discrepancy implies that the crustal thinning caused by a large number of normal faulting earthquakes in southern California has been canceled out by reverse faulting earthquakes which are less common but have higher magnitudes.
As will be shown in Section 3.3.3, the majority of this reverse fault component exists in the Los Angeles Basin.

The results of the bootstrap analyzes in this and subsequent sections suggest uncertainties that are larger for $\hat{P}_{ij}^{TOT}$ than for $\hat{P}_{ij}^{SM}$. This is due to the sensitivity of $\hat{P}_{ij}^{TOT}$ to a small number of large earthquakes within the population. The bootstrap analysis does not take into account the variation in quality of individual mechanisms, and we address this in separate quality analyzes (Section 3.3.5).

We can isolate the total geometrical differences between two normalized tensors, $\hat{P}_{ij}^{A}$ and $\hat{P}_{ij}^{B}$, using the angular measure (Frohlich & Davis, 1999; Sipkin & Silver, 2003),

$$\Theta = \cos^{-1}(\hat{P}_{ij}^{A} \hat{P}_{ij}^{B})$$

(3.1)

which ranges from 0 to 180° and is called $\Omega_{9v}$ by Frohlich & Davis (1999). This angle does not correspond to $\Omega$, which measures the difference in strain axes orientation, since it is defined in the space of normalized 9-component tensors, and hence is affected by the size of the CLVD components in both tensors. For two summed tensors with the same orientation of strain axes but $r_{CLVD}$ values of 0 and 0.5, $\Theta = 30^\circ$. For the summations shown in Figure 3.1, $\Theta = 18^\circ$.

We compute uncertainty bounds for both of the difference angles by simultaneous bootstrap resampling of the two summed tensors, computing $\Omega$ and $\Theta$ between a re-sampled $\hat{P}_{ij}^{A}$ and a re-sampled $\hat{P}_{ij}^{B}$ 10,000 times. As in the bootstrap analysis of $r_{CLVD}$, we compute 95% confidence intervals based on the resulting distribution of both angles. This analysis gives a range of 4.7–12.0° for $\Omega$ and 14.7–22.0° for $\Theta$. By comparison, a bootstrap analysis of $\hat{P}_{ij}^{TOT}$ for southern California indicates that 95% of the resampled summed tensors are within $\Omega = 5.0^\circ$ and $\Theta = 7.1^\circ$ of each other. Hence compared to the uncertainty of the summation results, the difference between $\hat{P}_{ij}^{TOT}$ and $\hat{P}_{ij}^{SM}$ is not that large in terms of orientation differences given by $\Omega$, but it is in terms of $\Theta$ where the CLVD components are taken into account.

### 3.3.2 Summations Over Different Magnitude Bins

We investigate partitioning of the overall potency release into earthquakes of different magnitudes by separating the catalog into five magnitude subsets, using the ranges: $0 < M_L \leq 1, 1 < M_L \leq 45$
2, ..., 4 < M_L ≤ 5, and computing \( \hat{P}_{ij}^{TOT} \) and \( \hat{P}_{ij}^{SM} \) for each. Figure 3.1 displays \( \hat{P}_{ij}^{TOT} \) and \( \hat{P}_{ij}^{SM} \) for each magnitude bin of the entire catalog, using the same representation as in Figure 3.1. As suggested by the comparison of \( \hat{P}_{ij}^{TOT} \) with \( \hat{P}_{ij}^{SM} \) for the entire catalog, we find that the orientations of the strain axes are very similar for different magnitude ranges (Figure 3.1a). We have tested this in more detail by computing \( \Omega \) between all pairs of tensors for magnitude bins, and results are given in Appendix F. Based on 95% confidence limits given by bootstrap analyzes, we find no evidence for principle strain axes orientation differences greater than \( 7^\circ \).

The CLVD component is larger and increasingly negative for lower magnitude bins (Figure 3.1b) in both summation types. Given the constant strain axis orientations which imply strike-slip dominance, this suggests a tendency towards an increased normal faulting/vertical thinning component in smaller earthquakes. The values of \( r_{CLVD} \) suggest a change in the CLVD component with magnitude that is steady and outside of the 95% confidence limits from bootstrap analysis for any of the magnitude bins. The largest value of \( r_{CLVD} \) is 0.32 for the 0 < M_L ≤ 1 bin, and although such a CLVD component was generated by \( \sim 20\% \) of random simulations, we would expect more variation in the principle axes orientations if the effect was due to randomness. Computation of \( \Theta \) between all pairs of tensors for magnitude bins and comparison of 95% confidence limits indicate robust differences between magnitude bins of up to \( 20^\circ \) (Appendix F). This is in contrast to the same comparisons based on the angle \( \Omega \), indicating that taking the CLVD component into account is necessary to show robust differences between summed tensors of different magnitude bins.

The persistence in the orientations of principle strain axes indicates that dominant faulting is the same for all earthquakes, but the CLVD differences highlight that the tensors are not self-similar and imply that the CLVD-related heterogeneity is greater for smaller earthquake populations.

### 3.3.3 Summations Over Different Tectonic Regions

In order to understand how the potency release of different fault zones contributes to the overall potency release, we compute \( P_{ij}^{TOT} \) and \( P_{ij}^{SM} \) for seven subregions outlined by polygons with dimensions \( \sim 50 - 250 \text{ km} \) (Figure 3.1). These polygons are subjectively chosen to outline regions of high seismicity and recognized faulting structures while minimizing the total number of regions used.
In the following, we provide a brief qualitative summary of each region, though more quantitative analysis of the relative number and potency release of each region, as well as comparisons of the temporal seismic behavior are given in Appendix G. The comparisons in the Appendix are removed from the strain-based focus of this study, but provide further support for distinct differences between the seismic behavior of such large-scale tectonic domains.

Region SJ is largely dominated by seismicity associated with the San Jacinto and Elsinore Faults, both of which are NW trending strike-slip faults that may be considered complex due to their many branches, step-overs and gaps. The region contains one small aftershock sequence associated
with the 1987 Superstition Hills earthquake, but is otherwise dominated by a fairly constant level of seismicity that conforms to a Gutenberg-Richter distribution. Region SSAF contains the most southern extent of the San Andreas Fault as well as the pull-apart basin of the Salton Sea. It is the smallest region considered and contributes the smallest amount of potency release and number of earthquakes to the southern California summed tensors in Section 3.1. Aftershocks of the 1987 Superstition Hills earthquake are present in both regions SJ and SSAF. Region LA includes the Los Angeles Basin (dominantly compressional reverse faulting) as well as some of the surrounding seismicity (both left- and right-lateral strike-slip faults). Potency release in this region is dominated by aftershocks of the 1994 Northridge earthquake, which contribute relatively more to the southern California total potency release than to the total number of events. Region SBM includes the San Bernardino mountains, where the San Andreas fault bends to a more westerly strike and intersects the northern extent of the San Jacinto fault. Seismicity in the northeast of this region is dominated by aftershocks of the 1994 Big Bear earthquake. Region LAN is almost entirely dominated by aftershocks of the 1992 Landers and 1999 Hector Mine earthquakes, the two largest events occurring in the region during the time period of our catalog. These aftershock sequences result in the region being the largest contributor of potency release to the entire catalog. Both mainshocks may be considered as complex ruptures due to the multiple faults that sustained slip, and these features are also represented in the aftershocks. Region KERN includes a large component of seismicity where faults traces are not mapped in the Kern County/Lake Isabella area, which includes the southern edge of the Sierra Nevada. Hydrothermal activity is present in the region, which may relate to the large number of small normal faulting earthquakes that contribute little in terms of overall potency release. Region OWV includes the southern part of Owens Valley and the Coso geothermal region. Owens valley is bounded by large normal faults, but also contains a large strike-slip fault. The large amount of microseismic activity means that the region contributes more earthquakes to the catalog than any other region, while the small size of these earthquakes leads to a total potency contribution that is only the fourth largest.

Figure 3.1a shows the results of both potency tensor summation types for each of the seven regional subsets of our catalog. In all results except $P^T_{ij}$ for region LA, the B-axis is the most
vertical axis, indicating a dominance of strike-slip deformation at these ~ 50–250 km scales. Despite the consistent style of faulting, variations in the azimuth of P- and T-axes lead to distinct differences in the strain-axes orientations between different regions. We quantify these differences and show that they are robust by computing Ω for all pairs of regionally summed tensors as well as 95% confidence intervals in Appendix F. From a visual comparison with the regional fault map, the P- and T-axis orientations are consistently close to 45° from the dominant fault strikes of each region, but rarely close to the orientations for the southern California summed tensors in Figure 3.1. This indicates that for these regions, the dominant length scales underpinning the deformation have shifted from plate boundary scales to those of the largest nearby fault structure.

Figure 3.1b shows that the nature of the CLVD component also varies strongly from region to region, indicating differences in the nature of fault heterogeneity within them. All regions except SBM have a larger absolute CLVD component than that of the entire region for the source mechanism summations, and in general the absolute \( r_{CLVD} \) value is smaller for \( \hat{P}_{ij}^{TOT} \) than \( \hat{P}_{ij}^{SM} \). Of the different regions, LA is the only case where \( r_{CLVD} \) is positive, implying that the small positive CLVD component of \( \hat{P}_{ij}^{TOT} \) for the entire southern California region results largely from reverse faulting in the Los Angeles region. The other regions have a greater tendency for normal faulting, though the larger \( |r_{CLVD}| \) values for \( \hat{P}_{ij}^{SM} \) suggest that this tendency is greater for smaller magnitude earthquakes. In general, the CLVD components are highest in the regions where the axes orientations are most different from those of southern California as a whole (i.e., KERN and LA).

### 3.3.4 Magnitude Bins for Tectonic Regions

In order to unite the results for magnitude and regional binning, we further subdivide each of the regions using the four largest magnitude bins. Results for the range \( 0 < M_L \leq 1 \) are omitted because of strong spatial variations in catalog completeness at these magnitudes.

Figure 3.2 displays results associated with \( \hat{P}_{ij}^{TOT} \) and \( \hat{P}_{ij}^{SM} \) for different magnitude bins in each of the seven regions. In order to overlay several results we do not plot the beachballs, but the same information is provided in terms of (a) the strain axis orientations using different sized symbols for different magnitude bins, and (b) the values for \( r_{CLVD} \). The results complement those for the
Figure 3.1: (a) Summations of potency tensors and source mechanism tensors for seven tectonically defined regions, using earthquakes in the range $0 < M_L \leq 5$. The radii of the beachball plots are equal, such that these represent the normalized tensors, $\hat{P}_{ij}^{TOT}$ and $\hat{P}_{ij}^{SM}$ for each bin. P-, B- and T-axis orientations are overlain as in Figure 3.1. (b) Values of $r_{CLVP}$ for the different regions. Error bars show the 95% confidence limits predicted by bootstrap analysis. For the source mechanism summations, these limits are all smaller than the symbol size.
magnitude separation of the entire catalog (Figure 3.1), showing that the strain axis orientations in each region are closely aligned over multiple magnitude ranges, while the CLVD component tends to become larger for summations over populations of smaller earthquakes. Region KERN appears to show unstable B- and P-axis orientations, but this can be related to the large negative value of $r_{CLVD}$, which means that there is little difference between the size of the intermediate and compressive strains. In order to define the orientation of a pure CLVD mechanism with $r_{CLVD} = -0.5$, only the T-axis direction is necessary, which in this case remains stable. A similar effect is shown in the LA region, where the P-axis remains stable while the T- and B-axis orientations switch for the $1 < M_L \leq 2$.

### 3.3.5 Quality-Based Analysis

We address the effects of variable data quality by repeating the magnitude and regional partitioning for catalogs with different quality restrictions. In the HASH algorithm, A, B, and C quality mechanisms are defined as those with FPU less than or equal to $25^\circ$, $35^\circ$, and $45^\circ$, respectively, where stations used for computation of all mechanisms must have a maximum azimuthal gap less than $90^\circ$ and maximum takeoff angle gap less than $60^\circ$. However, applying only the azimuthal and takeoff angle gap restrictions reduces the number of focal mechanisms to $\sim 24,000$ ($\sim 15\%$ of the number of data used in this study). We neglect these restrictions in our study and define quality entirely based on FPU. We define five levels of increasingly higher quality by imposing cutoff values for the maximum acceptable FPU: $42^\circ$, $35^\circ$, $31^\circ$, $28^\circ$, and $24^\circ$, chosen to return the best constrained $25\%$, $10\%$, $5\%$, $2.5\%$, and $1\%$ of the data, respectively. The preference of higher quality focal mechanisms leads to biases toward earthquakes of larger magnitude and those in well instrumented regions because they are recorded by more seismic stations. Since this study investigates the effect of magnitude and location-based selection upon summed tensors, we can expect these quality-related biases to influence our results. Because of this we do not necessarily interpret the results from higher quality data as showing the true nature of the deformation, but identify the changes in results due to step-by-step increases in quality, using these changes to indicate possible
Figure 3.2: (a) Equal area projections showing orientation of the principle strain axes (different shapes) for summed tensors in different magnitude bins (different symbol sizes). (b) $r_{CLVD}$ over different magnitude ranges for each of the seven regions (different colors). Dashed lines show the 95% confidence intervals for $r_{CLVD}$ in each of the summations based on a bootstrap analysis.
artifacts. To better constrain the effects of earthquake magnitude biases in the summations, we concentrate our analyzes for higher quality catalogs on the results for source mechanism summations. We also impose a requirement of at least 30 events for each summation to retain reasonably robust estimates of average properties.

Figure 3.3 shows results for the southern California source mechanism summations of Section 3.3.2 using magnitude bins with increasingly higher quality restrictions. The results are displayed in terms of (a) the axis orientations and (b) $r_{CLVD}$ for bins in the largest four magnitude ranges, where colors represent different quality levels. The orientations are most stable for the $4 < M_L \leq 5$ subset, but in general, orientations for all magnitudes are similar until the highest quality restriction (FPU $\leq 24^\circ$) is applied. Out of the three axes, the B-axis and T-axis orientations vary more strongly than the P-axis, which relates to a CLVD component that becomes more positive with higher quality in Figure 3.3b. The values of $r_{CLVD}$ show that the correlation between magnitude and negative CLVD component becomes less significant when intermediate quality restrictions are applied, disappearing when FPU $\leq 31^\circ$ and switching to become strongly positive for higher quality data sets. The results of Section 3.3.3 indicate that most reverse faulting is concentrated in LA, which is a well instrumented region, implying that earthquakes from this region are preferentially sampled with the higher quality restrictions. It is therefore unclear if the disappearing trend in $r_{CLVD}$ for intermediate quality restrictions is due to a growing dominance of LA data, or if the correlation between magnitude and CLVD component is an artifact of low quality data. However, given the subsequent results for regions SJ and SSAF where correlations are preserved at higher quality, we favor the former interpretation.

Application of the same quality analysis to the seven tectonic regions reproduces similar results in that the CLVD values are more sensitive than the principle strain axis orientations to data quality. In Figure 3.4 we show the values of $r_{CLVD}$ when these restrictions are applied to four of the regions: LA, SBM, SJ and OWV (LAN and SSAF are not well sampled at lower magnitudes, KERN is not at high magnitudes). The black lines reproduce the results for minimal quality restrictions shown in Figure 3.2 while colored lines correspond to the same quality restrictions used in Figure 3.3. The quality restrictions have different effects in the different regions, but these are likely to result from
the strong sensitivity of the CL VD component to spatial sampling rather than an increased number of low quality data. For the SJ and SBM cases, the CL VD trend is largely preserved for higher quality catalogs, indicating no evidence for an explanation based on catalog uncertainties. In the OWV case, the CL VD component remains large, but the correlation with magnitude becomes less prominent as quality increases. In the LA case, summations based on higher quality data show a smaller CL VD component for all magnitudes. Combined, these results are ambiguous in terms of the relationship between the CL VD component and data quality, though there is no clear correlation of our CL VD observations being explained by data uncertainty. It is likely that reverse faulting in the Los Angeles Basin is better sampled than the surrounding strike-slip faulting, and higher quality catalogs are simply reflecting different spatial biases.

3.3.6 Gridded Tensor Summations for 1 km-Scale Spatial Analysis

Methods

In a second approach to spatial analysis, we investigate source mechanism summations at a 1 km resolution for two regions. The first is the trifurcation region of the San Jacinto fault in the San Jacinto Mountains (TRIF in Figure 3.1). We define a $75 \times 75$ km square to encompass the region and compute the summed tensor $\hat{P}_{ij}^{SM}$ for $1 \text{ km}^2$ grid cells that contain earthquakes in our catalog. We choose this region because the relatively dense network coverage has led to a high number ($N = 12,708$) of data, including a large number of small events and thus wide spatial sampling. The grid cells are defined by choosing the most southwesterly point of the region and denoting boundaries at 1 km intervals to the north and east. We choose 1 km as a scale, since it is larger than 98% of horizontal location errors within the catalog. The depth extent of each grid cell is set uniformly to 15 km, such that the volume of crust is approximately equal for all cells. The second region is SBM in Figure 3.1, which has dimensions $140 \times 90$ km and is centered over the intersection of the San Jacinto and San Andreas faults in the San Bernadino Mountains. A horizontal grid is defined in the same way as for TRIF, though the grid itself is rotated anti-clockwise by $10^\circ$ from a north-south, east-west orientation. This second region provides a useful comparison, since it
Figure 3.3: (a) Orientations of P (circle), B (diamond) and T (square) axes of $\hat{P}_{SM}^{ij}$ for the magnitude bins (different stereonets) of the entire catalog when different quality restrictions (different colors) are imposed on the data. (b) $r_{CLVD}$ for the source mechanism summations over different magnitude bins for different quality thresholds. Results are only displayed for summations where the number of events is greater than 30.

is larger than TRIF and includes data that sample a greater diversity of faulting structures as well as including the 1992 Big Bear aftershock sequence.

We neglect the depth information of earthquakes for both regions in order to simplify the analysis and retain a reasonable number of events in grid cells. As the regions are both strike-slip dominated regimes, we assume that faults have a greater influence on horizontal variations in the earthquake orientations, and combining the entire range of depths increases the number of data we can use. We relate our results to fault geometries using the map of Jennings (1975).

We allocate earthquakes to each grid cell using two separate techniques. In the first, referred to as a “gridded summation”, we compute $\hat{P}_{SM}^{ij}$ based on earthquakes within the grid cell boundaries.
Figure 3.4: $r_{CLVD}$ for the normalized summations over different magnitude bins, given different quality thresholds. Line colors are the same as for Figure 3.3. The lowest quality threshold (black line) is equivalent to no quality restrictions, as shown in Figure 3.2. Results are only displayed where the number of events is greater than 30. (a) Region LA. (b) Region SBM. (c) Region SJ. (d) Region OWV.

At these scales, the number and quality of data in each grid cell can vary significantly, and we wish to assess the robustness of results by taking this variation into account. We therefore consider a second “quality-adaptive summation” method, where rather than including only the $N_{cell}$ earthquakes located within the grid cell, we include the nearest $N$ events to the center of the grid cell. We define $N$ such that $N \geq N_{cell}$ and the summed quality, $\sum_{i=1}^{N} q_i$, is greater than 15, where $q_i$ is given by the HASH measure PROB. Hence each summation is based on at least 15 tensors. We choose 15 as a cutoff since this leads to most summations being based on $\sim 30$ events which is a typical number used to validate asymptotic behavior in statistical studies. The quality adaptive summation cannot be interpreted on its own, since the method has the effect of smoothing out spatial variations in areas
of sparse data. This may not represent true variations, but it allows us to understand how much resolution is allowed by the data for a given location and assess the robustness of the gridded summation results.

For these results where we compare large number of tensor summations, it becomes impractical to use the beachball plots to display summed tensor properties. We therefore describe the orientation of the principle strain axes using the horizontal projection of the P-axis and a measure of faulting style, $\gamma_{FS}$, that quantifies which of the principle strain axes is most vertical. The size of the CLVD component in each grid cell is represented by the value of $r_{CLVD}$.

The dominant faulting style may be described by a function of the vertical components of the three eigenvectors,

$$
\gamma_{FS} = \arctan\left(\frac{|e_{33}| - |e_{13}|}{\sqrt{2}|e_{23}|}\right),
$$

(3.1)

where $e_{ij}$ refers to the vertical component of the $i^{th}$ eigenvector, as defined in Section 3.3.1. The angle $\gamma_{FS}$ ranges from $-90^\circ$ to $90^\circ$, and in the case of a DC mechanism it returns the rake angle. If $\gamma_{FS} = -90^\circ$, $|e_{13}| = 1$ and $e_1$ must be vertical so the tensor corresponds to a dominance of normal faulting. If $\gamma_{FS} = 90^\circ$, $e_3$ must be vertical and the tensor corresponds to a dominance of reverse faulting. If $\gamma_{FS} = 0$, it is either because $e_2$ is vertical or because $e_1$ and $e_3$ have the same plunge, which implies a dominance of strike-slip faulting. Because of the fault plane ambiguity, no distinction can be made about right-lateral vs. left-lateral faulting based on this parameter.

**Results for TRIF**

Results for gridded and quality adaptive summations across region TRIF using the entire magnitude range are shown in Figure 3.1. Fault traces are shown by light gray lines and the fault trifurcation itself is indicated by a black box in Figure 3.1b. Each grid cell in Figure 3.1a is colored according to the value of $\gamma_{FS}$ (red for dominant reverse faulting, yellow for strike-slip, blue for normal), and in every second grid cell we display the horizontal projection of the P-axis using black bars. In Figure 3.1b, we display the associated value of $r_{CLVD}$ for the summed tensor in grid cells with at least 3 events, since the distribution of $r_{CLVD}$ in our random simulations (Appendix D) is the same for $N \geq 3$. Corresponding results based on quality-adaptive summations are shown in Figures 3.1c and d.
The large yellow regions in Figures 3.1a and c highlight a dominance of strike-slip faulting ($\gamma_{FS} \sim 0$) over the region, consistent with the overall deformation. We also observe grid cells with dominance of both reverse ($\gamma_{FS} \sim 90^\circ$) and normal faulting ($\gamma_{FS} \sim -90^\circ$), and in Figure 3.1a these two extremes exist in adjacent grid cells at certain locations. However, the quality-adaptive summations in Figure 3.1c allow us to show that much of this apparent spatial heterogeneity is not robust given the number and quality of data. Similar heterogeneity in the P-axis azimuths is strongly reduced by the quality-adaptive summation, such that most variation is within a few degrees of the north-south trend shown by the San Jacinto summed tensor in Figure 3.2. Departures from the background strain axes orientation appear to be robust in a number of locations; most notably for an ~8 km region to the southwest of the fault trifurcation where we find that normal faulting dominates. This anomaly may relate to the complexity of the nearby trifurcation or to its position within a step-over, though the right-lateral nature of the strike-slip faults would imply compression in such a step-over. We also observe several smaller-scale regions with tendencies towards normal faulting, and five or six regions where the P-axis azimuth undergoes distinct orientation changes. From visual inspection, many of the changes in azimuth orientation correspond to nearby changes in the fault geometries, indicating that faults locally exert control over earthquake orientations.

We find an apparent correspondence between regions where the summed tensor axis orientations differ from the background in Figures 3.1a and c, and locations of non-zero values of $r_{CLVD}$ in Figures 3.1b and d. This suggests that the CLVD component becomes more significant in localized regions where the faulting style deviates from the regionally dominant sense of deformation.

**Results for TRIF magnitude bins**

Figures 3.2 and 3.3 exhibit results of both gridded summations and quality-adaptive summations for magnitude subsets of region TRIF in terms of $\gamma_{FS}$ and $r_{CLVD}$. In Figure 3.2 the results are presented for grid cells that contain at least one event, while in Figure 3.3 the results are only displayed for grid cells with at least three events. The lack of data in the upper magnitude ranges means that we only display results for $0 < M_L \leq 4$ in Figure 3.2 and $0 < M_L \leq 3$ in Figure 3.3. Beachball plots
Gridded Summations:

(a) 

![Image of gridded summations](image)

(b) 

![Image of gridded summations](image)

Quality Adaptive Summations:

(c) 

![Image of quality adaptive summations](image)

(d) 

![Image of quality adaptive summations](image)

Figure 3.1: Results of gridded tensor summations for region TRIF (see Figure 3.1) based on a $1 \times 1$ km horizontally defined grid, displayed in terms of (a) the dominant faulting style, $\gamma_{FS}$, of each grid cell and (b) the value of $r_{CLVD}$ in those cells with at least three events. The quality-adaptive summation results for the same data are shown in (c) and (d). For the parameter $\gamma_{FS}$, red corresponds to dominant reverse faulting, yellow to strike-slip and blue to normal faulting. The black rectangle in (b) indicates the location of the San Jacinto Fault trifurcation referred to in the text. Red polygons in (c) indicate areas where the strain axis orientations of grid cell summed tensors persistently differ from the orientations of the regional summed tensor.

showing the results of $\hat{P}_{ij}^{SM}$ and the associated values of $r_{CLVD}$ for each magnitude range over the entire region are also displayed in Figure 3.2.

We see no clear differences between spatial patterns of the strain axis orientations for different magnitude bins, such that the robust non-strike-slip features observed in Figure 3.1 are found for all magnitude ranges when data exist in those locations. This implies that persistent length-scales control earthquake behavior regardless of magnitude over a range of at least four magnitude units.
Most of these persistent length-scales are robust according to the quality-adaptive summation except in the range $3 < M_L \leq 4$, where the number of data is not sufficient to robustly image any local features shown by the gridded summation. However, the persistence of these features at multiple magnitude levels indicates that they are not a result of random noise, and this highlights the problem of small data sets when studies are restricted to larger earthquakes. The major difference between data from different magnitude ranges is shown to be the spatial coverage, and this feature affects which faulting structures are sampled by the regional summations in previous sections. Events in all magnitude ranges appear most likely to occur close to the dominating fault structures, but the greater likelihood of imaging off fault structures is provided by earthquakes in the $1 < M_L \leq 2$ range.

In Figure 3.3, we observe that the spatial variation in $r_{CLVD}$ of Figure 3.1 is broadly reproduced at all three magnitude levels. The histograms plotted in Figure 3.3a show the distribution of $r_{CLVD}$ across the grid cells for each magnitude range. The distributions are distinctly skewed to the left, reflecting the negative values for $r_{CLVD}$ of the regionally summed tensors of Figure 3.2. However, noticeable peaks at $r_{CLVD} = 0$ show that for a large proportion of the grid cells, the source mechanism summation is close to a DC. The histograms demonstrate narrower distributions of $r_{CLVD}$ for higher magnitude, correlating with the smaller values of $r_{CLVD}$ given in Figure 3.2. This may be explained by decreased sampling of more complex structures with populations of larger earthquakes. However, the number of data in grid cells is relatively small and the histograms may alternatively reflect increased uncertainties in focal mechanism data for lower magnitude populations.

Results for SBM magnitude bins

In the case of region SBM, the network does not provide enough spatial coverage for us to analyze the lowest magnitude range and we therefore show only results for $1 < M_L \leq 4$ (Figures 3.4 and 3.5). The dominant potency release for region SBM (Figure 3.1) has a near vertical B-axis (i.e. strike-slip faulting) and north-south P-axis orientations corresponding to the strike of the San Andreas Fault in the region, despite a lack of seismicity there. This is reflected in Figure 3.4 by the large yellow colored regions and generally north-south trending azimuth bars. Most of the region conforms to this pattern, showing that the SBM summed tensors in Figure 3.1 reflect spatial coverage that extends...
Figure 3.2: The (a) gridded summation and (b) quality adaptive summation results of $P_{ij}^{SM}$ for region TRIF over four magnitude bins displayed in terms of dominant fault style (colors) and P-axis azimuths (bars). Source mechanism summations for all earthquakes in the region are shown by beachball plots and $r_{CLVD}$ values to the right.
Figure 3.3: Values of $r_{CLVD}$ computed from those gridded summations used for Figure 3.2 that include at least three events. (a) Histograms showing $r_{CLVD}$ distributions for the grids ($N$ is the number of grid cells), (b) gridded summation results, and (c) quality adaptive summation results for three magnitude bins.

Beyond that of the aftershocks of the Big Bear earthquake. As observed for region TRIF, we find that a high degree of spatial heterogeneity indicated by the gridded summations is greatly reduced by the quality-adaptive summations. However, we identify three robust regions of non-strike-slip faulting with fixed lengths of $\sim 20–50$ km (shown by red boxes in Figure 3.4a) which correspond to fault kinks or fault intersections. As well as changes in fault style, the P-axis azimuths highlight distinct regions of changing fault orientations, which are in most cases in agreement to the fault traces displayed on the map.
Figure 3.4: The (a) gridded summation and (b) quality adaptive summation results of $P_{SM}$ for region SBM over the middle three magnitude bins, displayed in terms of dominant fault style (colors) and P-axis azimuths (bars). The black rectangle in the $1 < M_L \leq 2$ range of (b) highlights the region dominated by aftershocks of the 1992 Big Bear earthquake. The red rectangles in the $1 < M_L \leq 2$ range of (a) highlight the robust persistent features of non-strike-slip dominated faulting.

Figure 3.5 shows that regions of non-strike-slip faulting correspond to regions of high CLVD component, as observed for the TRIF region (Figure 3.3). Histograms of the $r_{CLVD}$ values for grid cells also show strong peaks close to $r_{CLVD} = 0$ for all magnitude ranges, and broadening distributions for lower magnitude ranges. The trend for an increasingly negative CLVD component in Figures 3.2b and 3.4b in lower magnitude data sets is reflected by the broadening of histograms in Figure 3.5 and a slight increase in asymmetry of the distribution.
Figure 3.5: Values of $r_{CLVD}$ are computed from those gridded summations used for Figure 3.4 that include at least three events. (a) Histograms of $r_{CLVD}$ for the number of grid cells in the gridded summation, (b) gridded summation results, and (c) quality adaptive summation results for three magnitude bins.

### 3.4 Discussion

For nearly all spatial scales and regions considered, we find that the orientations of the principle strain axes for summed potency tensors are independent of the magnitude range considered, regardless of the dominant faulting style and temporal nature of seismicity. This observation agrees with the results for other regions of Fischer & Jordan (1991), Amelung & King (1997), and Sheridan (1997). Unlike Amelung & King (1997) we do not consider this to be evidence of a self-similar deformation process, but rather a magnitude-independent response of earthquakes to dominant types of deformation and associated length-scales that are stationary over the time period considered.
At the largest scale (∼700 km), the alignment of earthquake orientations with plate motion directions suggests that the length-scales of plate tectonics influence the dominant earthquake behavior. At the scale of our seven tectonic regions (∼50 – 250 km), the orientations of summed tensors are more consistent with the dominant strike of faults in the region than with the plate motion direction, suggesting that the relevant length-scales are those of the largest faults. At the smallest scale considered, our 1 km spatial grid analyzes indicate that most of the potency release within a region is consistent with the largest faults, although there are clear departures over smaller persistent length-scales with a range of sizes (∼5 – 50 km). We may physically interpret the largest length-scale as the overriding response of the whole region to plate loading, the second-order length-scales as the large fault-zones characterized by persistent dominating fault orientations (e.g., Becker et al., 2005), and the third-order length-scales as the geometrical complexities within those fault zones.

When taking the CLVD components into account, we find clear differences between summed tensors for different magnitude and spatial bins. These differences indicate a lack of self-similarity in the faulting structures/length-scales that influence seismic behavior. Out of the seven tectonic regions, the CLVD components are highest in regions KERN and LA, where the principle strain axis orientations are most different from those of the southern California summations. These regions have a significant component of normal and reverse faulting, respectively, and the CLVD components indicate that these faulting styles mix with the strike-slip style that dominates the southern California region. We therefore suggest that the CLVD components represent heterogeneity that is a function of the difference between the orientation of dominant fault structures and the orientation of the overlying plate motion directions. The same interpretation may be applied to observations of CLVD components at smaller scales.

Within regions TRIF and SBM, most of the grid cells have small CLVD components and axis orientations consistent with the region’s summed tensor. Areas where the CLVD component is large are localized and correspond to locations where the axis orientations are different. This indicates a mixing of earthquake slip corresponding to local faulting structures with slip corresponding to the dominant orientation of the fault zone, plate boundary or both. Such mixing may relate to complexities in the stress field generated by geometrical features of the fault zone. At the scale
of these geometrical complexities, which we interpret as third-order length scales, the nature of
summed tensors are therefore more heterogeneous as they include responses to the plate boundary
scale stress, the dominant fault zone orientation as well as very local orientations of stress and
faulting. Since there is some non-uniqueness in the heterogeneity of faulting that may generate a
CLVD component, it is unclear from the analysis whether this heterogeneity reflects individual faults
that are rougher than the dominant faults, or whether deformation is accommodated by a set of faults
with highly varied orientations. However, accounting for a number of small length-scales associated
with heterogeneity of strain axis orientations can explain the CLVD components associated with
large fault zones and hence the geometrical differences between them.

The analysis of regions TRIF and SBM also allows us to provide an explanation for the apparent
magnitude-dependence of CLVD components. The results for specific grid cell localities over
different magnitude ranges do not show much variation, but the number of grid cells containing
earthquakes, and hence amount of spatial sampling in the analysis, is much larger for smaller earth-
quakes. As noted above, most grid cells have a small CLVD component, while high CLVD com-
ponents are concentrated in small localized areas of heterogeneity. As smaller magnitude ranges
are considered, the spatial sampling increases to include more of these areas, increasing the CLVD
component for the overall summed tensor. We may ask whether these localities are not represented
in populations of large earthquakes because there are not enough data or because there are physical
limitations to the occurrence of large events there. A physical limitation is plausible since larger
and smoother potential rupture surfaces will aid the chances of propagation over large distances
to generate larger magnitude earthquakes. If we consider the faulting structures of the localities
with enhanced heterogeneities to be immature relative to the larger fault zones, this ties together
with the interpretation of Ben-Zion & Sammis (2003) that faults evolve from complex to homoge-
neous structures. However, further investigation of this point is hindered by the lack of seismicity
on the San Andreas Fault which is the region’s most mature fault. The focal mechanism analysis
of Hardebeck (2006) found that within the data uncertainty, earthquakes generally behave homoge-
neously over \( \sim 10 \) km scales. Our findings support this observation, since most small earthquakes
occur in regions of relative homogeneity. We further find that the level and nature of heterogeneity is not always the same, and heterogeneous behavior can also be localized at a $\sim 10$ km scale, though a range of scales are observed. These heterogeneous regions are better sampled by smaller earthquakes.

As highlighted by the study of Hardebeck (2006), there can be considerable ambiguity between true earthquake heterogeneity and uncertainty within the data. Our analysis of successively higher quality catalogs shows that differences between regional CLVD components are robust, and artifacts due to low quality data do not explain our observations. However, we also found that values of $r_{CLVD}$ are more sensitive to data quality than the orientations of the principle strain axes, implying that the CLVD component is the least well constrained component of the summed tensors. This may be explained in view of our interpretations that regional CLVD components are generated by small localized structures within the regions. Focal mechanism quality is largely dependent on the station distribution with respect to the earthquake location (Kilb & Hardebeck, 2006), and hence this strong spatial aspect will affect how many of the CLVD contributing localities are sampled by data, given a certain quality cutoff. The quality dependence of the CLVD component therefore supports our interpretations of spatial variation in the heterogeneity of structures. Analysis of likely biases that considers the exact distribution of the employed seismic stations would provide a more complete understanding of the effects of data quality.

Our results and interpretations have implications for the study of regional strain partitioning, scale-independent aspects of deformation and stress inversions. Our analyzes suggest that over $\sim 20$ year time periods in a complex plate boundary region such as southern California, the tectonic strain is not partitioned into distinct regions of pure strike-slip, normal and reverse faulting mechanisms. Instead, the deformation is partitioned into regions of relative homogeneity, governed by the largest nearby fault structure, and regions of relative heterogeneity, where the dominant faulting style mixes with local complexities. Our results from various spatial scales suggest that the heterogeneous regions are often confined to small scale faulting structures, while the overall deformation of a plate boundary can be understood to first order in terms of a combination of the relatively homogeneous large-scale ($\sim 50 – 250$ km) fault zones.
The lack of self-similarity observed for different scales and different faulting structures indicates that several specific length-scales are important to how earthquake populations behave. The association of earthquake deformation with scale-invariance is often based on the observation of many power-law distributions in earthquake data (e.g., Turcotte, 1997, ch. 4). However, power-law distributions may be produced on planar structures (e.g., Hillers et al., 2007), and can also reflect mixing of many populations influenced by a wide range of length-scales (e.g., Ben-Zion, 2008), as we infer for our results. The other end of the spectrum is a system for which there is one characteristic dominant scale. The interaction of several different length-scales shows that this description also cannot be used to explain our regional results. Although the large-scale dynamics of the entire system may be captured by taking into account the large faults and the overriding stress field, accounting for the differences between those faults requires taking into account heterogeneities associated with many smaller length-scales that have a wide range of sizes.

Tectonic stress is typically calculated by inversion of focal mechanism data, where focal mechanisms are used as a proxy for fault slip direction (e.g., Angelier et al., 1982; Michael, 1987). These inversion methods are based on the assumption that the stress is homogeneous for the population of earthquakes to which the inversion is applied. It is therefore important to understand the spatial variations in earthquake behavior so as to define suitable spatial bins for the inversion process. Our inference is that the large-scale fault zones control the dominant behavior of earthquakes across southern California. This may explain how fundamental characteristics of the stress field can fit GPS observations using a fault-defined block model, as done by Becker et al. (2005). Fitting the smaller scale variations of stress requires identification of the second and third-order structures interpreted in this study. An approach using appropriate spatial bins based on cluster analyses of the earthquake locations, such as that of Hardebeck & Michael (2004), may be well suited to this goal.
Chapter 4

Quantifying Focal Mechanism Heterogeneity for Fault Zones in Southern and Central California

SUMMARY

We present a statistical analysis of focal mechanism orientations for nine California fault zones with the goal of quantifying variations of fault zone heterogeneity at seismogenic depths. The focal mechanism data are generated from first motion polarities for earthquakes in the time period 1983–2004, magnitude range 0–5, and depth range 0–15 km. Only mechanisms with good quality solutions are used. We define fault zones based on a 15 km zone around the fault traces according to the USGS Quaternary fault map, and use summations of normalized potency tensors to extract information about the focal mechanism heterogeneity for each fault zone. This is quantified using two measures that are roughly analogous to the standard deviation and skewness of the double-couple orientation distributions. We find a decreasing trend in relative focal mechanism variation as a function of geologically mapped fault trace complexity, indicating a link between the long term evolution of a fault and its seismic behavior over a 20 year time period. The nature of heterogeneity and partitioning of faulting styles are affected by the dominant orientation of the fault zone. This indicates that the heterogeneity of earthquake orientations in a fault zone is correlated with geometrical properties of the main fault and the efficiency of that fault at releasing plate motion. These correlations are observed despite clustering of small earthquakes in zones of complexity along the main faults.
4.1 Introduction

The aim of this study is to quantify fault zone heterogeneity using focal mechanism data from $0 < M_L \leq 5$ earthquakes. Heterogeneity of a fault zone here refers to any departure of the fault from a single planar surface within material that has homogeneous mechanical properties. Geometrical heterogeneity can take many forms; for example, breaks in the fault (stepovers), changes in the fault orientation due to kinks or bends, roughness of the fault surface, and overlapping fault segments. Slip on geometrically rough surfaces leads to complex distributions of stress in the surrounding medium which can influence the resulting distribution of seismicity (e.g., Dieterich & Smith, 2009). King & Nábělek (1985) related the initiation and termination of large earthquakes to “barriers” resulting from bends in the associated faults, while Wesnousky (2006) compiled data on large earthquake ruptures which indicate that many earthquakes start and finish at fault stepovers. Heterogeneity of material properties, in particular the frictional parameters, can have similar effects to those of geometrical heterogeneity. Ben-Zion & Rice (1993) represented fault zone heterogeneity by variations in frictional parameters across a grid of discrete slip surfaces to study seismicity patterns on a single fault. This study and others (e.g., Zöller et al., 2005; Hillers et al., 2006, 2007; Ben-Zion, 2008) showed that the range of size scales in the distribution of heterogeneity is a key controlling parameter for the statistical properties of seismicity. Bailey & Ben-Zion (2009) showed that evolving stress heterogeneity resulting from heterogeneous fault properties leads to a $\sim 70\%$ reduction in the expected stress drop for large earthquakes. It is therefore important from both theoretical and seismic hazard perspectives to understand the nature and amount of heterogeneity on existing faults.

Maps of fault traces provide one data source for investigating the geometrical heterogeneity of a fault zone. Previous studies have quantified this heterogeneity by the number of stepovers per unit length (e.g., Wesnousky, 1988; Stirling et al., 1996), the number and relative direction of fault splays (e.g., Ando et al., 2009), and the distribution of azimuths in the fault sections (e.g., Wechsler & Ben-Zion, 2009). The results of Wesnousky (1988); Stirling et al. (1996); Wechsler & Ben-Zion (2009) indicate a decrease in heterogeneity with increasing cumulative slip on a fault, consistent with Ben-Zion & Sammis (2003) who summarize multi-disciplinary studies indicating an evolution
of fault zones over time from fractal geometries towards smoother, more continuous structures. Fault traces develop due to displacement from a large number of earthquakes and therefore represent the current state of a slowly evolving process as manifested at the free surface. However, these data only provide a 2D representation of the fault at the Earth’s surface. The decreasing normal stress close to the free surface is expected to lead to more heterogeneity in fault surface traces than exists at seismogenic depths (e.g., Rockwell & Ben-Zion, 2007). Hence it is important to verify that the surface heterogeneity is representative of heterogeneity at depth and has relevance over timescales important for seismic hazard.

For a well instrumented region, earthquake data provide information about brittle failures over the depth range of \( \sim 0–20 \) km. Previous studies have quantified fault heterogeneity with earthquake data using the distribution of earthquake hypocenters around faults (e.g., Powers & Jordan, 2009) or the distribution of focal mechanism orientations (e.g., Kagan, 1990; Rivera & Kanamori, 2002; Hardebeck, 2006; Bailey et al., 2009). Although the number of focal mechanism data is smaller than for hypocenter locations, focal mechanisms contain information on the orientation of subsurface faulting. Variation in the orientation of focal mechanisms provides information on the combined heterogeneity of stress, fault geometry and material properties. Several studies use inversions of focal mechanism data to specifically investigate one of these features, typically the regional stress field (e.g. Michael, 1987; Hardebeck & Hauksson, 2001; Townend & Zoback, 2004). Rivera & Kanamori (2002) inferred based on inversions of southern California focal mechanism data that there is likely to be heterogeneity in both the stress field and frictional properties of the faults. However, Hardebeck (2006) showed that a large amount of this heterogeneity can be explained by uncertainties in the data. Other studies have shown that a smoothly varying stress field can explain variations in focal mechanism data (e.g., Becker et al., 2005; Hardebeck & Michael, 2006), but such inversions do not quantify geometrical heterogeneity of faults. In this study we concentrate on variation of focal mechanism orientations as a measure of heterogeneity in itself. This approach was also used by Kagan (1990) and Hardebeck (2006), who described heterogeneity by a distribution of rotation angles between focal mechanism orientations. We use an approach based on summation of normalized potency tensors (e.g., Kostrov, 1974; Fischer & Jordan, 1991; Amelung & King, 1997;
Sipkin & Silver, 2003; Bailey et al., 2009) that requires less computation time yet still quantifies aspects of the rotation distributions. The summations provide two quantities that describe the heterogeneity: one related to the amount of scatter in orientations and one related to the rotational asymmetry of this scatter.

In this study we specifically target faults in central and southern California for a comparative study of heterogeneity, due to the availability of data there. We select nine strike-slip fault zones based on their fault traces, quantify the heterogeneity of good quality double-couple (DC) focal mechanism orientations within those zones, and then compare the differences in heterogeneity between the different fault zones. We find that the amount of scatter in DC orientations correlates with the variation in azimuths of the fault trace segments within the zones. This indicates that focal mechanism heterogeneity is representative of the long term evolutionary characteristics of the fault, as reflected by the fault trace. The second component of focal mechanism heterogeneity (asymmetry in the scatter) correlates with the dominant fault trace direction, regardless of fault trace heterogeneity. This result implies that long term strike-slip motion in a direction not aligned with plate motion leads to secondary components of reverse or normal faulting, as expected from the direction of misalignment.

Data from local networks of seismic stations are usually limited by the distribution of seismicity over the last ∼ 20–40 years. Even for small events, it is questionable if these data provide a representative sample of the entire fault zone in space, and they may also be affected by temporal changes in the instrumentation network. We investigate potential biases due to the restricted sampling of focal mechanism data by examining the partitioning of a population into different faulting styles as a function of hypocenter location, time and magnitude. We find that many of the focal mechanism data are clustered in space, and the clusters of seismicity are themselves heterogeneous. The applicability of focal mechanism results to an entire fault zone therefore depends on how representative those clusters are for the fault zone as a whole. Since large sections of the faults with largest cumulative slip are relatively straight and produce little low magnitude seismicity, the differences among the examined fault zones are likely to be larger than that shown by our analysis of the limited available data.
The remainder of this paper is organized as follows: in Section 4.2 we describe the data used in this study. Our analysis can be divided into two parts. The first part concerns investigation of the focal mechanism heterogeneity for the nine fault zones and the second is an examination of the partitioning into different faulting styles for specific fault zones. We describe the methods of both parts in Section 4.3, the results in Section 4.4, and discuss implications in Section 4.5. Some details of the methods are given in Appendices.

4.2 Data

4.2.1 Fault Trace Data

Digitized fault trace data are defined based on the USGS quaternary fault map (California Geological Survey & U.S. Geological Survey, 2008). In these data faults are divided into groups of connected and disconnected straight line segments of length between ∼2 m and 9 km. We select nine different fault zones from southern and central California (Figure 4.1) that are strike-slip dominated and have earthquake data available in the vicinity. We concentrate on strike-slip fault zones to minimize the effect of dipping faults on extrapolating surface properties to those at depth.

We extract surface traces for each of the zones based on the fault names in the database. We then define the zone using a 15 km horizontal distance from the fault traces, reducing the distance to halfway between faults in cases where there would be overlap between zones. The choice of a 15 km width around the vertices of fault traces is roughly equal to the depth extent of the seismogenic zone, and allows for some flexibility with respect to location errors in the earthquake catalog as well as dipping faults while providing an adequate number of focal mechanism data for analysis. Seven of the zones relate to large, well known faults in southern California: the San Andreas (SAF; represented by four different sections), the San Jacinto, the Elsinore and the Garlock faults. The other two zones are part of the Eastern California Shear Zone (ECSZ), and are both collections of smaller faults that contain the surface ruptures of the 1992 $M_W = 6.2$ Joshua Tree and $M_W = 7.3$ Landers earthquakes (ECSZ 1), and the 1999 $M_W = 7.1$ Hector Mine earthquake (ECSZ 2),
respectively. In the ECSZ cases and the Coachella section of the SAF, we modify the database definitions of the fault zones to incorporate nearby faults based on their proximity to other faults and to avoid placing boundaries through areas of dense seismicity. More detail concerning the selection of fault zones is given in Appendix H.

4.2.2 Focal Mechanism Data

We assume that a point-source double-couple (DC) representation is adequate for $0 < M_L \leq 5$ earthquakes. The DC solutions used in this study are computed from first motion polarities using
the program HASH (Hardebeck & Shearer, 2002). The program computes a set of acceptable fault plane solutions that fit the phase data within some predefined misfit threshold and returns the preferred solution as the average of this set. The uncertainty of the solution is quantified by the range of acceptable solutions. Effects of uncertainty in the hypocenter location on the solution are incorporated by recomputing acceptable solutions using a number of velocity models. We use only A and B quality mechanisms from the catalogs for which the root mean square angular difference in acceptable solutions (FPU) is $\leq 35^\circ$, 60% of acceptable solutions are within $30^\circ$ of the preferred solution (i.e., $\text{PROB}=0.6$), the misfit of the fault plane inversion is less than 20%, and the ratio describing how well stations sample the focal sphere (STDR) is $\geq 0.4$. In Appendix I, we compare heterogeneity results based on the A and B quality data with those based on relaxed quality constraints. The lower quality constraints allow the use of more data, but changes in the heterogeneity can largely be explained by increased uncertainty in those data. The multiple quality restrictions given by the A and B catalogs are more likely to generate distributions of uncertainties that are similar for the fault zones, such that the relative differences in heterogeneity are expected to indicate physical differences.

For all fault zones except the Parkfield section of the SAF, we use focal mechanisms from the catalog of Hardebeck et al. (2005), available via the Southern California Earthquake Data Center. This covers the time period January 1984 – December 2003. We reassign the hypocenter locations using the LSH (1.12) relocated catalog of Lin et al. (2007). For the Parkfield section of the SAF, we use the catalog generated for the study of Thurber et al. (2006). This catalog covers the period May 1979–August 2005, and is limited to a rectangular region shown in Figure 4.1. For both catalogs, restrict magnitude to $0 < M_L \leq 5$ and depth to $z \leq 15$ km.

### 4.3 Methods
4.3.1 Fault Trace Heterogeneity

We quantify geometrical characteristics of mapped fault traces using the distribution of azimuths weighted by fault length, rather than number of stepovers per unit length (as in Wesnousky, 1988; Stirling et al., 1996), since this approach is less subjective given the digital data, and less sensitive to uncertainties in mapping of fault terminations. For all fault trace data assigned to a given fault zone, we extract the length \( l_i \) and azimuth \( \alpha_i \) of each segment. Since some faults have a staircase pattern, we resample cases where the azimuths of neighboring segments are \( \pm 90^\circ \text{ or } 0^\circ \) by ignoring the vertex between them. We sort the segments based on azimuth and add the cumulative length to describe the distribution of the fault trace orientations (illustrated in Figure 4.1). We take care to account for the \( 180^\circ \) periodicity in \( \alpha_i \) by setting the zero point of the distribution to halfway between the segments with the greatest difference in azimuth. From the distribution we extract the azimuths corresponding to 25%, 50% and 75% of the total length. We quantify the dominant orientation by the 50% value \( \alpha_{50} \). We quantify the scatter in orientations by the difference between 25% and 75% values \( \alpha_{I/Q} \). Rank-based statistics are used to describe the azimuths since they are less dependent on outliers than the mean and circular standard deviation, though results based on these measures are similar.

Our method of quantifying complexity ignores the location of fault segments within zones, and thus makes no distinction between smooth and sudden changes in fault direction which would have different implications for rupture mechanics. We also weight each segment of the fault trace based only on \( l_i \), regardless of the amount of slip or slip rate which are not provided in the data set. Despite these limitations, \( \alpha_{I/Q} \) represents a simple measure of fault variability within the zone and values are consistent with a visual assessment of the fault maps.

4.3.2 Focal Mechanism Heterogeneity

We describe the orientation of focal mechanisms using a source mechanism tensor \( \hat{P}_{ij} \) (Riedesel & Jordan, 1989) which is effectively a normalized potency tensor. This describes the orientation of the non-elastic earthquake strain that generates seismic radiation, though by the nature of our data this is always constrained to be of DC type. We convert from the strike, dip and rake of the fault
Figure 4.1: Cumulative length $L$ of the San Jacinto fault trace segments as a function of azimuth $\alpha$. The azimuth corresponding to zero length is defined as the azimuth half way between the two most widely separated azimuths. We quantify the dominant fault orientation by $\alpha_{50}$ separating half of the fault length from the other. We quantify the complexity of the fault zone using the angular range between the 25% and 75% values, $\alpha_{IQ}$.

plane solution using equations given in Appendix A of Bailey et al. (2009). The orientation of a DC has three degrees of freedom and as such the source mechanism tensor is symmetric, has zero trace, zero determinant and unit Euclidean norm. The advantage of this representation over other representations (see Kagan, 2005) is that one DC orientation is unambiguously defined by only one tensor. By using the normalized potency tensor, we disregard magnitude information associated with the earthquake and concentrate on the orientation rather than the amount of strain release. A comparison of results for analysis methods based on the potency and normalized potency is given by Bailey et al. (2009).

We describe the combined orientation properties for a population of $N$ tensors using the summed source mechanism tensor (e.g. Kostrov, 1974; Bailey et al., 2009).

$$E_{ij} = \sum_{k} \hat{P}_{ij}^{(k)} ,$$

which is symmetric and has zero trace, leaving five degrees of freedom. For a summation of homogeneously oriented DCs $E_{ij} = (N/\sqrt{2})\hat{P}_{ij}$, and the two degrees of freedom that quantify departure of $E_{ij}$ from this homogeneous result relate to heterogeneity of orientations in the population. We
estimate the homogeneous part of $E_{ij}$, or the dominant direction for the population, by computing the orientation of the DC component of $E_{ij}$

$$E_{ij}^{DC} = \frac{1}{\sqrt{2}} (t_{ij} - p_{ij}) ,$$

(4.2)

where $t_i$ and $p_i$ are the eigenvectors corresponding to the maximum and minimum eigenvalues of $E_{ij}$, $\lambda_3$ and $\lambda_1$, respectively (taking extension to be positive). For a summation of $N$ tensors aligned with $E_{ij}^{DC}$, the Euclidean norm is $\sqrt{E_{ij}E_{ij}} = N$ and the intermediate eigenvalue of $E_{ij}$ is $\lambda_2 = 0$. A value of the norm of $E_{ij}$ smaller than $N$ indicates scatter in the DC orientations, and a non-zero value of $\lambda_2$ indicates rotational asymmetry in that scatter.

We quantify the tensor norm related measure of heterogeneity using the metric

$$\Delta_{rNORM} = 1 - \frac{\sqrt{E_{ij}E_{ij}}}{N} ,$$

(4.3)

which quantifies the inconsistency in orientations of the population ($1 - \Delta_{rNORM}$ is termed the seismic consistency by Frohlich & Apperson, 1992), where $0 \leq \Delta_{rNORM} \leq 1$. It is zero when all DCs have the same orientation, unity when all DCs cancel each other out and $\sim 1/\sqrt{N}$ for a summation of uniformly random orientations.

For a DC tensor, the zero value of $\lambda_2$ means that there is no net strain in the direction of the intermediate strain axis (B-axis). For $E_{ij}$ the net strain must be re-stated in terms of number of events, and a non-zero $\lambda_2$ results from an asymmetric distribution in DC orientations. We quantify this measure of heterogeneity using

$$r_{CLVD} = \frac{\sqrt{6}}{2} \frac{\lambda_2}{\sqrt{E_{ij}E_{ij}}} ,$$

(4.4)

where the CLVD subscript refers to the compensated linear vector dipole component that is typically used to describe the non-DC part of a deviatoric tensor (Knopoff & Randall, 1970; Julian et al., 1998). The range of possible values is $-0.5 \leq r_{CLVD} \leq 0.5$, where $r_{CLVD} = 0$ for a pure DC. Positive or negative values of $r_{CLVD}$ imply a difference between the variation of P and T axes.
orientations, as summarized with examples in Table 4.1. If $r_{CVLD} = 0$ and $\Delta r_{NORM}$ is small, the DC orientations must all be homogeneously aligned. If $r_{CVLD} = 0$ and $\Delta r_{NORM}$ is large, the DC orientations are likely to be heterogeneous but the P and T axis must vary in the same manner such that deformation in the direction of the intermediate axis is canceled out. For summation over $N$ uniformly random DC orientations, the probability distribution for $r_{CLVD}$ has the form $\cos(\pi r_{CLVD})$ when $N \geq 3$ (Bailey et al., 2009). Comparisons of $\Delta r_{NORM}$ and $r_{CLVD}$ to other measures of focal mechanism heterogeneity are given in Appendix J.

Distributions of rotation angles (e.g., Kagan, 1990; Hardebeck, 2006) and the associated axes of rotation (Kagan, 2009) provide a more complete description of focal mechanism heterogeneity, but there are two advantages of using quantities based on the summation of source mechanism tensors: firstly the fault plane ambiguity has no impact on how we measure the difference between mechanisms, and secondly, it is unnecessary to run computations for all pairs in a population which leads to a rapid ($N^2$) increase in computational time with number of data. In Appendix K, we show that $r_{NORM}$ relates to the mean of the minimum rotation angles between all pairs of DC orientations, and $r_{CLVD}$ relates to the distribution of axes about which those rotations occur. A further advantage of the summation method is that the same summation technique could be applied to the analysis of moment tensor catalogs where a DC mechanism is not necessarily assumed.
Table 4.1: The CLVD component of $E_{ij}$ interpreted in terms of mixing of strike-slip, reverse and normal faulting styles, assuming that one of the principle axes for $E_{ij}$ is vertical. Four cases of a non-zero CLVD component are shown in the third column. The associated faulting styles and example distributions of P-axes (red crosses) and T-Axes (blue circles) are shown in the fourth and fifth columns.

### 4.3.3 Strain Partitioning

The partitioning of the focal mechanism data into different deformational styles provides insight into the underlying processes that lead to non-zero values of $\Delta r_{\text{NORM}}$ and $r_{\text{CLVD}}$. We can define six end-member cases of DC orientation for a given $E_{ij}$ using

$$
\hat{E}_{ij}^A = \hat{E}_{ij}^{DC}, \\
\hat{E}_{ij}^B = \frac{1}{\sqrt{2}}(b_ib_j - p_ip_j), \\
\hat{E}_{ij}^C = \frac{1}{\sqrt{2}}(t_it_j - b_ib_j), \\
\hat{E}_{ij}^D = \frac{1}{\sqrt{2}}(p_ip_j - b_ib_j), \\
\hat{E}_{ij}^E = \frac{1}{\sqrt{2}}(b_ib_j - t_it_j), \\
\hat{E}_{ij}^F = \frac{1}{\sqrt{2}}(p_ip_j - t_it_j),
$$

(4.1)

where $\hat{E}_{ij}^{DC}$ is defined in Eq. 4.2 and $b_i$ is the intermediate strain axis of $E_{ij}$, and compute which is closest in orientation to each $\hat{E}_{ij}^{(k)}$ based on which gives the largest value for the inner product. $\hat{E}_{ij}^A$ indicates the dominant faulting orientation for the population since it has the same P and T axis orientations as $E_{ij}$. $\hat{E}_{ij}^B$ and $\hat{E}_{ij}^C$ share either a P or T axis with $E_{ij}$. Assuming that slip occurs in the
direction of maximum shear stress resolved onto a pre-existing fault plane, DC focal mechanisms with orientations of \( \hat{E}^A_{ij} \) or \( \hat{E}^B_{ij} \) can result from a loading tensor with principle stress orientations parallel to the P, B and T axes of \( E_{ij} \) (McKenzie, 1969). \( \hat{E}^D_{ij}, \hat{E}^E_{ij} \) and \( \hat{E}^F_{ij} \) do not share either P or T axes with \( E_{ij} \), and any partitioning of focal mechanisms into these directions indicates strong variation in the loading tensor orientation for the population. For the San Jacinto fault zone which has the most focal mechanism data, we investigate the partitioning as a function of epicenter location, hypocenter depth, magnitude and time.

In order to better visualize the spatial location of focal mechanisms from the different deformational styles, we use a kernel smoothing approach to plot the density of epicenters and density of hypocenter depths along strike. We define a 2D Gaussian function with standard deviation of 1 km centered on each earthquake location. The density of earthquakes at each point in a grid encompassing the fault zone is then computed as the sum of all Gaussians evaluated at that grid point.

### 4.4 Results

#### 4.4.1 Source Mechanism Summations

The results for \( E_{ij} \) in each of the nine fault zones are shown by “beachball” plots which indicate the distribution of compressive and extensive axes in Figure 4.1, with angular histograms of fault trace azimuths overlaying each. We find that the distribution of fault traces is quite variable, with some fault zones (e.g., Coachella and ECSZ 1) showing asymmetry. Two possible dominant fault planes are shown by black lines which indicate \( \hat{E}^{DC}_{ij} \). The histograms overlap one of these directions in all cases except ECSZ 1 and the Mojave Section, for which the dominant faulting styles are not strike-slip. This indicates a relationship between \( \hat{E}^{DC}_{ij} \), when strike-slip, and the direction of faulting at the surface.
Figure 4.1: Summed source mechanism tensor $E_{ij}$ results and angular histograms of the fault traces for each of the fault zones. The beachball plots indicate orientations of compression and extension for $E_{ij}$ by white and colored regions of an equal-area projection, respectively. The radius of each beachball is scaled by $1 - \Delta r_{NORM}$ to indicate the relative consistency in orientations. Lines within the beachballs show the double-couple part of the result. The angular histograms divide the digitized segments of the fault traces into $5^\circ$ bins of azimuth in the range $-90^\circ < \alpha \leq 90^\circ$ and compute the total length of fault segments in each bin. The histogram bins are all normalized by the maximum bin size such that the scale is arbitrary.

4.4.2 Focal Mechanism Heterogeneity

The relative heterogeneity of each fault zone as quantified by $\Delta r_{NORM}$ and $r_{CLVD}$ is shown in Figure 4.1. For each fault zone we perform a bootstrap resampling (Press et al., 1992, p. 691) of the data 10,000 times and compute the range of $\Delta r_{NORM}$ and $r_{CLVD}$, which reflect the robustness of the values given the number of available data. We find no correlation between the number of data and the values of $\Delta r_{NORM}$ and $r_{CLVD}$. The bootstrap errors do not take into account the uncertainties associated with the focal mechanisms (see Appendix I) and we concentrate on relative differences between fault zones.
We find that the Parkfield section of the SAF lies noticeably closer to zero for both measures, indicating more homogeneity than for the other fault zones. For the zones in southern California, values of $\Delta r_{NORM}$ are in the range 0.29–0.51 with several overlapping error bounds. The different fault zones are better distinguished by values of $r_{CLVD}$ which span close to the entire $-0.5$ to 0.5 range. This indicates that asymmetry in focal mechanism orientations is fundamental to a description of the heterogeneity. The most heterogeneous fault zones are the Garlock fault and ECSZ 2. In these two zones (for which we have the smallest number of data) we find a low value of $r_{CLVD}$ due to populations of normal and reverse faulting that cancel each other out.

4.4.3 Fault Trace Focal Mechanism Relationships

Figure 4.2 shows the relationships between (a) $\alpha_{50}$ and $r_{CLVD}$, and (b) $\alpha_{IQ}$ and $\Delta r_{NORM}$. In the case of the Garlock fault which is the only left-lateral fault considered, we subtract $90^\circ$ from $\alpha_{50}$ to make it comparable with the other fault zones. We find a negative correlation between the $\alpha_{50}$ and $r_{CLVD}$ with a Pearson correlation coefficient of $r_P = -0.74$ (Spearman Rank correlation coefficient
of $r_s = -0.68$). The errors bars for $\alpha_{50}$ correspond to the $\alpha_{25}$ and $\alpha_{75}$ used in computation of $\alpha_{IQ}$. The three points with the lowest values of $r_{CLVD}$ have larger error bars to the right than the left, and moving those points to the right would make the correlation stronger. The azimuth of relative plate motion for the Pacific and North American plates ranges between $-40^\circ$ and $-36^\circ$ for longitude and latitude ranges of the region (using NUVEL-1A, DeMets et al., 1994), and we show this as a gray bar in Figure 4.2(a). However, we find that an azimuth of $\sim -50^\circ$ would better separate the positive from negative values of $r_{CLVD}$.

Figure 4.2(b) shows a positive correlation of $r_p = 0.53$ ($r_s = 0.62$) between $\alpha_{IQ}$ and $\Delta r_{NORM}$. The correlation is improved to $r_p = 0.79$ ($r_s = 0.94$) if we remove the Garlock, Coachella and ECSZ 2 zones which have the smallest number of data.

Based on a Student’s t-test which takes into account the number of degrees of freedom, the correlation between $\alpha_{IQ}$ and $\Delta r_{NORM}$ is not significant at a 90% level unless the three fault zones are removed, when it becomes significant at the 95% level. The correlation between $\alpha_{50}$ and $r_{CLVD}$ is significant at a 95% level without removing any data. However, the assumption of an underlying normal distribution is unlikely due to the limited range of the quantities and we therefore cannot place too much weight on these significance values.

4.4.4 Strain Partitioning

Figure 4.3 shows the partitioning of focal mechanism data into the six deformation styles defined by Eq. 4.1. We show results only for the fault zones where $N > 100$ (San Jacinto, ECSZ 1, San Bernardino and Parkfield). In all regions except the ECSZ 1, we find that the number of data belonging to categories $\hat{E}_{ij}^P$, $\hat{E}_{ij}^E$ and $\hat{E}_{ij}^F$ is negligible, implying that the principle axes of the deviatoric loading tensor for the fault zones are close in direction to the strike-slip consistent strain axes of $\hat{E}_{ij}^A$.

For the Parkfield section, we find that very few of the focal mechanisms are classified outside of $\hat{E}_{ij}^A$, implying that all earthquakes are minor variations about the dominant strike-slip sense of deformation, as suggested by Thurber et al. (2006). The San Jacinto and San Bernardino fault zones have dominant strike-slip components and second-order components consistent with the sign of $r_{CLVD}$.
We also find third-order components that indicate further, non-trivial complexity in the variation of fault plane and stress tensor orientations within the fault zone.

For ECSZ 1, the DC part of $E_{ij}$ has a near-vertical P-axis and therefore $\hat{E}^A_{ij}$ and $\hat{E}^B_{ij}$ are both normal faulting styles. The presence of focal mechanisms in both of the normal faulting categories indicates the existence of normal faulting with highly variable strike. Although variation in the strike of dipping faults could generate a CLVD component in $E_{ij}$ (Example 4 in Table 4.1), the CLVD component for ECSZ 1 is generated by a mixing of $\hat{E}^C_{ij}$ strike-slip and $\hat{E}^A_{ij}$ normal faulting mechanisms.

The dominant DC direction is of normal faulting type when most of the mapped faults are of right-lateral strike slip type because of a sampling issue related to the distribution of seismic stations.

Although the aftershocks of the 1992 Landers earthquake dominate a lower quality catalog for the southern California region, selecting only mechanisms of A and B quality significantly reduces the number of data and produces bias towards events in the south, which are most likely related to the pre-Landers 1992 Joshua Tree earthquake. It is likely that better sampling of the region would lead to a dominant strike-slip component, but that a high amount of heterogeneity would remain due to the complex fault geometry. The existence of mechanisms in both $\hat{E}^B_{ij}$ and $\hat{E}^E_{ij}$ categories, where
compressive and extensive axes oppose each other, indicate a fairly heterogeneous stress field for the fault zone. Nearly all focal mechanisms in this region can be attributed to one of three aftershock sequences (the 1992 Joshua Tree, the 1992 Landers and the 1999 Hector Mine earthquakes) and may explain the higher degree of heterogeneity (c.f., Hardebeck & Hauksson, 2001). However, we might also expect spatial, temporal and magnitude partitioning to show features distinct to aftershock sequences, and there is little difference between the results in these zones and the San Jacinto fault zone, where there is no large aftershock sequence.
In Figures 4.4–4.6 we show the partitioning of focal mechanisms for the San Jacinto fault zone into $\hat{E}_{ij}^A$, $\hat{E}_{ij}^B$ and $\hat{E}_{ij}^C$ as a function of epicenter location, hypocenter depth, magnitude and earthquake time, respectively. We discard the other three categories in Eq. 4.1 since the number of earthquakes are negligible.

### 4.4.5 San Jacinto Spatial Dependence

Figures 4.4 and 4.5 illustrate the spatial partitioning of the San Jacinto fault zone focal mechanisms into the three dominant faulting styles. This is shown by plots of the 2D density of epicenters and of hypocenter strike-depth locations for each of the three faulting style categories. The density maps highlight that most of the seismicity is confined to a few clusters. We find that the four largest clusters of strike-slip earthquakes correspond in location to either a relative cluster of reverse faulting mechanisms or normal faulting mechanisms or both. These heterogeneous clusters have differing shapes and are not always associated with complexity of the fault’s surface trace.

### 4.4.6 San Jacinto Time Dependence

Figure 4.6 shows the number of focal mechanisms in each of the three faulting style categories as a function of time. We show the time-dependence in terms of cumulative number as well as a histogram binned into half year intervals. By normalizing the cumulative distributions, the slope corresponding to a given number of events per year will be different for each line. However, this allows us to compare the times at which changes in the rates occur for the three groups. We find that for the first $\sim 8$ years of data, the three lines are relatively straight, indicating a roughly constant rate of earthquake occurrence and equal partitioning between the different faulting styles. After $\sim 1992$, there is an increase in the rate of normal faulting earthquakes and a decrease in the rate of reverse faulting earthquakes, though the reverse faulting line has a sudden step at the end of 1994. At the end of 2001, all three faulting styles show a sudden increase in seismicity rate, but the partitioning levels appear roughly constant. This increase occurs after a $M_L = 5.1$ earthquake on 31st October, 2001 (Figure 4.5). However, the curvature of the cumulative distributions do not appear to show the typical power law decay expected for aftershock sequences.
Figure 4.4: Smoothed density of epicenter locations in the San Jacinto fault zone (see Figure 4.1) for each of the three DC categories $\hat{E}_A^{ij}$ (strike-slip faulting), $\hat{E}_B^{ij}$ (normal faulting) and $\hat{E}_C^{ij}$ (reverse faulting). The beachballs in the top right of the maps, which are rotated along with the map projection, indicate the category. Smoothing of the density is performed using Gaussian kernels with standard deviation of 1 km (note variable color scale).
Figure 4.5: Smoothed density of hypocenter strike-depth coordinates in the San Jacinto fault zone for each of the three DC categories $E^A_{ij}$ (strike-slip faulting), $E^B_{ij}$ (normal faulting) and $E^C_{ij}$ (reverse faulting), with a vertical exaggeration factor of 2. The along strike distance is computed by projecting onto the dotted line shown in the lower map. Black, red and blue dots respectively represent the epicenters of the strike-slip, reverse and normal faulting earthquakes on that map. Yellow stars in the lower plots indicate the hypocenters of the three $M_L > 5$ earthquakes that occurred in the zone during the time period, with magnitude and year noted next to each.

Most of the changes in temporal partitioning can be related to the spatial clusters in Figure 4.4 and Figure 4.5. The largest cluster of seismicity, which mixes all three faulting types and is in the middle of the fault, is active throughout the entire time period with an increase in rate that accounts for the changes in 1992. The northern cluster where we observe a mixing of strike-slip and normal faulting is most active during the mid to late 1990’s. The cluster between these, where we observe a mixing of reverse and strike-slip faulting contributes more events after 2001.

4.4.7 San Jacinto Magnitude Dependence

Figure 4.7 shows that earthquakes in the three faulting styles follow a Gutenberg-Richter distribution of magnitudes with similar $b$-values of $\sim 1$. The roll-off of the power law distribution at lower
magnitudes is usually associated with the magnitude of completeness, which in this case would be the lowest magnitude at which A and B quality focal mechanisms can be computed for all earthquakes. However, comparison with a the LSH catalog for the same time period and fault zone indicates that the focal mechanism data are incomplete at all magnitude levels, despite a similar $b$-value. We find that only about 20–30% of earthquakes throughout the range $2.2 \leq M_L \leq 5$ are represented in the San Jacinto catalog, and fewer for $M_L < 2.2$ earthquakes. The distribution roll-off appears to be different for the three faulting styles. This indicates that given the relative locations of earthquakes and seismic stations, it is easier to constrain the nodal planes for the mechanisms of dip-slip than strike-slip faults when there are a small number of recordings. The faulting style with the lowest magnitude roll-off differs for comparisons in other fault zones which we do not show here, indicating that this feature is more a function of the local network than of the HASH algorithm. This may have an effect on our results if the number of strike-slip faults in our data are proportionally smaller than in reality. If this is the case, the values of $\Delta r_{NORM}$ and $r_{CLVD}$ should actually be smaller.
Figure 4.7: Frequency-size distributions of focal mechanisms in the categories $\hat{E}_i^A$ (strike-slip faulting, black), $\hat{E}_i^B$ (normal faulting, blue) and $\hat{E}_i^C$ (reverse faulting, red). Distributions are in reversed cumulative form, with percent of the sub-population shown on the y-axis.

However, the results for the San Jacinto Fault zone presented in Figure 4.1 are within the bootstrap errors when we recompute $E_{ij}$ using only $M_L \geq 2.4$ earthquakes.

We find that for all fault zones, there are more large magnitude earthquakes of strike-slip type than either of the dip-slip types. Most differences can be reasonably explained by different $a$-values for the three faulting types. However, there is likely to be a physical limit to the size of dip-slip earthquakes since there are no large through-going dipping faults in any of the zones, and we would expect this to be reflected in a longer time period of data.

### 4.5 Discussion

There are five major results from this study:

1. Individual fault zones have distinct characteristics of focal mechanism heterogeneity, and the asymmetry in the distribution of orientations is key to quantifying this heterogeneity.

2. The quantity $\Delta r_{NORM}$ which describes scatter of focal mechanism orientations correlates with the variation in fault zones’ trace azimuths.
3. The quantity $r_{CLVD}$ which describes asymmetry in focal mechanism orientations correlates with the dominant direction of the fault zone trace.

4. The partitioning of focal mechanisms into different faulting styles implies a wide range of fault orientations and indicates that fault zone heterogeneity cannot be described by simple distributions of minor variation about a mean.

5. Seismicity tends to cluster in regions of heterogeneity. The heterogeneity that is observed appears to depend more on the location of the seismicity than the magnitude or time.

In the following, we discuss the reliability, relevance and limitations of these results.

### 4.5.1 Fault Zone Differences

The existence of relative differences in focal mechanism heterogeneity for different fault zones which are spatially defined by independent data implies that there are characteristics unique to a given fault zone. As discussed below, this is likely to relate to the evolution of fault zones towards smoother structures as they accumulate more slip. Since there are observable differences between faults, there are limits to the use of scale-invariance to describe fault networks, as done by King (1983); Turcotte (1997), which treat all faults as essentially the same.

The usefulness of the CLVD component of $E_{ij}$ in quantifying differences between fault zone populations of focal mechanisms indicates the importance of measuring asymmetry in focal mechanism orientation distributions. Previous studies that quantify heterogeneity of a set of focal mechanism orientations use the distribution of minimum rotation angles between all pairs of DC orientations (e.g., Kagan, 1990; Hardebeck, 2006), or the distribution of the vertical components of the three eigenvectors (Frohlich, 1992). Although these methods give a more complete description of focal mechanism heterogeneity than our two measures, they cannot always constrain the asymmetry that would lead to a CLVD component. For example, the use of a rotational Cauchy distribution by Kagan (1990) to describe focal mechanism heterogeneity describes only scatter in orientations and not asymmetry. In order to describe asymmetry using DC rotations, analysis of the distribution of rotation axes is important (e.g., Kagan, 2009). However, displaying the distribution of rotation
axes together with the amount of rotation for each axis is not feasible for moderately large data sets, and information is lost by displaying the two components separately. The triangle diagrams of Frohlich (1992) assess the distribution of a set of DC orientations based on the vertical components of their principle strain axes, which is related to the dominant faulting style. These diagrams are capable of displaying the type of heterogeneity that generates a CLVD component from mixing of different fault types. However, they neglect variation in azimuths of the strain axes and as a result some aspects of focal mechanism variation cannot be observed, such as the second and third cases in Table 4.1. This study highlights the advantage of pursuing several approaches in order to understand DC heterogeneity.

4.5.2 Focal Mechanism Scatter and Fault Trace Complexity

We show that the amount of scatter in focal mechanism orientations is correlated with the variations in the fault geometry as mapped at the surface. This indicates that the ∼ 20 years of small earthquake data used are representative of the long term state of the fault zone. Ben-Zion & Sammis (2003) summarized evidence for an evolution of faults over geologic timescales towards smoother geometries with a narrower range of size scales. The reduction of ΔrNORM with αIQ is likely to relate to this evolution.

Characteristics of fault traces have previously been related to the maturity of fault zones as quantified by their cumulative displacement (e.g., Stirling et al., 1996). Our two measures of fault complexity are plotted against different estimates of cumulative displacement (see Appendix L ) in Figure 4.1. We find a negative correlation for both measures that is best seen using the log of the displacement. The correlation is better for the measure based on fault trace data due to the high value of αIQ and low value of displacement for ECSZ 1. In both cases, the three sections of the SAF in southern California have more heterogeneity than would be expected based on their total displacement. Out of two estimates for the total displacement on the Garlock fault, we find that the smaller estimate, based on reconstruction of Quaternary displacements that are consistent for the entire SAF system (Powell, 1993), correlates better with ΔrNORM. This indicates that earthquakes on the Garlock fault behave in a manner that is more consistent with the role of the fault in the
Figure 4.1: (a) Fault trace heterogeneity measured by $\alpha_{IQ}$ and (b) focal mechanism heterogeneity measured by $\Delta r_{NORM}$ as a function of total cumulative slip in each of the fault zones. Estimates of cumulative slip are taken from Petersen & Wesnousky (1994) and Powell et al. (1993), and are shown for each estimate as a colored bar over the entire reported range with a vertical black lines denoting the median. In cases where estimates do not overlap, we plot more than one bar for the fault zone. Error bars in (b) correspond to the minimum and maximum from a bootstrap resampling of the focal mechanism data.

4.5.3 Focal Mechanism Asymmetry and Dominant Fault Zone Direction

The relation between the dominant fault azimuth and asymmetry in the distribution of focal mechanism orientations may be explained by considering the direction of the fault relative to the plate loading. As illustrated in Figure 4.2(a), slip on faults parallel to the direction of relative plate motion can efficiently release the elastic strain energy built up by plate loading. However, slip on a fault that is misaligned will lead to a deficiency in extension if the plate motion pulls away from the fault (Figure 4.2b), or compression if the plate motion pushes into the fault (Figure 4.2c). As indicated by the figures, these deficiencies can be released by reverse or normal faults with the appropriate orientations.

displacement of whole SAF network than with the offset geological markers, and may explain why the Garlock does not fit with the correlation in Figure 4.2(b). This is also consistent low to zero estimates of present day slip rates for the Garlock fault from geodesy (e.g., Meade & Hager, 2005; Becker et al., 2005).
Figure 4.2: Cartoons showing how (a) faults parallel to plate motion lead to homogeneous faulting (b) plate motion that pulls away from the fault leads to second-order normal faulting and (c) plate motion that pushes into the fault leads to second order reverse faulting.

The above considerations imply that there is a fundamental difference between the strain release in the large earthquakes which rupture along the entire fault zone, and in the population of small earthquakes when the main fault is effectively locked. As indicated by geodetic and geologic data, large through-going faults take the dominant role in releasing elastic strains built up by ongoing relative plate motions. The results in this study indicate that when those large faults are locked, the earthquakes on the smaller faults deform the crust in response to the large scale strain release events, rather than simply reflecting the overriding plate motion. This is observable in both the fault zones where there has been a moderate to large earthquake during the time period of this study (Parkfield, ECSZ 1 and ECSZ 2) as well as other fault zones where there has not been a large earthquake for over a hundred years. The response of smaller faults in such a manner may be explained by a regionally variable loading tensor resulting from slip on the large faults, or by a single loading tensor acting on a pre-existing distribution of fault orientations that has evolved in response to the orientation of the dominant fault surface.

The existence of dominant fault surfaces with orientations that are not aligned with plate motions may seem to indicate an inefficiency in the manner that potential strain energy is released over a plate boundary. However, heterogeneity in the initial strength of a set of faults combined with an evolution towards smoother structures as total slip increases can explain how some faults become
focuses of strain energy release, despite the existence of more optimally aligned faults in the same zone.

The explanation of the correlation between \( r_{CLVD} \) and \( \alpha_{50} \) by Figure 4.2 is not supported by values for the San Jacinto Fault and Coachella Section of the SAF, which both have negative \( r_{CLVD} \) but have \( \alpha_{50} \) that are smaller than the azimuth of relative plate motion according to the NUVEL 1A model (Figure 4.2a). These results could be reconciled with the explanation if the NUVEL model is not applicable to the time period studied and the plate motion direction is closer to \( \sim -50^\circ \), if \( \alpha_{50} \) is not a good representation of dominant fault direction which should be greater than \( -40^\circ \), or if the earthquake data do not provide a good sample of the fault zone and \( r_{CLVD} \) should be greater than zero. All three explanations are possible, but geodetic plate motions are in agreement with long term plate motions (e.g. Meade & Hager, 2005). It is also possible the stress field in these regions is affected by the transition to the triple junction to the south, such that there is more extension and an increased number of normal faulting mechanisms to the east of the main plate boundary (Platt et al., 2008).

### 4.5.4 Fault Style Partitioning

A CLVD component in \( E_{ij} \) could be generated by mixing two fault populations with different dominant orientations (see Figure 4.2 and Appendix J), but the partitioning results in Section 4.4.4 indicate a division into three or four populations. The existence of focal mechanisms in \( \hat{E}_A^{ij} \), \( \hat{E}_B^{ij} \) and \( \hat{E}_C^{ij} \) categories indicates variation of both the P and T axes orientations over a 90\(^\circ\) range. The existence of wide ranging variations as well as clear asymmetry implies that the distribution of fault orientations in a fault zone is often quite complex, and cannot be described by minor symmetric variations about a mean.

The absence of many mechanisms with \( \hat{E}_D^{ij} \), \( \hat{E}_E^{ij} \) and \( \hat{F}_C^{ij} \) orientations indicates that the mechanisms are not as diverse as found in some studies (e.g., Zoback et al., 1993; Rivera & Kanamori, 2002). Since mechanisms of \( \hat{E}_A^{ij} \), \( \hat{E}_B^{ij} \) and \( \hat{E}_C^{ij} \) can be generated by different fault orientations responding to the same loading tensor, relatively smooth variations in the stress field would be adequate to explain our results (c.f. Hardebeck, 2006).
4.5.5 Spatial Clustering in Zones of Fault Heterogeneity

The results presented in Section 4.4.5 serve as a caveat to the interpretation of the focal mechanism data as representing heterogeneity over the entire examined fault zones. Most focal mechanisms in this study belong to spatial clusters of earthquakes. These spatial clusters exist in what would be termed aftershock sequences as well as background seismicity. It is likely that these spatial clusters occur because of concentrations of stress that lead to a locally heterogeneous stress field or activate many smaller faults in the surroundings that have variable orientations. The focal mechanism data would therefore represent a bias towards the heterogeneous parts of the fault zones. If there are “locked” parts of the fault zone, which only fail in large ($M_L \geq 6$) earthquakes, it is likely that much of the strain release would be more homogeneous in those regions than the interseismic earthquake focal mechanisms indicate. This is consistent with the observation that all of the largest earthquakes are of strike-slip type (Section 4.4.7), and that $|r_{CLVD}|$ values are generally smaller for populations of larger earthquakes (Bailey et al., 2009). However, the relative differences in heterogeneity levels are still applicable to the entire fault zones, as implied by the correlations with $\alpha_{IQ}$ and $\alpha_{50}$. Due to the relative lack of seismicity on the smooth sections of the faults, we consider it likely that a more complete sample of the fault zone which is not limited to heterogeneous clusters would make the differences in heterogeneity more apparent.

The results in Sections 4.4.5–4.4.7 also highlight the effect of network sampling on our results. Both temporal and spatial variations in the sampling of earthquakes can lead to biases towards different focal mechanism styles when imposing quality restrictions on the focal mechanism catalog. As such we are not only limited in the spatial sampling of the fault zone, but even in the sampling of the earthquakes that occurred. However, results in Appendix I showed that differences between fault zones are apparent regardless of the data quality. Although it is difficult to constrain the biases towards spatial sampling and particular focal mechanism types, it is unlikely that these biases can explain the quantifiable differences between fault zones.
4.5.6 Objective Identification of Faults

Bailey et al. (2009) showed that a wide range of length-scales influence the deformation characteristics of a given region, and in this sense there appears to be a hierarchical structure of non-similar length-scales with a wide-range of sizes that influence deformation at plate boundaries. This combination of features indicative of different geometrical frameworks explains part of the difficulty in objectively defining a given fault zone. It would be difficult to perform our analysis in reverse and define fault zones from the focal mechanism differences, since the focal mechanism data are spatially distributed into variable sized clusters that have no unifying characteristic length-scale. The fault trace data provide an alternative approach for defining fault zones based on prior information and independent auxiliary data, which we have followed in this study. The use of independent data to define the fault zones strengthens our results, showing that two distinct methods show differences in the complexity of individual fault zones. However, we are limited by the coverage and resolution of the fault mapping as well as decisions made during mapping. Uncertainties related to these subjective parts of the mapping process, such as the reliability of offset geological markers, are difficult to quantify. Our result that there are differences between fault zones is therefore conditional on the assumption that the examined fault zones are properly defined (to first order) by the regions we use.

4.5.7 Fault Complexity vs. Fault Zone Complexity

Our results are applicable to fault zones rather than individual faults. Some studies indicating fault evolution are based on roughness of fault surfaces (e.g., Sagy et al., 2007; Candela et al., 2009) or width of a damage zone, which refer only to properties of single faults. It is reasonable to argue that the reverse and normal faulting mechanisms we have included in the SAF sections are off-fault seismicity not associated with the main fault trace. However, it is impractical to identify which earthquakes occurred exactly upon a given fault surface since the subsurface geometry cannot be well constrained. Furthermore, we would greatly restrict the number of data in our analysis. Earthquakes such as the 2002 Denali event can originate on smaller faults then propagate over great distances via other faults, indicating that it is important to consider off fault structures in the same context. Powers & Jordan (2009) showed that seismicity in southern California appears controlled
by faults to distances of $\sim 3$ km, which might be a better justification for the extent of fault zones. However the 15 km buffer in this study allows us to incorporate more data and gives some flexibility with respect to dipping faults.

4.6 Conclusions

Source mechanism summations are useful in quantifying variation in populations of focal mechanisms. Our results indicate that long term properties of fault zones represented by their fault traces correlate with focal mechanism heterogeneity which differs between fault zones. The scatter in focal mechanism orientations shows a reduction of complexity of fault traces which we relate to the evolutionary maturity of the fault zone. The asymmetry in the focal mechanism orientations, related to the fault zone azimuth, leads to a component of heterogeneity that is controlled by the overall orientation of the fault zone with respect to plate motions. We find that the spatial location of seismicity strongly affects on the distribution of focal mechanism orientations, and the use of focal mechanism data in our analysis is limited by how representative spatial clusters are in terms of the entire fault zone.
References


Hardebeck, J. L., Shearer, P. M., & Hauksson, E., 2005. A new earthquake focal mechanism
catalog for southern California, in 2005 SCEC Annual Meeting Abstracts, p. 130, Southern
California Earthquake Center, accessed at http://www.data.scec.org/research/altcatalogs.html,
July 2008.


Heaton, T. H. & Heaton, R. E., 1989. Static deformations from point sources and force couples
located in welded elastic Poissonian half-spaces: Implications for seismic moment tensors, Bull.

Heermann, R., Shipton, Z. K., & Evans, J. P., 2003. Fault structure control on fault slip and
ground motion during the 1999 rupture of the Chelungpu fault, Taiwan, Bull. Seismol. Soc. Am.,
93, 1034–1050.

Hill, M. L. & Dibblee, T. W., 1953. San Andreas, Garlock, and Big Pine faults, California; a study
of the character, history, and tectonic significance of their displacements, Bull. Geol. Soc. Am.,
64(4), 443–458.

Hillers, G., Ben-Zion, Y., & Mai, P. M., 2006. Seismicity on a fault controlled by rate-and
state-dependent friction with spatial variations of the critical slip distance, J. Geophys. Res.,

Hillers, G., Mai, P. M., Ben-Zion, Y., & Ampuero, J.-P., 2007. Statistical properties of seismicity of
fault zones at different evolutionary stages, Geophys. J. Int., 169(19), 515–533,

Holschneider, M. & Ben-Zion, Y., 2006. Bayesian estimation of faults geometry based on seismic


Jennings, C. W., 1975. Fault map of California with locations of volcanoes, thermal springs, and
thermal wells, no. 1 in Geologic Data Map, California Division of Mines and Geology,
Sacramento CA.


659–691.

102, 573–583.


A Computation of the Source Mechanism Tensor from a Fault Plane Solution

For a DC source mechanism there are three degrees of freedom since $P_{ij}$ is symmetric, has both trace and determinant constrained to be zero, and has a Euclidean norm of $\|P_{ij}\| = \sqrt{P_{ij}P_{ij}} = 1$. The zero determinant corresponds to the requirement that there is always an axis (the null axis) in the direction of which there is zero net deformation. The three parameters of a DC mechanism may also be given in terms of a slip direction given by the rake, $\gamma$, on a fault plane given by a strike, $\varphi$, and dip, $\delta$, (Aki & Richards, 2002, p.101). Since we assume that there is no net rotation of the source region during slip, corresponding values of $\varphi$, $\delta$ and $\gamma$ define compensating slip in a direction perpendicular to the fault slip along a auxiliary plane perpendicular to the fault plane. Which of the planes is the fault plane and which is the auxiliary plane is arbitrary in the framework of the DC description, producing the so called “fault plane ambiguity”.

The DC focal mechanism orientation (or fault plane solution), given in terms of $\varphi$, $\delta$ and $\gamma$, can be computed by inversion of the pattern of P- and S-wave polarities detected by the surrounding seismic stations (Aki & Richards, 2002). We convert values of $\varphi$, $\delta$ and $\gamma$ to $P_{ij}$ using the following relations, which are adapted from Aki & Richards (2002), p.112 using $x_1 = E$, $x_2 = N$, and $x_3 = up$:

\[
\begin{align*}
\hat{P}_{11} &= \frac{1}{\sqrt{2}} (\sin \delta \cos \gamma \sin 2\varphi - \sin 2\delta \sin \gamma \cos^2 \varphi) , \\
\hat{P}_{12} &= \frac{1}{\sqrt{2}} (\sin \delta \cos \gamma \cos 2\varphi + \frac{1}{\sqrt{2}} \sin 2\delta \sin \gamma \sin 2\varphi) , \\
\hat{P}_{13} &= \frac{1}{\sqrt{2}} (\cos \delta \cos \gamma \sin \varphi - \cos 2\delta \sin \gamma \cos \varphi) , \\
\hat{P}_{22} &= -\frac{1}{\sqrt{2}} (\sin \delta \cos \gamma \sin 2\varphi + \sin 2\delta \sin \gamma \sin^2 \varphi) , \\
\hat{P}_{23} &= \frac{1}{\sqrt{2}} (\cos \delta \cos \gamma \cos \varphi + \cos 2\delta \sin \gamma \sin \varphi) , \\
\hat{P}_{33} &= \frac{1}{\sqrt{2}} (\sin 2\delta \sin \gamma) .
\end{align*}
\]
In order to convert this to the potency tensor, each component should be multiplied by $P_0 / \sqrt{2}$, while conversion to the moment tensor requires multiplication by $\sqrt{2}M_0$, where $M_0$ is the scalar moment (e.g., Riedesel & Jordan, 1989; Aki & Richards, 2002). The difference between the position of the $\sqrt{2}$ in these conversions results from summation over elastic constants when calculating the moment tensor. This is consistent with the relation, $M_0 = \mu P_0$, where $\mu$ is the rigidity.

### B Quantification and display of the CLVD component

For DC source mechanisms, the eigenvalues of $\hat{P}_{ij}$ are constrained to be $\lambda_1 = -1/\sqrt{2}$, $\lambda_2 = 0$ and $\lambda_3 = 1/\sqrt{2}$, respectively, such that the eigenvalues for $P_{ij}$ are $-P_0/2$, 0 and $P_0/2$. To quantify the non-DC nature of a given summed tensor, we make use of the decomposition into DC and CLVD component tensors (Knopoff & Randall, 1970; Julian et al., 1998),

$$
P_{ij} = P^{DC}_{ij} + P^{CLVD}_{ij}
$$

$$
= \frac{1}{\sqrt{2}} (P^{DC}_{ij} \hat{P}^{DC}_{ij} + P^{CLVD}_{ij} \hat{P}^{CLVD}_{ij}) ,
$$

where the eigenvectors of $P_{ij}$, $\hat{P}^{DC}_{ij}$ and $\hat{P}^{CLVD}_{ij}$ are all the same. The eigenvalues of $\hat{P}^{CLVD}_{ij}$ are given by one of two special cases,

$$
\lambda_1 = \frac{-2}{\sqrt{6}}, \quad \lambda_2 = \frac{1}{\sqrt{6}}, \quad \lambda_3 = \frac{1}{\sqrt{6}},
$$

or

$$
\lambda_1 = \frac{-1}{\sqrt{6}}, \quad \lambda_2 = \frac{-1}{\sqrt{6}}, \quad \lambda_3 = \frac{2}{\sqrt{6}}.
$$

We use a convention that compression is negative, and hence the first case is analogous to uniaxial compression while the second is analogous to uniaxial extension.

Since $\lambda_2 = 0$ for the DC component tensor, a non-zero value of $P^{CLVD}_0$ can be interpreted in terms of the value of $\lambda_2$, i.e. the sense of deformation in the direction of the B-axis. For potency tensors generated by summation of DC potency tensors the CLVD component is described by the sign of $\lambda_2$ and the relative size of $P^{CLVD}_0$. 

111
A pure DC tensor is displayed graphically by a beachball plot where the two extensional and compressional regions intersect in the direction of the null axis. The lack of any such intersection indicates deformation in the B-axis direction, so it can no longer be termed the null axis, and this indicates the presence of a CLVD component. In our method of display, the B-axis symbol will plot in either a region of compression if $\lambda_2 < 0$ or extension if $\lambda_2 > 0$. For a deviatoric tensor, the areas denoting extension and compression must be equal and the absolute size of the CLVD component is then indicated by size of the smallest angular distance between the B-axis and the extensional/compressional boundary.

**C Comparison of measures for the CLVD component**

The measure of the CLVD component used in this paper is

$$r_{CLVD} = \frac{\sqrt{6}}{2} \lambda_2 .$$  \hspace{1cm} (C1)

In Figure C1, we compare this to the Gamma-index (Kagan & Knopoff, 1985),

$$\Gamma = - \frac{3}{2} \sqrt{6} \lambda_1 \lambda_2 \lambda_3 ,$$  \hspace{1cm} (C2)

and (Giardini, 1984),

$$f_{CLVD} = \frac{-\lambda_2}{\max(|\lambda_1|,|\lambda_3|)} .$$  \hspace{1cm} (C3)

**D Simulations of Random DC Orientations**

For summed tensors of DC mechanisms that are uniformly distributed in orientation we can expect that $r_{CLVD} = 0$. This is illustrated in simulations by selecting $N$ DC potency tensors from a uniform random distribution of orientations using the method of Kagan (2005), and computing the source
mechanism summation. The distribution of $r_{CLVD}$ based on 10,000 simulations for $N > 2$ follows a cosine function. We display four example values of $N$ in Figure D1a. The cumulative histograms in Figure D1b indicate that when $|r_{CLVD}| \gtrsim 0.35$ for $\hat{P}_{ij}^{SM}$, a hypothesis of the CLVD component being generated by random noise can be rejected at a confidence level of 90%. This implies that although $|r_{CLVD}|$ is a measure of fault complexity, it does not represent a degree of randomness.

For $\hat{P}_{ij}^{TOT}$ the value of $r_{CLVD}$ does not directly compare to these simulations since each potency tensor is weighted by $P_0/\sqrt{2}$ in the summation. While $r_{CLVD}$ for $\hat{P}_{ij}^{SM}$ relates to the amount of complexity in terms of number of events, $r_{CLVD}$ for $\hat{P}_{ij}^{TOT}$ relates more to the deformation of the considered volume and how significant that complexity is in terms of the overall deformation.

### E Equations for the computation of $\Omega$

Based on the eigenvectors, $e_1$, $e_2$ and $e_3$, of the source mechanism tensors, the minimum rotation angle from one orientation of principle strain axes to another is given by (Kuipers, 2002)

$$\Omega = \acos \left[ \frac{\max(Tr(R_1), Tr(R_2), Tr(R_3), Tr(R_4)) - 1}{2} \right], \quad (E1)$$
Figure D1: (a) Histograms of $r_{CLVD}$ based on $M = 10,000$ simulations where $N$ randomly oriented DC potency tensors have been summed, shown for $N = 3$, $N = 30$ and $N = 100$. The bins are of width $\Delta r_{CLVD} = 0.2$, and number of simulations in each bin is divided by $M \Delta r_{CLVD}$ to normalize the area under each histogram. The function $(2/\pi) \cos(\pi r_{CLVD})$ is overlain to illustrate the consistent shape. (b) Cumulative histograms of the same simulations, indicating the 2.5, 5, 12.5, 87.5, 95 and 97.5% values, which correspond to estimates of 75%, 90% and 95% confidence intervals for non-randomness given an absolute value of $r_{CLVD}$ in a source mechanism summation.

where

$$
\begin{align*}
R_1 &= (e_1^A, e_2^A, e_3^A)(e_1^B, e_2^B, e_3^B)^T \\
R_2 &= (-e_1^A, e_2^A, -e_3^A)(e_1^B, e_2^B, e_3^B)^T \\
R_3 &= (e_1^A, -e_2^A, -e_3^A)(e_1^B, e_2^B, e_3^B)^T \\
R_4 &= (-e_1^A, -e_2^A, e_3^A)(e_1^B, e_2^B, e_3^B)^T.
\end{align*}
$$

F Quantitative Comparison of Magnitude and Regional Summed Tensors

The difference angles $\Omega$ and $\Theta$ for comparisons between all magnitude-based subsets of Section 3.3.2 and all regional subsets of Section 3.3.3 are displayed in Figure F1 and Figure F2, respectively. Although both angles are given in degrees, the scales are not analogous, since $\Theta$ measures an angle between two points on a 9-dimensional hypersphere, whereas $\Omega$ quantifies an angle of rotation in three-dimensional space.
Figure F1: The differences between summed tensors for different magnitude bins given by the rotation angles, $\Omega$ and tensor difference angles, $\Theta$. Each plot shows one of the magnitude bins compared with the other four. Red symbols correspond to differences in $\hat{P}_{TOT}$, while green symbols correspond to differences in $\hat{P}_{SM}$. Each plot shows the median, 75% and 95% confidence intervals based on bootstrap analysis of the summed tensor pairs. Confidence intervals are so narrow for differences in $\hat{P}_{SM}$ in (b) that the box colors are not visible. A dashed line is plotted at $20^\circ$ for reference.
Figure F2: The differences between summed tensors for region bins given by the rotation angles, $\Omega$ and tensor difference angles, $\Theta$. Each plot shows one of the regions compared with the other six. The regions are ordered bottom–top/left–right based on the smallest to largest orientation differences, $\Omega$, between the $\hat{P}^{TOT}$ for the region and $\hat{P}^{TOT}$ for southern California. Red symbols correspond to differences in $\hat{P}^{TOT}$, while green symbols correspond to differences in $\hat{P}^{SM}$. Each plot shows the median, 50% and 95% confidence intervals based on bootstrap analysis of the summed tensor pairs. A dashed line is plotted at $20^\circ$ for reference.
Further Details about the Regional Subsets

As shown in Figure G1, both the number of earthquakes and total potency release for the regions vary significantly. Unlike the comparison for magnitude bins (Figure 3.1), log-scaling of the y-axis is not needed to compare regions, indicating that all regions have a significant contribution to the total summations in Figure 3.1. However, the relative differences in column height between the two histograms in Figure G1 imply that the relative contributions of each region are different for the two summation types, \( P_{ij}^{SM} \) and \( P_{ij}^{TOT} \). For example, the aftershocks of the Landers and Hector Mine earthquakes in region LAN dominate in terms of potency release and thus contribute more to \( P_{ij}^{TOT} \), while regions OWV and LAN dominate in number and jointly contribute more to \( P_{ij}^{SM} \). When considering the entire southern California region, region LA has considerably more influence in potency release than number, while the pattern for KERN is opposite. Given the respective significance of reverse and normal faulting in these regions, this may explain why \( r_{CLVD} \) is positive for \( P_{ij}^{TOT} \) and negative for \( P_{ij}^{SM} \) in Figure 3.1.

Figure G2 illustrates the regional differences in temporal seismic behavior by showing the increase with time of cumulative potency for \( M_L \leq 5 \) earthquakes in each region. A constant potency rate is indicated by the straight diagonal line, but all regions progress in a step-like fashion corresponding to the relatively large earthquakes. Regions where large earthquakes occurred (e.g. the 1987 Superstition Hills earthquake in SJ, the 1992 Landers and Big Bear earthquakes in LAN and SBM, the 1994 Northridge earthquake in LA and the 1999 Hector Mine earthquake in LAN) have the strongest divergence from a constant rate of potency release due to the contributions of large aftershocks (the mainshock events mentioned above are not represented in our \( M_L \leq 5 \) data). For regions having few notably large earthquakes (SSAF, KERN and OWV), the lines remain closer to the constant potency release rate. A further difference is seen in the curvature of lines after large steps, indicating decay in the number of aftershocks. Although the amount of curvature is affected by the scale of the y-axis, some regions (e.g. SJ and SSAF) show much less tendency for this curvature than others (e.g. LAN and OWV).
Figure G1: (a) Summed scalar potency and (b) total number of events in each of the seven regions shown in Figure 3.1.

H Fault zone selection

Table H1 shows the number of focal mechanisms and specifies the database names of faults used to define each of the fault zones. We consider all fault categories from the California Geological Survey & U.S. Geological Survey (2008) database and extract all fault segments by searching for the keywords given in Column 3 of Table H1. For the Elsinore fault zone, we exclude the southern-most fault section due to lack of seismic data. For the Garlock Fault, the eastern section is removed from the analysis since the change in fault strike is not sampled by the focal mechanism data. For the two ECSZ zones, we include all fault traces such that the Joshua Tree and Landers aftershocks are included in ECSZ 1, and the Hector Mine aftershocks are included in ECSZ 2. For the Coachella section of the San Andreas Fault, four nearby faults are incorporated so as to include the nearby focal mechanisms and increase the number of data. For the Parkfield section of the San Andreas Fault, the longitude and latitude limits are the extent of the catalog data.

I Tradeoff Between Number and Quality of Focal Mechanisms

Since most earthquakes are small they are not recorded by many seismic stations, but fewer recordings of a given event make it more difficult to constrain the focal mechanism orientation. Hence
<table>
<thead>
<tr>
<th>Year</th>
<th>Cumulative Potency Release ($P_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>0 cm km$^2$</td>
</tr>
<tr>
<td>1990</td>
<td>50 cm km$^2$</td>
</tr>
<tr>
<td>1995</td>
<td>100 cm km$^2$</td>
</tr>
<tr>
<td>2000</td>
<td>150 cm km$^2$</td>
</tr>
</tbody>
</table>

**Figure G2:** Cumulative potency release over the catalog time interval for $0 < M_L \leq 5$ earthquakes in each subregion used in this study. Locations of these regions are shown in Figure 3.1.
Table H1: Summary of data for the fault zones used in this study. The number $N$ of A and B quality focal mechanisms in each fault zone and the names of faults in the USGS fault database used to generate the zones.

<table>
<thead>
<tr>
<th>Name</th>
<th>$N$</th>
<th>Fault Database Search Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Jacinto</td>
<td>870</td>
<td>San Jacinto fault zone</td>
</tr>
<tr>
<td>Elsinore</td>
<td>84</td>
<td>Elsinore fault zone - all sections except the Laguna Salada section</td>
</tr>
<tr>
<td>Garlock</td>
<td>42</td>
<td>Garlock fault zone ($\text{lon.}, \lambda \leq -117.54^\circ$)</td>
</tr>
<tr>
<td>ECSZ 1</td>
<td>440</td>
<td>Johnson Valley fault zone</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Camp Rock-Emerson-Copper Mountain fault zone</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Homestead Valley fault zone</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Burnt Mountain fault</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Eureka Peak fault and Unnamed faults in Eureka Valley</td>
</tr>
<tr>
<td>ECSZ 2</td>
<td>51</td>
<td>Pisgah-Bullion fault zone</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mesquite Lake fault</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lavic Lake fault</td>
</tr>
<tr>
<td>SAF - Coachella</td>
<td>55</td>
<td>San Andreas fault zone, Coachella section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unnamed fault, Coachella Section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mecca Hills fault</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hidden Springs fault</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Powerline fault</td>
</tr>
<tr>
<td>SAF - Bernardino</td>
<td>543</td>
<td>San Andreas fault zone, San Bernardino Mountains section</td>
</tr>
<tr>
<td>SAF - Mojave</td>
<td>50</td>
<td>San Andreas fault zone, Carrizo section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>San Andreas fault zone, Mojave section</td>
</tr>
<tr>
<td>SAF - Parkfield</td>
<td>358</td>
<td>San Andreas fault zone, Creeping section ($\lambda \geq -120.89^\circ, \varphi \leq 36.30^\circ$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>San Andreas fault zone, Parkfield section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>San Andreas fault zone, Cholame-Carrizo section ($\lambda \leq -120.18^\circ, \varphi \geq 36.65^\circ$)</td>
</tr>
</tbody>
</table>

There is a tradeoff between the number of earthquakes and the quality level of a data set. This is illustrated by Figure I1, which compares the histograms of fault plane uncertainty ($FPU$) values for focal mechanisms in the nine fault zones from low and higher quality catalogs. The higher quality catalog uses only mechanisms of A and B quality, for which restrictions are outlined in Section 4.2.2, and the low quality catalog uses all mechanisms where $FPU < 45^\circ$. It is clear that many more mechanisms are available if quality restrictions are relaxed.

The advantages of using a higher number of data are that statistics are less affected by outliers, and that more of the fault zone is likely to be sampled by the data. However, greater uncertainty in the focal mechanism data will lead to greater variations in the focal mechanism orientations. This is important with relation to our study since uncertainties may overshadow the physical heterogeneity sampled by the focal mechanisms. We investigate this potential artifact by comparing results for
catalogs with our two different quality restrictions (Figure I2). For seven out of nine of the fault zones, the decrease in quality restrictions lead to lower values of $|r_{CLVD}|$ and increasing values of $\Delta r_{NORM}$. Exceptions to this pattern are the Garlock Fault and the Mojave Section of the SAF, but both have few data in the higher quality catalog. The bootstrap error bounds which show the maximum and minimum values from 10,000 resampled populations and constrain the robustness of the values, get smaller in all cases due to the increased number of data, though there is no relation between these error bounds and the results. Although the decrease in data quality leads to a general reduction in the relative differences of focal mechanism heterogeneity between fault zones, particularly for the Parkfield section of the SAF, distinct differences remain for the different fault sections despite high data uncertainties.

In order to understand how much of the differences in Figure I2 can be explained by data uncertainty and how much may result from changes in the spatial sampling, we consider an approach that uses a kernel representation of the focal mechanisms. By representing each DC orientation as a kernel function with width related to uncertainty, then randomly sampling from the combined kernel density using $N$ points many times, we incorporate uncertainties related to the number of data $N$ and the data quality into the heterogeneity results.
Figure I2: Comparison of heterogeneity results for two different quality levels in the catalog. (a) Colored squares show the results from Figure 4.1 for higher quality data in the different fault zones, while (b) triangles show the results from a catalog where $FPU < 45^\circ$. In both cases, the error bars show the minimum and maximum from a bootstrap resampling of the results.

To simulate uncertainty of a single DC orientation, we use a kernel function of the form

$$ f(\Omega) = Ce^{\kappa \cos(\Omega)}, $$

where $\Omega$ is the minimum rotation angle of principle strain axes from a mean orientation, $\kappa$ is a concentration parameter, and $C$ is a normalizing factor.

Kilb & Hardebeck (2006) found that $FPU$ is the most representative quantification of focal mechanism quality for HASH generated data. We relate $\kappa$ to $FPU$ by generating synthetic populations of DC orientations for different values of $\kappa$, then computing $FPU$ from the variation in the population using the codes made available by Hardebeck & Shearer (2002). We can generate a population of DC orientations that sample Eq. I1 for a given value of $\kappa$ by the Monte Carlo method: we select a DC orientation from a uniform random distribution using the equations of Kagan (2005),
Values of $FPU$ are computed for populations of DCs generated by random sampling the von Mises-Fisher kernel 10,000 times using different values of $\kappa$. A linear relationship between $\log(FPU)$ and $-\log(\kappa)$ is fitted for $FPU < 45^\circ$ by least squares and overlain as a solid blue line. The red dashed line shows the cutoff of the scaling relationship.

Figure I3: Values of $FPU$ are computed for populations of DCs generated by random sampling the von Mises-Fisher kernel 10,000 times using different values of $\kappa$. A linear relationship between $\log(FPU)$ and $-\log(\kappa)$ is fitted for $FPU < 45^\circ$ by least squares and overlain as a solid blue line. The red dashed line shows the cutoff of the scaling relationship.

compute the minimum rotation angle $\Omega_i$ to the mean orientation, compute a uniform random variable $Y_i \sim U(0, 1)$ and retain the DC orientation in the sample if $Y_i \leq \exp[\kappa \cos \Omega_i]$. The results of our simulations (Figure I3) suggest a scaling relationship of the form

$$\kappa = A e^{-B \log(FPU)},$$

(12)

when $FPU \leq 45^\circ$, with $A = 2.688 \times 10^4$ and $B = 2.011$. The scaling relation appears to break down for $FPU > 45^\circ$ and tends towards a uniform distribution ($\kappa = 0$). This indicates that there is little informational value in using focal mechanisms with $FPU > 45^\circ$.

We combine Eq. I1 and Eq. I2 to generate a density of kernels for both levels of catalog quality in each fault zone. Comparing results based on the kernel densities for different quality levels takes into account that the distribution of uncertainties for the two populations is not the same, which is not done in Figure I2. We can take uncertainties related to the number of data into account by random sampling of each kernel density with a number of DC orientations equal to the number of
data \( N \). We compute \( \Delta r_{\text{NORM}} \) and \( r_{\text{CLVD}} \) from \( N \) sampled DC orientations, repeating the sampling 1,000 times to obtain a distribution for the two measures. The range in \( \Delta r_{\text{NORM}} \) and \( r_{\text{CLVD}} \) quantifies how well constrained the kernel density can be given the number of data available.

We generate a third kernel density where the mean DC orientations of the kernels are randomly selected from the higher quality catalog, but using the same number of kernels and values of \( FPU \) as the lower quality catalog. This allows us to test whether uncertainties as represented by Eq. I1 account for the differences in observed heterogeneity for the two catalog quality levels.

Figure I4 shows the results for \( \Delta r_{\text{NORM}} \) and \( r_{\text{CLVD}} \) from samples of the three kernel densities in each of the fault zones. The differences between results for the low and higher quality catalogs are similar to those observed in Figure I2, such that \( |r_{\text{CLVD}}| \) decreases and \( \Delta r_{\text{NORM}} \) increases as quality decreases. However, differences in \( \Delta r_{\text{NORM}} \) are much greater due to the increased uncertainty in lower quality data.

The results for \( \Delta r_{\text{NORM}} \) from the case where uncertainty is added to the higher quality data are similar to values for the lower quality data. This indicates that changes in \( \Delta r_{\text{NORM}} \) due to the increased number of data can be explained by data uncertainty. Hence the addition of data provide little extra information on the \( \Delta r_{\text{NORM}} \) part of the physical heterogeneity of the fault zones that does not exist in the higher quality data. Since uncertainty has such a large effect on \( \Delta r_{\text{NORM}} \), it is important to have as similar distribution of uncertainties as possible for the populations when comparing relative differences in the heterogeneity of populations. The multiple conditions used in the A and B category definitions are preferred in this regard since the acceptable variation in the data set is less. For this reason, we concentrate only on the higher quality data in our analysis.

The changes in \( r_{\text{CLVD}} \) due to the simulated effects of increased uncertainty are small, and do not explain the differences between results for the low and higher quality data. This shows that the systematic decrease in \( |r_{\text{CLVD}}| \) with the addition of lower quality data cannot be explained by our representation of uncertainty by Eq. I1. Hence, either the kernel function is inappropriate to model the uncertainties or the heterogeneity of the lower quality data is different. Non-zero values of \( r_{\text{CLVD}} \) are generated by anisotropic variation in DC orientations, and therefore cannot be generated from an isotropic distribution such as in our kernel representation Eq. I1. In this respect, any kernel
Figure I4: Comparison of heterogeneity for data that sample three kernel densities representing different quality levels in the catalog. Squares show median values of $r_{CLVD}$ and $\Delta r_{NORM}$ for the higher quality catalog, triangles correspond to the lower quality catalog and circles correspond to the resampled high quality data with a simulated effect of uncertainty. Dashed lines show the changes as lower quality data are included. Error bars show the 75% confidence limits for the bootstrap resampling of $r_{CLVD}$ and $\Delta r_{NORM}$ which are in many cases smaller than the symbols. We use 75% confidence limits, rather than the minimum and maximum used in Figure 4.1 and Figure I2, to make comparison of values easier.

function which depends only on $\Omega$ is likely to produce similar patterns. It is likely that anisotropy may result from limitations in the distribution of the seismic stations to sample the focal sphere of the earthquake. The distribution of stations has a strong influence on the quality of focal mechanism solutions, and quality restrictions may lead to a bias towards certain DC orientations which may that are easier to constrain in certain locations. We see evidence for such a bias in Section 4.4.7, where we show that dip-slip events are better sampled than strike-slip events at low magnitudes for the San Jacinto fault zone. It is therefore more likely that higher quality data result in biases towards more anisotropic variations than low quality data. Accounting for such a bias is beyond the scope of this study, but we assume that the relative differences in the bias do not differ strongly for the nine fault zones, and accept that true values of $r_{CLVD}$ are likely to be closer to zero than suggested by the higher quality data.
Interpretation of Heterogeneity Measures Using Numerically Simulated Variation of DC Orientations

The measures $\Delta r_{NORM}$ and $r_{CLVD}$ provide useful measures of focal mechanism heterogeneity that are easy to compute, but their physical interpretation is non-intuitive. In order to better understand the type and amount of variation that can lead to different values of each quantity, we simulate variation in populations of 1,000 DC focal mechanisms and compute the values of $\Delta r_{NORM}$ and $r_{CLVD}$ from each simulation. We impose variation on the population in two ways: by selecting focal mechanisms from a Fisher distribution (Eq. I1) with changing values for the concentration parameter $\kappa$, and by mixing different proportions of populations with different mean orientations.

A Fisher distribution as defined in Eq. I1 leads to axially symmetric and equal variation of all three strain axes about a pre-defined mean DC orientation. We arbitrarily select a strike-slip mechanism as our mean orientation and investigate four different values of $\kappa$: 10, 20, 50, 100 and 200. The synthetic data sets are generated by Monte Carlo sampling, as described in Appendix I.

Figure J1 displays the variation of principle strain axes for different values of $\kappa$. In these cases the $\kappa$ values 10, 20, 50, 100 and 200 correspond to variation such that 90% of the principle strain axes are within approximately $74^\circ$, $55^\circ$, $34^\circ$, $25^\circ$ and $17^\circ$ of their pre-defined mean values, and values of $\Delta r_{NORM}$ that are 0.71, 0.46, 0.21, 0.11 and 0.06, respectively (Figure J1c).

In order to simulate asymmetry in the population we divide the 1,000 focal mechanisms into subsets of first order, second order and third order faulting/DC orientations. If we denote the mean DC orientation of the first order faulting population $\hat{E}_{DC}^{ij}$, as defined in Eq. 4.2, the mean DC orientations of the second and third order faulting populations correspond to $\hat{E}_{ij}^{B}$ and $\hat{E}_{ij}^{C}$ in Eq. 4.1. Since we define $\hat{E}_{ij}^{DC}$ to be a strike-slip mechanism as our dominant orientation, the second and third order populations in our simulations correspond to reverse and normal faulting mechanisms, respectively, though this decision is arbitrary and does not affect the results. In each simulation the value of $\kappa$ is constant for all three subsets. We investigate cases where the number of reverse faults $N_R$ is 0, 100, 200, 300, 400 and 500, and the number of normal faults $N_N$ is 0, 100, 200 and 300.
Figure J1: Angular variation of the P and T axes and values of $\Delta r_{NORM}$ for a single dominant faulting type with different values of $\kappa$. (a) Red and blue circles show the orientation of 10,000 P and T axes, respectively when $\kappa = 200$. A black dashed line circle shows $\theta_{90}$, the angle from the mean within which 90% of the simulated data are located. (b) Red and blue circles show the orientation of 10,000 P and T axes, respectively when $\kappa = 10$. Black dashed line circle shows $\theta_{90}$ for five different values of $\kappa$. (c) The relationship between $\theta_{90}$ and $\Delta r_{NORM}$ for the simulated data sets where there is no mixing of fault types.

For each of the 120 synthetic data sets, we compute $E_{ij}$ and then $\Delta r_{NORM}$ and $r_{CLVD}$. We find that $\Delta r_{NORM}$ is inversely related to the value of $\kappa$ as well as being inversely related to the percentage of strike-slip events (Figure J2a). If $\kappa$ and the percentage of strike-slip events are constant, $\Delta r_{NORM}$ increases as the percentages of normal and reverse faulting mechanisms become closer together. The lowest value of $\Delta r_{NORM}$ for each value of $\kappa$ corresponds to cases where there is no fault mixing (Figure J1).

Figure J2(b) shows the positive correlation of $r_{CLVD}$ with $N_R - N_N$, regardless of $\kappa$, such that $r_{CLVD}$ is roughly equal to $(N_R - N_N)/N_{TOTAL}$. This may be stated more generally as $r_{CLVD} \approx (N_B - N_C)/(N_A + N_B + N_C)$, where $N_A \geq |N_B - N_C|$ and is the number of DCs with the first order faulting orientation, and $N_B$ and $N_C$ correspond to the number in the second and third order faulting populations. However, this relationship breaks down for lower values of $\kappa$ since the mechanisms become so scattered that $E_{ij}$ is no longer aligned with the mean strike-slip orientation. We therefore do not plot the result for $\kappa = 10$ in Figure J2.

The cases presented here relate to variation of the entire focal mechanism, such that we are effectively simulating variation of the fault orientation and slip vector simultaneously. Alternative
cases can be considered where the deviatoric loading stress orientation is kept constant, and DC orientations are computed due to maximum shear stress resolved onto fault planes of varying orientations, consistent with the assumptions made for stress inversion (Michael, 1987). In the results of simulations for these cases, which are not presented here, $\Delta r_{NORM}$ is related to the variation in fault plane orientations. A non-zero CLVD component can be generated in two ways: either by two separate populations of fault orientations, such that resolved shear stress leads to slip in perpendicular directions, or by introducing a CLVD component to the loading tensor. In the first case, reduction in the variation of fault orientations restricts the directions of slip possible, and the value of $r_{CLVD}$ becomes larger as $\Delta r_{NORM}$ becomes smaller. In the second case, increased variation of the fault orientations leads to better sampling of the overlying loading tensor, and $r_{CLVD}$ becomes larger as $\Delta r_{NORM}$ becomes larger.
**Figure K1:** Average minimum rotation angle between all DC pairs in the fault zones as a function of $\Delta r_{NORM}$.

**K    Relation between distribution of minimum DC rotations and heterogeneity measures**

Using the equations of Kagan (2005) we compute the minimum rotation angles $\Omega$ and the rotation axes between all pairs of DC orientations in each of our fault zones for the A and B quality focal mechanisms. Figure K1 shows that the average of $\Omega$ correlates with $\Delta r_{NORM}$. The measures are not exactly analogous since $\Omega$ measures the orientation difference in three dimensional space while $\Delta r_{NORM}$ relates to differences in the space of $\hat{P}_{ij}$. Figure K2 shows the orientation density of rotation axes for the minimum rotations and illustrates visually that the distribution is more clustered where $|r_{CIVD}|$ is larger.

**L    Total Cumulative Offset for Fault Zones**

Table L2 shows a compilation of total cumulative offset for the nine fault zones in this study. We take values from two separate sources (Petersen & Wesnousky, 1994; Powell et al., 1993) and give the relevant citations for each value.
Figure K2: Density of rotation axes orientations for minimum rotations between all DC pairs in the fault zones using equal area lower-hemisphere projections with 5° binning. The density distribution in each case is normalized such that 1 indicates a uniform distribution, but the color scale is truncated at 5 to allow for comparison. DC orientations for $E_{ij}$ are overlain as red lines, with the P and T axes shown by white and black circles, respectively. The value of $r_{CLVD}$ is given below each of the circles.
<table>
<thead>
<tr>
<th>Name</th>
<th>PetWes94 Slip (km)</th>
<th>Notes/Reference</th>
<th>PowWelMat93 Slip (km)</th>
<th>Notes/Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Jacinto</td>
<td>24</td>
<td>Sharp (1967)</td>
<td>24–32</td>
<td>Offset crystalline rocks (see Powell et al., 1993, p. 55)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>23–26</td>
<td>Offset lacustrian units: Matti &amp; Morton (1975)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>18–24</td>
<td>Offset faults (see Powell et al., 1993, p. 55)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>28</td>
<td>Powell (1993)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>Powell (1993)</td>
</tr>
<tr>
<td>Garlock</td>
<td>48–64</td>
<td>Davis &amp; Burchfiel (1973)</td>
<td>48–64</td>
<td>Offset lith./struct. features (see Powell et al., 1993, p. 71)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8–12</td>
<td>Powell (1993)</td>
</tr>
<tr>
<td>ECSZ - Zone 1</td>
<td>1.6–4.0</td>
<td>Camp Rock fault – Dokka (1983)</td>
<td>4</td>
<td>Camp Rock-Emerson fault – Powell (1993)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>Blackwater-Calico-Mesquite Lake fault, Powell (1993)</td>
</tr>
<tr>
<td>SAF - S. of Tejon Pass</td>
<td>-</td>
<td></td>
<td>210–240</td>
<td>Offset crystalline rocks (see Powell et al., 1993, p. 117)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>204–210</td>
<td>Powell (1993)</td>
</tr>
<tr>
<td>(Mojave, Bernardino,</td>
<td></td>
<td></td>
<td>159–165</td>
<td>Matti &amp; Morton (1993)</td>
</tr>
<tr>
<td>Coachella)</td>
<td></td>
<td></td>
<td>165–205</td>
<td>Dillon &amp; Ehlig (1993)</td>
</tr>
<tr>
<td>SAF - Coachella</td>
<td>-</td>
<td></td>
<td></td>
<td>Offset crystalline rocks (see Powell et al., 1993, p.47)</td>
</tr>
<tr>
<td>SAF - N of Tejon Pass</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Parkfield)</td>
<td></td>
<td></td>
<td>260</td>
<td>Hill &amp; Dibblee (1953)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>295</td>
<td>Powell (1993)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>315</td>
<td>Sims (1993)</td>
</tr>
</tbody>
</table>

**Table L2:** Compiled results for total cumulative offset of the faults relevant to this study. PetWe94 refers to the reference Petersen & Wesnousky (1994), and PowWelMat93 refers to the reference Powell et al. (1993).