Mechanics of Oblique Convergence

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The distribution and magnitude of the strike-parallel component of velocity in an obliquely converging thrust wedge or accretionary prism are determined by the geometry and mechanical properties of the wedge. A mechanical analysis based on the assumption of a critical or stable geometry of the wedge, for which the rate of cross-strike deformation is zero, leads to the following conclusions for different bulk rheologies. (1) In a linear viscous wedge, the strike-parallel motion relative to the underthrust slab decreases exponentially away from the rear and is effectively concentrated in a shear zone with a width comparable to the thickness of the wedge at the rear. The wedge also deforms by corner flow, producing a circulation in the cross-strike plane. The strike-parallel and corner flow velocities depend on the thickness and viscosity of the wedge and on the shear stresses applied to its lower and rear boundaries. Convergence at the wedge front is normal to strike. (2) A critically tapered perfect plastic wedge moves coherently without internal deformation. For low and moderate obliquities of the convergence vector, the wedge moves at the same velocity as the backstop (upper plate). For high angles of obliquity, the wedge moves laterally relative to the underthrust slab at a maximum velocity dependent on its dimensions and the stress conditions on its boundaries, so that it is separated from the upper plate by a strike-slip fault, defining a forearc sliver. No geometrical configuration exists that allows the strike-parallel motion to be distributed through the wedge. (3) A noncohesive Coulomb wedge behaves in much the same way as a plastic wedge, but the geometry and velocity depend only on its mechanical properties and the shear stresses on its boundaries, and they are independent of scale.

INTRODUCTION

The way in which oblique convergence at plate margins is partitioned into components of strike-parallel and strike-normal motion has been a topic of active discussion for some time. Observational evidence is strongly in favor of a high degree of partitioning: the pattern of seismicity in obliquely converging accretionary wedges suggests that the relative slip vector in the frontal region of the forearc is commonly significantly less oblique than the overall relative plate motion at the boundary [De Mets et al., 1990; Ekström and Engdahl, 1989; McCaffrey, 1991; R. McCaffrey, Constraints on mechanical behavior of forearcs from slip vector azimuths and oblique plate convergence, submitted to Journal of Geophysical Research, 1992; G. Yu et al., Slip partitioning along major convergent plate boundaries, submitted to Journal of Geophysical Research, 1992] and many such margins show active strike-slip faults at the rear of the system, within or behind the volcanic arc, taking up a significant proportion of the strike-parallel motion [Fitch, 1972; Beck, 1983, 1986; Lettis and Hanson, 1992]. In some cases the strike-parallel component of motion is distributed over a zone of finite width some distance behind the surface trace of the subduction zone [Dewey and Lamb, 1992; England and Wells, 1991]. Structural observations from older mountain chains also suggest a degree of partitioning [Ellis and Watkinson, 1987; Cashman et al., 1992; Cobbold et al., 1991; Corsini et al., 1992]. Lineations on faults in the Makran, for example, lie between the plate convergence vector and the strike-normal direction or show a bimodal distribution [Platt et al., 1988]; and in arcuate belts such as the Alps, lineations show a radial pattern in the more external parts of the arc, trending at high angles to the chain, whereas strike-parallel lineations are more common in the internal parts of the system [Platt et al., 1989].

Mechanical analysis of the problem has been based on three alternative approaches. Beck [1983, 1986] calculated the degree of partitioning between a thrust fault beneath the forearc region and a strike-slip fault behind it that would dissipate the least energy. McCaffrey [1992] compared results using this method with those using a force balance approach, calculated by integrating the shear stresses on the two boundaries. Both methods assume frictional behavior on the faults and treat the forearc region as a nondeforming block with dimensions that have to be independently specified. The third method is based on a continuum mechanics approach, treating the forearc lithosphere as a thin viscous sheet [England et al., 1985]. Beck's and McCaffrey's calculations suggest that a degree of partitioning is likely, and McCaffrey [1992] shows that under these assumptions there is a maximum degree of obliquity, independent of the obliquity of the plate motion, that will be observable at the front of the forearc. This obliquity is the arcsine of the ratio of the resistances on the lower and rear boundaries of the forearc, and the seismological evidence suggests that this angle varies from about 15 to 40° [McCaffrey, 1992]. The analysis by England et al. [1985], on the other hand, predicts an exponential increase in the strike-parallel component of motion towards the plate boundary.

The approach I have taken here differs in several respects from these analyses. The frontal regions of forearcs are commonly occupied by accretionary wedges or thrust belts that undergo high rates of internal deformation related to the relative plate motion. A body of recent work has shown that these wedges adopt a geometry that is a function of the stresses applied at their boundaries and their internal bulk rheology [Elliott, 1976; Chapple, 1978; Stockmal, 1983, Emerman and Turcotte, 1983; Davis et al., 1983; Dahlen et al., 1984; Dahlen, 1984; Platt, 1986]. All these analyses were applied to the two-dimensional case, assuming arc-normal convergence. In this paper I extend this approach into three dimensions for the case of a linear system. I have done this for viscous, plastic, and Coulomb rheologies and for a variety of boundary stress conditions, including frictional. Depending on the rheo-
Theorology, some or all of the geometrical properties of the system can be predicted from the internal mechanics and do not have to be independently specified. Predictions of the distribution and magnitude of the strike-parallel component of velocity arise from these calculations and vary according to the assumed rheology. The results therefore provide potential tests for the bulk rheology of accretionary wedges.

The region considered in my analysis is strictly the deforming and deformable part of the frontal forearc region, which is largely composed of accreted crustal material lying above a relatively rigid subducting slab. The rear of this system is likely to lie in the region of the forearc high or the forearc basin behind it. This is a more restricted region than that considered by Beck [1983, 1986], McCaffrey [1992], or England et al. [1985], who consider the whole forearc, including the underlying lithosphere, at least as far back as the magmatic arc. Both approaches may be valid, but the results may not be directly comparable.

**MECHANICAL ASSUMPTIONS**

The analysis is based on the following assumptions:

1. The deforming material is assumed to behave as a medium without internal mechanical discontinuities, in which the stresses and rates of deformation vary continuously (the continuum approximation). This is clearly not true: accretionary wedges and thrust belts contain large faults, with displacements of kilometers or tens of kilometers. Many previous successful studies, however, suggest that the mechanical behavior of these systems, viewed on a sufficiently large scale, can be predicted on the basis of the continuum approximation. It is important to note, however, that the predictions are only valid on a scale larger than that of the discontinuities.

2. The analysis presented here is for a very specific mechanical configuration, on which a considerable amount of research has already been done. This has allowed me to make considerable use of work already done by others. The configuration (Figure 1) is based on an idealized concept of accretionary wedges and thrust belts, with a deforming wedge of material bounded below by a rigid underthrust slab and behind by a rigid backstop. The wedge is also assumed to be linear and of infinite extent. This last assumption means that the analysis is essentially two-dimensional, although it predicts three-dimensional velocity distributions and stress states. The orientation of the backstop (shown as vertical in Figure 1) is not critical at the scale of this analysis, although it will affect the details of flow in its vicinity.

3. The wedge is assumed to have a critical geometry. Previous mechanical analyses of accretionary wedges and thrust belts suggest that the state of stress in the wedge is a function of its geometry and the tractions on its boundaries. If the stress state exceeds the mechanical strength of the material, the wedge deforms internally. The change of wedge geometry caused by this deformation results in a decrease in the intensity of deviatoric stress. Deformation therefore continues until the wedge reaches a configuration such that the deviatoric stress intensity is no longer sufficient to cause any change in its geometry. If the wedge is composed of a material with a finite strength (e.g., a Coulomb or plastic material), this will happen when the deviatoric stress intensity at any point is such that the material is just on the point of failure or plastic yield. Such configurations are known as critical geometries. Coulomb or plastic wedges have two critical geometries: they can be on the point of horizontal compressional deformation, in which case they have a critical geometry for compression; or they can be on the point of horizontal extension, in which case they have a critical geometry for extension. A viscous wedge will deform until the stretching rates (and hence the deviatoric normal stresses) normal and parallel to the boundaries of the wedge are zero, although shear deformation parallel to the boundaries may continue. Such a configuration is known as a stable geometry, and there is only one such geometry. In this paper I will generally assume that Coulomb and plastic wedges have reached their critical geometry for compression and that viscous wedges have reached the stable geometry.

4. The analyses presented here have been carried out for three different isotropic bulk rheologies for the wedge, which generate the tractions that control its behavior. Emerman and Turcotte [1983] used a no-slip boundary condition in their analysis of the corner-flow problem in viscous accretionary wedges. This is a common assumption in hydrodynamic analyses of problems involving fluids moving over solid surfaces and assumes molecular coupling between the two media. Chapple [1978] used a constant shear stress boundary condition in his analysis of thrust wedges: this is an appropriate assumption for a plastic wedge, equivalent to assuming that there is a thin layer of plastic material (assumed to be weaker than the wedge itself) along its boundaries with the rigid slab and backstop. Davis et al. [1983] used a normal-stress-dependent boundary condition for a wedge with Coulomb behavior, equivalent to assuming frictional behavior along the boundaries. Another possibility is to assume that the shear stress on a boundary is a function of the velocity difference across it. This is equivalent to assuming that there is a thin layer of fluid with viscous properties along the boundary. For various reasons that will be discussed below, I found velocity-dependent shear stress boundary condition the most appropriate assumption for the analysis of a viscous wedge.

**FORCE BALANCE EQUATIONS**

In this section I derive the force balance equations for the wedge configuration illustrated in Figure 1, as these are common to all the analyses. The Cartesian coordinate system and reference frame used are illustrated in Figure 1. The $x$ axis is taken up the dip of the underthrust slab, which is assumed to be planar. The $y$ axis is horizontal and parallel to the intersection of the underthrust slab and the backstop and is the invariant direction in the wedge geometry. The $z$ axis is normal to $x$ and $y$ and is assumed to lie in the plane of the backstop. Note that $x$ and $y$ are not horizontal and vertical, but the difference ($\beta$) is generally small, and throughout this paper "vertical" and "horizontal" should be understood as meaning parallel to $z$ and to the $xy$ plane, respectively. The wedge has surface slope $\alpha$, basal slope $\beta$, an angle of taper $\theta = \alpha + \beta$, and...
Fig. 1. (Top) mechanical and (bottom) kinematic configuration of an obliquely converging forearc wedge, as discussed in this paper. Note that the x axis is inclined to the horizontal by the angle \( \beta \). The wedge is assumed to be linear and of effectively infinite extent in the x direction. Top inset shows the components of the stress tensor: note that the near face is the negative x face and that the stress components acting on it are equal and opposite to those that act on the positive x face (hidden). Bottom inset shows a plan view of the velocity components: the backstop moves at \( V \) and points in the deforming wedge at \( v \) with respect to the lower plate.

Local thickness \( h \) measured in the z direction. \( \theta \), \( \alpha \), and \( \beta \) are assumed to be small angles for the purposes of approximations. The underthrust slab is taken as the reference frame for velocities. The backstop moves at velocity \( V \) in the xy plane at an angle of obliquity \( \gamma_0 \) to x. The wedge itself may be deforming, and material points within it have a velocity \( v \) that is variable. The local obliquity of the velocity is given by \( \tan \gamma = v_y/v_x \) (Figure 1). For stresses, I use the engineering sign convention (tensile stresses positive) throughout. The wedge is assumed to have an average density \( \rho \), and the effect of partial or complete immersion in water is not considered here.

The essential starting point for all analyses of this type are the stress equilibrium equations

\[
\nabla \cdot \mathbf{T} + \rho g = 0
\]  

(1)

where \( \mathbf{T} \) is the equilibrium stress tensor. If \( y \) is horizontal, these can be written

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} - \rho g \sin \beta = 0, 
\]

(2a)

\[
\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0, 
\]

(2b)

\[
\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} - \rho g \cos \beta = 0. 
\]

(2c)

Two additional assumptions allow them to be simplified further. First, if the system extends infinitely in the y direction, all gradients in that direction must on average be zero and can be neglected. Second, because the wedge is thin compared with its horizontal dimensions, the stress gradient \( \partial \sigma_{zz}/\partial x \) is likely to be small compared to \( \partial \sigma_{zz}/\partial z \) and can be neglected. Hence

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} - \rho g \sin \beta = 0, 
\]

(3a)
\[ \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0, \quad (3b) \]
\[ \frac{\partial \sigma_{zz}}{\partial z} - \rho g \cos \beta = 0. \quad (3c) \]

Some statements can be made about the other stress components. Because the wedge is invariant in y, the stretching rate \( D_{yy} \) in this direction must be zero. For viscous wedges, and for critical plastic or Coulomb wedges, this condition requires that the deviatoric stress component \( \sigma_{yy} \) must also be zero. This point has to be demonstrated separately for each rheology, so it will be simply stated here as a proposition. Hence \( \sigma_{yy} = \sigma_m \), where \( \sigma_m \) is the mean stress, given by
\[ \sigma_m = \left( \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \right)^{\frac{1}{3}}. \quad (4) \]

It follows that \( \sigma'_{xx} = -\sigma'_{zz} \), and hence
\[ \sigma_{xx} = \sigma_{zz} + 2\sigma'_{xx}. \quad (5) \]

Assuming that the vertical stress is simply due to the weight of the overlying rock and that \( \beta \) is small, the stress equilibrium equation (3c) can be integrated with respect to \( z \) and substituted into (5) to give
\[ \sigma_{xx} = \rho g (z - h) + 2\sigma'_{xx}. \quad (6) \]

Stress equilibrium equation (3a) can now be integrated with respect to \( z \), taking a depth averaged value for \( \overline{\sigma}_{xx} \) (indicated by the overbar). Note also that \( dh/dx = -\tan \theta \), and also that \( \sigma_{xx} = (\tau_b)_x \) (the x component of the basal shear traction; Figure 1) at the base of the wedge and is zero on its free upper surface. Using small-angle approximations, this leads directly to the force balance equation in the \( x \) direction on a vertical slice through the wedge:
\[ (\tau_b)_x - \rho g h \alpha x + 2\overline{\sigma}_{xx} \theta - \frac{2h\partial \overline{\sigma}_{xx}}{\partial x} = 0. \quad (7) \]

Similarly, integrating stress equilibrium equation (3b) with respect to \( z \), taking a depth averaged value for \( \overline{\sigma}_{xy} \), gives the force balance equation in the \( y \) direction:
\[ (\tau_b)_y + \overline{\sigma}_{xy} \theta - \frac{h \partial \overline{\sigma}_{xy}}{\partial x} = 0. \quad (8) \]

These two equations relate the geometry of the wedge to the shear stress on its base and to the state of stress within it. The latter is a function of the bulk mechanical properties of the wedge, so application of these equations depends on assumptions about the rheology.

**Linear Viscous Wedge**

**Rheology and Assumptions**

The governing relationships are assumed to be
\[ T'' = 2\mu D \quad (9a) \]
where \( \mu \) is a constant viscosity, and
\[ D_{zz} = 0. \quad (9b) \]
(incompressibility criterion).

The components of the rate of deformation tensor are related to the velocity gradients [Malvern, 1969]:
\[ D_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \quad (10) \]
in this region. Much the same problem applies to the normal-
stress-dependent shear stress condition appropriate to friction-
al behavior.

The boundary condition I have chosen to use here is a
velocity-dependent shear stress condition. This is the most
appropriate for a viscous wedge, as it is equivalent to assuming
that there is a thin layer of relatively low viscosity fluid along
the boundary. This layer would correspond in real terms, for
example, to a décollement horizon at the base of the wedge
composed of weak or overpressured sediment. The condition is
applied by assuming that the shear stress on the boundary is
related to the velocity difference between the material of the
wedge and the boundary by a coupling constant. The shear
stresses along the base are then

\[
\begin{align*}
\tau_b &= (\sigma_{bx})_b = \rho (v_x)_b, \\
\sigma_{by} &= \rho (v_y)_b,
\end{align*}
\]

and on the backstop they are

\[
\begin{align*}
\sigma_{xy} &= q(v_y)_b - \nu \cos \gamma_0, \\
\sigma_{xz} &= q(v_z)_b.
\end{align*}
\]

where \(p\) and \(q\) are coupling constants on the base and backstop
respectively.

The significance of the coupling constants can be appreciated
as follows. If a velocity \(v\) is imposed across a layer with
thickness \(h\) of fluid with viscosity \(\mu\) between parallel plates
with a no-slip boundary condition, the shear stress parallel to
the velocity vector is \(\sigma = \mu v/h\). If on the other hand a
coupling constant \(p\) on a fluid boundary produces a shear stress
\(\tau\) in response to a velocity difference \(v\), then \(\tau = pv\). For \(\tau\) to
equal \(\sigma\), \(p\) must equal \(\mu/h\). Hence if a coupling constant
\(p = \mu/h\) acts on a fluid layer of thickness \(h\) and viscosity \(\mu\),
with a velocity contrast \(v\) across the boundary, the resulting
shear stress adjacent to the boundary is the same as the shear
stress that would result from the layer being sheared with a no-
slip boundary condition between plates moving at \(v\) relative to
each other.

A direct comparison between the effects of the coupling
constants and the no-slip boundary condition is not possible,
as the coupling constant would have to be infinite to allow a
shear stress to exist in the absence of a velocity contrast.
Insertion of an infinite coupling constant in the expressions
for the velocities in the \(y\) direction derived below leads to
unconstrained values. I show below, however, that if the
coupling constant \(p\) on the base is small compared with \(\mu/h\),
the rate of corner flow circulation is reduced from the value it
would have for the no-slip condition by approximately the
factor \(ph/3\mu\).

**Corner Flow Circulation**

The circulation of material in the \(xz\) plane due to the shear
stresses on the boundaries of the wedge is the corner flow
problem and can be approached directly from the Navier-Stokes
equations for an incompressible fluid of high viscosity:

\[
-\frac{1}{\rho} \nabla \sigma_m + \mu \nabla^2 \nu - g = 0.
\]

Neglecting velocity gradients in the \(y\) direction, the equations
in the \(x\) and \(z\) directions reduce to

\[
\begin{align*}
-\frac{1}{\rho} \frac{\partial \sigma_m}{\partial x} + \mu \left( \frac{\partial^2 \nu}{\partial x^2} + \frac{\partial^2 \nu}{\partial z^2} \right) - g \sin \beta &= 0, \\
\frac{1}{\rho} \frac{\partial \sigma_m}{\partial z} - g \cos \beta &= 0.
\end{align*}
\]

These equations have precisely the same form as the Navier
Stokes equations for plane incompressible creeping flow. The
biharmonic equation

\[
\nabla^4 \psi = 0,
\]

where \(\psi\) is the stream function in \(x\) and \(z\), can be derived
directly from them [Malvern, 1969]. This means that the
corner flow problem in a three-dimensional accretionary wedge
that is invariant in the \(y\) direction can be treated as a plane flow
problem in the \(xz\) plane, even though the velocities in the \(y\)
direction vary with both \(x\) and \(z\).

For the purposes of the corner flow analysis, the second
derivatives of velocity in the \(x\) direction can be neglected
because the high length/thickness ratio of an accretionary
wedge means that these terms will be small compared with the
second derivatives in the \(z\) direction [Emerman and Turcotte,
1983]. In equation (14b) \(\partial^2 \nu_z / \partial z^2\) can also be neglected, as
in a slowly creeping fluid it will be small compared to the
gravity term. Hence

\[
\frac{1}{\rho} \frac{\partial \sigma_m}{\partial x} + \mu \frac{\partial^2 \nu}{\partial x^2} - g \sin \beta = 0,
\]

\[
\frac{1}{\rho} \frac{\partial \sigma_m}{\partial z} - g \cos \beta = 0.
\]

Differentiating (16a) with respect to \(z\) and (16b) with respect to \(x\) and combining,

\[
\frac{\partial^3 \nu_z}{\partial x^2 \partial z} = 0.
\]

Integrating with respect to \(x\) and then \(z\),

\[
\frac{\partial \nu_z}{\partial z} = C_1 z + C_2,
\]

where \(C_1\) and \(C_2\) are constants of integration. These constants
can be evaluated by setting upper and lower boundary
conditions, assuming that the wedge has a stable configuration
as defined above, that there is no accretion or removal of
material from the upper or lower surfaces, and that the surface
slope is small enough to be neglected. This means that \(v_z = 0\)
everywhere along the base of the wedge (\(z = 0\)), so \(\partial \nu_z / \partial z = 0\),
and hence from (10) \(D_z = (\partial \nu_z / \partial z) = 0\). Then from (9) and
(11),

\[
\frac{\partial \nu_z}{\partial z} = \frac{p(v_z)_b}{\mu} = C_2,
\]

Similarly, \(v_z = 0\) everywhere along the upper surface (\(z = h\)),
and hence \(\partial \nu_z / \partial z = 0\). The surface of the wedge is also free of
shear stress, so \(\sigma_{xz} = 0\). This gives the upper boundary condition

\[
\frac{\partial \nu_z}{\partial z} = 0.
\]

so that \(C_1 = -C_2 / h\). Hence from (18) and (19),

\[
\frac{\partial \nu_z}{\partial z} = \frac{p(v_z)_b}{\mu} \left(1 - \frac{z}{h}\right).
\]
The assumptions listed above to obtain the boundary conditions are rather restrictive. Erosion or sedimentation on the upper surface and accretion or tectonic erosion on the lower surface will all invalidate them. The likely effects are that erosion on the upper surface and accretion on the lower surface will enhance the rates of corner flow derived below, whereas sedimentation on the upper surface and tectonic erosion on the lower surface will depress them. The presence of a significant surface slope, toward the front of the wedge for example, will enhance the rate of circulation.

In a stable steady state accretionary wedge the flow of material across any plane parallel to \( z \) through the wedge should be constant and equal to zero relative to the backstop. In the reference frame I have chosen here, relative to the underthrust plate (Figure 1), this flow is given by the difference in the stream function \( \psi \) across the wedge [Turcotte and Schubert, 1982]:

\[
\int_{0}^{h} d\psi = V \cos \gamma_0 h. \tag{23}
\]

Since \( \psi/\partial z = v_x \) by definition [Malvern, 1969],

\[
\int_{0}^{h} v_x dz = V \cos \gamma_0 h. \tag{24}
\]

Substituting (22) and integrating gives

\[
(v_x) = \frac{3V \cos \gamma_0}{3\mu + ph}, \tag{25}
\]

and hence from (22),

\[
(v_x) = \frac{3V \cos \gamma_0(2\mu + ph)}{2(3\mu + ph)}. \tag{26}
\]

The predicted pattern of velocities is illustrated in Figure 2. The corner flow circulation can best be visualized by referring the velocities to the backstop, however, which gives

\[
(v_x) = -\frac{ph \cos \gamma_0}{3\mu + ph}, \tag{27a}
\]

\[
(v_x) = \frac{ph \cos \gamma_0}{3\mu + ph}. \tag{27b}
\]

If \( ph << \mu \), these reduce to

\[
(v_x) = -\frac{ph \cos \gamma_0}{3\mu}, \tag{28a}
\]

\[
(v_x) = \frac{ph \cos \gamma_0}{6\mu}. \tag{28b}
\]

In this reference frame the corner flow velocities predicted by the no-slip condition at the base and on the surface would be \(-V \cos \gamma_0 \) and \((V \cos \gamma_0)/2\), respectively. Those predicted by the velocity-dependent shear stress condition are lower by the factor \( ph/3\mu \). As noted above, rates of corner flow indicated by the accumulation of sediment on the surfaces of accretionary wedges are low compared to plate convergence rates. If accretionary wedges can be treated as steady-state linear viscous systems, this suggests that \( ph << \mu \). In principle, if the rate of motion of the surface of an accretionary wedge away from the backstop could be measured (by satellite geodesy, for example) and compared with the rate of plate convergence, it should be possible to determine the ratio of the coupling constant at the base of the wedge to the viscosity of the wedge itself. Note, however, that mass circulation within many accretionary wedges is likely to be dominated by the effects of accretion of sediment to the base of the wedge [Platt, 1986].

Profile of a Viscous Wedge

If a viscous wedge has reached a stable configuration, as defined in the section on mechanical assumptions, \( D_{xx} \) and hence \( \sigma_{xx} \) must be zero. Equation (7) then reduces to

\[
(\tau_{x}) = \rho g h a. \tag{29}
\]

This is the characteristic expression for the profile adopted by any viscous medium flowing over a resistive surface. It describes the form of glaciers [Elliott, 1976] and may well be appropriate for thrust and accretionary wedges [Elliott, 1976; Platt, 1986]. Since from (11) \((\tau_{x}) = (\sigma_{xx}) = p(v_x)\) the wedge profile allows calibration of the coupling constant at the base.

To obtain a profile for the wedge, (29) must be converted into an expression for \( h \) as a function of \( x \). Assuming, for the reasons outlined in the previous paragraph, that \( ph << \mu \), then \((v_x)_b \) will everywhere be close to \( V \cos \gamma_0 \) and \((v_x)_h \) will be nearly constant and equal to \( pV \cos \gamma_0 \). The surface slope \( a = \beta/\mu \), and \( \beta = -\alpha/\partial x \). The basal slope \( \beta \) may vary with \( x \) and will depend on the elastic properties of the underthrust plate and on the various surface and sub-surface loads imposed on it. It can in principle be constrained by observation, and in many accretionary wedges it appears to be nearly constant over significant distances normal to the active margin [e.g. Davis et al., 1983; Westbrook and Smith, 1983; Moore et al., 1988; Karig, 1986]. Assuming constant \( \beta \), then (29) can be integrated to give

\[
h = h_0 - \frac{\tau_b}{\rho g \beta} \ln \left( \frac{\tau_b + \rho g h_0 \beta}{\tau_h + \rho g h \beta} \right) - \beta x. \tag{30}
\]

A profile of this form is shown in Figure 2.

In the rear part of a viscous wedge, \( h \to h_0 \) and \( x \to 0 \), so (30) simplifies to

\[
h = h_0 - \beta x. \tag{31}
\]

Velocity in the \( y \) Direction

In a stable wedge of infinite extent in the \( y \) direction, \( D_{xx} = D_{yy} = D_{zz} = 0 \), and hence from (9),
\[ \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0. \]

Shear stress components can be expressed in terms of the velocity gradients from (9) and (10), bearing in mind that gradients in y are zero, giving

\[ \sigma_{xy} = \mu \frac{\partial v_y}{\partial x}, \]  \hspace{1cm} \text{(33a)}

\[ \sigma_{yz} = \mu \frac{\partial v_y}{\partial z}. \]  \hspace{1cm} \text{(33b)}

From the stress equilibrium equation (3b),

\[ \frac{\partial \sigma_{xx}}{\partial x} = -\frac{\partial \sigma_{zz}}{\partial z}. \]  \hspace{1cm} \text{(34)}

\[ \frac{\partial^2 v_y}{\partial x^2} = -\frac{\partial^2 v_y}{\partial z^2}. \]  \hspace{1cm} \text{(35)}

Because the coupling constants relate stresses to velocities, some constraints can be placed on the stresses and stress gradients, and hence on the first- and second-order derivatives of velocity, along the base and backstop of the wedge. Some of the boundary conditions for velocities and velocity gradients are known. At this point it is more convenient to express the velocity in the y direction as dimensionless velocity \( u = v_y / V \sin \gamma_0 \). If the velocity at the base is \( u_b \) and on the backstop \( u_0 \), when \( x = z = 0 \), \( u_b = u_0 = u_{b0} \), and hence from (11) and (12),

\[ \frac{\partial u}{\partial x} = q(u_{b0} - 1)/\mu, \]  \hspace{1cm} \text{(36a)}

\[ \frac{\partial u}{\partial z} = pu_{b0}/\mu. \]  \hspace{1cm} \text{(36b)}

When \( x = \infty \),

\[ u_b = 0, \]

\[ \frac{\partial u}{\partial x} = 0, \]  \hspace{1cm} \text{(37)}

and because the upper surface of the wedge is free of shear stress, neglecting \( \alpha \), when \( z = h_0 \),

\[ \frac{\partial u}{\partial z} = 0. \]  \hspace{1cm} \text{(38)}

From (11), \( (\sigma_{zz})_b = p(\nu_y)_b \) at the base of the wedge, where \( p \) is the coupling constant, and \( \sigma_{zz} \) drops to zero at the free surface of the wedge. \( (\partial \sigma_{zz} / \partial z)_b \) will therefore be a function of \( (\sigma_{zz})_b \), and \( \partial^2 v_y / \partial z^2 \) will be a function of \( (\nu_y)_b \). \( (\partial \sigma_{zz} / \partial z)_b \) may also be a function of \( h \), which is itself a function of \( x \) (30). I show below that \( \nu_y \) is at a maximum on the top of the wedge at the backstop and diminishes rapidly away from it. In this region of the wedge \( h \) is large and varies slowly, so that the dependence of \( (\partial \sigma_{zz} / \partial z)_b \) on \( h \) can be neglected for the purpose of this analysis. I therefore set

\[ \frac{\partial^2 u}{\partial z^2} = -B^2 u_b. \]  \hspace{1cm} \text{(39)}

where \( B \) is an unknown constant. Substituting (35) in (39)

\[ \frac{\partial^2 u}{\partial z^2} = B^2 u_b. \]  \hspace{1cm} \text{(40)}

If \( B \) is independent of \( x \), (40) can be integrated, using the boundary conditions (37), to give

\[ u_b = u_{b0} e^{-Bx}. \]  \hspace{1cm} \text{(41)}

From the boundary conditions (36)

\[ u_{b0} = q/(q + B\mu). \]  \hspace{1cm} \text{(42)}

By a similar argument we can relate \( (\partial \sigma_{xy} / \partial z)_0 \) to \( (\nu_y)_0 \) along the backstop, so that using the dimensionless velocity \( u_0 = (\nu_y)_0 / V \sin \gamma_0 \),

\[ \frac{\partial^2 u_0}{\partial z^2} = A^2 (u_0 - 1). \]  \hspace{1cm} \text{(43)}

It is less easy in this case to justify treating \( A \) as a constant with \( z \), but \( \nu_y \) decays over a distance in the \( x \) direction significantly greater than \( h \), so that variations in \( A \) with \( z \) are likely to be no greater than variations of \( B \) with \( x \). There are not enough defined boundary conditions to integrate (43) uniquely, but it must have an exponential form, and \( \partial u_0 / \partial z \) is zero when \( z = h_0 \). An expression for \( u_0 \) that satisfies these conditions is

\[ u_0 = 1 - \frac{1 - u_{b0}}{1 + e^{-A^{2} h_0 - 2 A h_0}}. \]  \hspace{1cm} \text{(44)}

Equation (44) predicts a velocity \( u_{b0} \) at the surface of the wedge against the backstop \( (x = 0, z = h_0) \),

\[ u_{b0} = 1 - \frac{1 - u_{b0}}{1 + e^{-A^{2} h_0 - 2 A h_0}}. \]  \hspace{1cm} \text{(45)}

If \( Ah_0 \) is large, \( u_{b0} \rightarrow 1 \); i.e., \( (\nu_y)_0 \rightarrow V \sin \gamma_0 \), the velocity of the backstop.

From (40) and (43),

\[ A^2 = \frac{B^2 u_{b0}}{1 - u_{b0}}, \]  \hspace{1cm} \text{(46)}

and from boundary condition (36),

\[ A = -\frac{pu_{b0}}{\mu(1 - u_{b0})}. \]  \hspace{1cm} \text{(47)}

From (42), (46), and (47), the constants \( A, B, \) and \( u_{b0} \) can be determined in terms of \( p, q, \) and \( \mu \):

\[ A = p/\mu q^{2} \mu^{-1}, \]

\[ B = p^{2} q^{2} \mu^{-1}, \]  \hspace{1cm} \text{(48)}

\[ u_{b0} = \left(1 + p^{2} q^{2} \mu^{-2}\right)^{-1}. \]

The type of velocity distribution predicted by (41) and (44) is illustrated schematically in Figure 3. Note that the length scale for the decay of \( u_0 \), with \( x \) to half its value at the backstop is \( (\ln 2)/B \). For a wedge with a weakly coupled base (e.g., \( \ln 2)/B \)), and a well-coupled backstop (e.g., \( qh_0 = \mu \)), \( u_0 \) decays over a distance \( x = h_0 \). The strike-parallel motion is therefore confined to a region near the backstop that is small compared to the across-strike width of the wedge, which justifies the assumption made initially that variations in \( h \) should not significantly affect the velocity distribution at the base of the wedge. It raises a problem, however, in that \( h \) decays over a distance that may not be much larger than the scale of discontinuities in the wedge, i.e., the scale at which the continuum approximation breaks down. In practice, therefore, the exponential decay of the velocity may be partly masked by noise induced by the discontinuities.

The velocity also increases up the contact with the backstop toward the surface, and if the wedge is thick or is well coupled at the rear, the velocity at the surface may approach that of the backstop itself. The velocity at the surface of the wedge decays in the \( x \) direction, but there are not enough constraints to describe this decay. This is unfortunate, as the velocity at the surface is the most easily measurable, by satellite geodesy, for
example. Some indication as to its behavior may be obtained by looking at the way the vertically averaged velocity varies. Because the steepest vertical velocity gradients occur near the base of the wedge, the vertically averaged velocity may not be too different from that at the surface.

The vertically averaged velocity \( \bar{v}_y \) can be obtained in principle from the force balance equation in the \( y \) direction (8). Substituting (11) and (33) and using the dimensionless velocity \( \bar{u} \) gives

\[
\mu \frac{\partial \bar{u}}{\partial x} - h_0 \frac{\partial^2 \bar{u}}{\partial x^2} + p\bar{u}_b = 0. \tag{49}
\]

\( \theta = -dh/dx \), and both \( h \) and \( \bar{u}_b \) are nonlinear functions of \( x \) (equations (30) and (41)), so this expression cannot be integrated directly. As discussed above, \( \bar{u} \) largely decays within a short distance of the backstop, where \( \alpha \) is small; hence \( \theta \) can be taken as constant, and a simplified expression for \( h \) (equation (31)) can be used. Then from (49) and (41):

\[
\mu \frac{\partial \bar{u}}{\partial x} - (h_0 - \beta x)\mu \frac{\partial^2 \bar{u}}{\partial x^2} + p\bar{u}_b e^{-\beta x} = 0. \tag{50}
\]

Unfortunately this expression cannot be integrated directly either, but \( \beta x \) in the second term can probably be neglected, because \( \beta \) is small and \( x \) is small near the backstop, which allows integration for \( \bar{u} \):

\[
\bar{u} = \frac{\bar{u}_b e^{-\beta x}}{\mu \bar{B}(h_0 - \beta)} . \tag{51}
\]

This expression differs from that for the basal velocity (41) only by a constant factor and suggests that the vertically averaged distribution of \( v_y \) has the same exponential form as \( v_y \) along the base. This in turn would suggest that the distribution of \( v_y \) has the same form throughout the wedge. A problem arises near the backstop, however, because (43) predicts that \( \partial^2 \bar{u}/\partial x^2 \) depends on \( (u_0 - 1) \), whereas (51) implies that it depends directly on \( u \). Both relationships are true at the intersection of the underthrust plate and the backstop and have been used already to solve for \( A \) and \( B \), but they cannot both be true everywhere. The problem becomes most noticeable at the top of the wedge by the backstop, where \( u_0 \) could approach unity, and (43) predicts that the second derivative approaches zero. An exponential relationship of the form of (51) would then predict a constant velocity of \( u = 1 \) over the entire upper surface of the wedge, which is clearly incorrect. The velocity distribution along the surface of the wedge therefore cannot have an exponential form.

The exponential form of the strike-parallel velocity distribution described by (41) and (51) is analogous to that calculated for obliquely converging or strike-slip boundaries for a thin viscous sheet model of the lithosphere by England et al. [1985]. The reasons for it, however, are significantly different. The calculations by England et al. [1985] predict a velocity distribution that is related to the length of the boundary, whereas the predictions here are not: the boundary is assumed to be of infinite length, and the velocity distribution is caused by the boundary-parallel shear stress at the base of the deforming wedge, which is assumed to be zero by England et al. [1985].

**PERFECTLY PLASTIC WEDGE**

**Rheology and Assumptions**

The governing relationships are assumed to be

\[
T' = \frac{kD}{\sqrt{\Pi_D}}, \quad \Pi_T = k^2, \quad D_{\theta\theta} = 0, \tag{52}
\]

where \( \Pi_D \) and \( \Pi_T \) are the second invariants (intensities) of the deformation rate and deviatoric stress tensors, respectively. The yield stress \( k \) is a material constant, but it can of course
vary spatially throughout the wedge. In the following, $k$ is assumed to be constant in space.

A perfectly plastic material has the characteristics that if the yield condition is not satisfied, the deformation rates are zero and the stresses are unconstrained. If the yield condition is satisfied, the deformation rates are proportionately related to the deviatoric stress components, but they are only known if $\Pi_0$ is independently specified. Hence for our purposes three possible states can be defined in which the wedge could exist: state 1, a subcritical wedge in which the deformation rates are zero and the stresses are unconstrained (this will not be discussed further); state 2, a critical wedge in which the yield condition is satisfied everywhere but the deformation rate is zero; and state 3, a supercritical wedge in which the yield condition is satisfied everywhere and which is deforming at rates that are dictated by the external boundary conditions.

The condition $D_{yy} = 0$ can be applied because of the assumption of infinite extent in the $y$ direction. If the wedge is deforming (state 3) it follows from the plasticity relation (52) that

$$\sigma_{yy}^\prime = 0.$$  
(53)

Equation (5) then applies, and (52b) reduces to

$$\sigma_x^2 + \sigma_y^2 + \sigma_z^2 = k^2.$$  
(54)

Equations (53) and (54) may also apply to nondeforming critical wedges (state 2), for less obvious reasons. In a geological situation, any critical wedge will experience frequent disturbances to its geometry, caused by accretion of material to the front, for example, that will cause it to become temporarily supercritical. It will then deform until it has returned to its critical geometry. While it is deforming it must obey the rule that $D_{yy} = 0$ and hence that $\sigma_{yy}^\prime = 0$. It is reasonable to assume, therefore, that when it has ceased deforming but is in the critical state, $\sigma_{yy}^\prime = 0$, and equation (54) applies.

**Boundary Conditions**

The relationships discussed here can be worked out using either the constant shear stress or the velocity-dependent shear stress boundary conditions discussed earlier, and the results are not significantly different. The constant shear stress condition gives the components of the basal shear stress as

$$\tau_{xx} = \tau_0 \cos \gamma,$$  
(55a)

$$\tau_{yy} = \tau_0 \sin \gamma,$$  
(55b)

where $\tan \gamma = (v_x)_0 / V \cos \gamma_0$ (Figure 1).

The shear stress on the backstop is taken as a constant, $-\tau_0$. If $\tau_0 = k$, this condition is roughly equivalent to assuming that there is no mechanical predisposition to slip on this boundary, but because the backstop is assumed to be undeformable, this boundary will always differ in character from the rest of the wedge.

The velocity-dependent shear stress boundary condition gives

$$\tau_{xx} = \rho(v_x)_0,$$  
(56a)

$$\tau_{yy} = \rho(v_y)_0,$$  
(56b)

and the shear stress on the backstop is

$$\tau_{yy} = q[(v_y)_0 - V \sin \gamma_0].$$  
(57)

The discussion below is conducted first in terms of the constant shear stress condition, which is probably more appropriate to a plastic wedge.

**Deforming Wedges (State 3)**

I deal with state 3 wedges first, so as to establish under what conditions the strike-parallel velocity $v_y$ can vary through a perfectly plastic wedge and what form that variation takes. In fact, this section demonstrates that the necessary conditions are unlikely or impossible in practice.

In state 3 supercritical wedges, deformation rates are proportional to the deviatoric stresses. If the longitudinal deviatoric stress $\sigma_{xx}^\prime \neq 0$, there will be a finite rate of shortening of the wedge. This shortening will continue until either it reaches the critical state (i.e., it evolves into a state 2 wedge), the conditions for which will be defined next, or $\sigma_{xx}^\prime \rightarrow 0$. The wedge will then still be supercritical but will deform only by strike-parallel shear: there will be no further shortening. Equation (54) then reduces to

$$\sigma_{xx}^\prime + \sigma_{yy}^\prime + \sigma_{zz}^\prime = k^2.$$  
(58)

For a constant shear stress boundary condition (55) the force balance equations (7) and (8) then become

$$\tau_0 \cos \gamma = \rho g h \alpha,$$  
(59)

$$\tau_0 \sin \gamma = -\sigma_{yy}^\prime \theta.$$  
(60)

Equation (59) predicts continuously increasing $\alpha$ toward the front of the wedge, whereas (60) predicts decreasing $\theta$ toward the wedge front, for any reasonable velocity distribution. The only geometry that can satisfy both constraints requires the basal slope $\beta$ to decrease toward the front at a greater rate than $\alpha$ is increasing, so that the maximum value of $\beta$ (at the rear of the wedge) must be larger than the maximum value of $\alpha$ (at the wedge front). No natural wedges have such a geometry. In nature, therefore, a plastic wedge that has a variable $v_y$ will fail to satisfy the force balance equations. Parts of the system will accelerate in such a way as to remove the variations in $v_y$, state 3 wedges therefore rapidly evolve into state 2 nondeforming critical wedges and can only have a transient existence, as a result of accretion for example.

**Nondeforming Critical Wedges (State 2)**

These wedges move coherently, with a constant strike-parallel velocity $v_y$. The purpose of this section is to define the controls on this velocity and to place some limits on the state of stress within the wedge. The latter will be defined by the force balance equations (7) and (8), and the plastic yield equation (58), but, like a viscous wedge, the stress state is also dependent on scale. A plastic wedge has one more degree of freedom than a viscous wedge, however, because of its finite yield strength. Although the longitudinal strain rate $D_{xx}$ must be zero when the wedge is in the critical state, this extra degree of freedom means that the longitudinal deviatoric normal stress $\sigma_{xx}^\prime$ need not be zero: the stress components can vary independently within the constraints of (58) in the $x$ direction, across the wedge. Hence the taper $\theta$ and surface slope $\alpha$ are also free to vary in the $x$ direction, and it is not possible to determine a profile for the wedge.

Integrating the force balance equation (8) horizontally through the wedge, noting that $\theta = -dh/dx$, gives
\[ \tau_b (x - L) \sin \gamma - \sigma_{xy} h = 0. \] (61)

where \( L \) is the width of the wedge in the \( x \) direction.

If the strike-parallel velocity of the wedge (\( v_y \)) of the wedge is less than that of the backstop (\( V \cos \gamma_0 \)), there will be slip along the rear boundary, and \( (\sigma_{xy})_0 = -\tau_0 \). Hence

\[ \tau_0 h_0 = L \tau_b \sin \gamma. \] (62)

Note that this expression gives the angle of obliquity of relative motion between the wedge and the underthrust plate as the arcsine of the ratio of the resistances on the lower and rear boundaries. This is essentially the same expression as that reached by McCaffrey [1992], using a force balance analysis applied to a forearc sliver of arbitrary geometry. The obliquity is independent of the obliquity of the overall relative plate motion. Equation (62) can be recast to give the strike-parallel velocity of the wedge in terms of its boundary conditions and geometry, bearing in mind that \( \tan \gamma = v_y / V \cos \gamma_0 \) (Figure 1):

\[ v_y = \frac{\tau_0 h_0 V \cos \gamma_0}{\sqrt{\tau_0^2 h_0^2 - L^2 \tau_b^2}}. \] (63)

Clearly, \( V \cos \gamma_0 \) cannot be less than \( v_y \), so (62) can be used to give a minimum value of the angle of obliquity of convergence for there to be slip along the backstop:

\[ \sin \gamma_0 = \frac{\tau_0 h_0}{L \tau_b}. \] (64)

If \( \sin \gamma_0 \) is less than this, \( |(\sigma_{xy})_0| < \tau_0 \), there will be no slip on the rear boundary, and the wedge will move at the same velocity as the backstop. If \( \sin \gamma_0 \) is greater than this, then there will be slip on the rear boundary, and the wedge will move laterally at a velocity given by (63). Two types of state 2 wedge can therefore be distinguished. State 2A wedges form at relatively low angles of obliquity of plate motion and move at the same velocity as the upper plate. State 2B wedges form at relatively high angles of obliquity and move as forearc slivers at a velocity intermediate between those of the two bounding plates. This behavior is illustrated schematically in Figure 4.

Some constraints can be placed on the geometry and internal stress state of a state 2 wedge. At the base of the wedge, (54) reduces to

\[ \sigma_{xx}^2 + \sigma_{xy}^2 + \sigma_{yy}^2 = k^2. \] (65)

Along the upper surface, shear stress components parallel to the \( xy \) plane will be negligible, and (54) reduces to

\[ \sigma_{xx}^2 + \sigma_{yy}^2 = k^2. \] (66)

In a strong wedge, for which \( k > \tau_b \), the stress components \( \sigma_{xx} \) and \( \sigma_{yy} \) can be neglected in the body of the wedge, and (66) becomes a useful approximation for the vertically averaged stress state. The components of the deviatoric stress tensor can therefore be expressed in terms of the two variables \( \sigma_{xx} \) and \( \sigma_{xy} \):

\[ \sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & 0 & 0 \\ 0 & 0 & \sigma_{xx} \end{bmatrix}. \] (67)

The principal axes of this stress tensor are vertical and horizontal. The most probable arrangement of the stresses for a compressive critical wedge, as is assumed in this paper, has the maximum extensional deviatoric stress \( \sigma_{ij}^1 \) vertical and the maximum compressive deviatoric stress \( \sigma_{ij}^2 \) horizontal in the \( xy \) plane. The values of the principal deviatoric stresses are then given by

\[ \sigma_{ij}^1 = -\sigma_{xx} \], \[ \sigma_{ij}^2 = \left( \sigma_{xx}^2 + \sqrt{\sigma_{xx}^4 + 4 \sigma_{yy}^2} \right)/2 \]. (68)

The three possible orientations of the principal stresses, and the values in each case, are illustrated with Mohr constructions in Figure 5.

At the rear of the wedge, \( \sigma_{xy} = -\tau_0 \), so that from (66),

\[ \sigma_{xx} = \frac{-\sqrt{k^2 - \tau_b^2}}{\tau_b}. \] (69)

These values can be substituted into (68) to give the values of the principal stresses at the rear of the wedge. The angle \( \delta \) between \( \sigma_{ij}^2 \) and the \( x \) direction is given by \( \tan 2\delta = \sigma_{xy}/\sigma_{xx} \) (Figure 5). Hence from (69), at the rear of the wedge,

\[ \tan 2\delta = \frac{2 \tau_0}{\sqrt{k^2 - \tau_b^2}}. \] (70)

Should the wedge begin to deform in this region, \( \delta \) will also be the orientation of the direction of maximum shortening rate, or of instantaneous shortening, for a compressive wedge.

At the front of the wedge, where \( h \to 0 \), the force balance equations (7) and (8) reduce to

\[ \tau_b \cos \gamma = -2 \sigma_{xx} \theta, \] (71)

\[ \tau_b \sin \gamma = -\sigma_{xy} \theta. \] (72)

These relations give

\[ \tan \gamma = \frac{\sigma_{xy}}{2 \sigma_{xx}}. \] (73)

If the wedge is strong, so that (66) applies, the maximum compressive stress lies in the horizontal plane at an angle \( \delta \) to \( x \), given by:

\[ \tan 2\delta = 4 \tan \gamma. \] (74)

(73) and (74) predict the stress state at the wedge front, and hence strain rates in the case of plastic yield, in terms of the obliquity of motion between the wedge and the underthrust plate (note that for a state 2B wedge this is not the same as the obliquity of plate motion).

From (66), (71), and (72), an expression for the taper at the front of the wedge can be obtained:

\[ \theta = \frac{5\tau_b}{2k} \sqrt{\cos^2 \gamma + 4 \sin^2 \gamma}. \] (75)

### Velocity-Dependent Boundary Condition

I first show that for this boundary condition also, the only stable condition for a critical plastic wedge is state 2, in which there is no variation in \( v_y \).

For a plastic wedge that has a critical geometry but is yielding by shear parallel to the \( y \) direction (state 3), so that \( v_y \) is variable, \( D_{xx} \) and hence \( \sigma_{xx} \) are zero, and \( v_x = V \cos \gamma_0 \). Then from (56),

\[ (\tau_b)_{xx} = p V \cos \gamma_0, \] (76a)

\[ (\tau_b)_{yy} = p (v_y) \theta. \] (76b)

If the wedge is strong with \( k > \tau_b \), from (66) \( \sigma_{xy} = k \), so from (8),

\[ p (v_y) \theta = -\sigma_{xy} \theta. \] (77)
State 2A: low angle of convergence

State 2B: high angle of convergence

Fig. 4. Schematic illustration of the behavior of obliquely converging plastic or Coulomb wedges in a stable and critical state (state 2). If the obliquity of plate convergence is below a certain limiting value, the wedge moves coherently with the upper plate (state 2A). If the obliquity is above that value, the wedge is separated from the upper plate by a strike-slip fault or shear zone (state 2B): the strike-parallel velocity of the resulting forearc sliver has a value dependent on its mechanical properties and geometry, and hence there is a limit to the obliquity of motion of the wedge relative to the underthrust plate. The convex-upward profile is characteristic of a plastic wedge.

\[ \frac{\partial (\nu_y)}{\partial x} = -\sigma_{xy} \frac{\partial \theta}{\partial x} \]  

(78)

If \((\nu_y)_b\) varies through the wedge, the dominant sign of the x gradient will be negative, and (78) therefore requires the taper \(\theta\) to decrease toward the wedge front.

From (7) and (76), bearing in mind that \(\sigma'_{xx} = 0\),

\[ p V \cos \gamma_0 = \rho g h x. \]  

(79)

\[ : h \frac{\partial \alpha}{\partial x} + \alpha \frac{\partial h}{\partial x} = 0. \]
and since $\partial h/\partial x = -\theta$, 

$$\frac{\partial a}{\partial x} = \frac{a\theta}{h}.$$  \hspace{1cm} (80) $$

$h$, $\alpha$, and $\theta$ are all positive, so $\partial a/\partial x$ must also be positive; that is, $\theta$ must increase toward the wedge front.

The conclusions reached in the two preceding paragraphs, based on the force balance equations, are clearly mutually incompatible. The conclusion must be that the assumption of...
variable \( v_y \) and hence of zero \( \sigma_{xx} \) are invalid and that a state 3 wedge cannot stably exist. This is the same conclusion as was reached using a constant shear stress boundary condition.

For a state 2 wedge, (61) becomes
\[
pv_y (x - L) - \tau h = 0.
\] (81)

and (62) becomes
\[
\tau h_0 = Lp v_y.
\] (82)

Hence from (57)
\[
v_y = \frac{gh_0}{Lp + q h_0} \sin \gamma_0.
\] (83)

\( v_y \) is always less than \( V \sin \gamma_0 \). This means that for a velocity-dependent boundary condition there will always be slip on the rear boundary: the condition effectively dictates that the rear boundary is a zone of weakness relative to the wedge. The wedge will therefore move as a forearc sliver, at a velocity intermediate between that of the two bounding plates, at all nonzero angles of obliquity.

Equation (70) for the orientation of the maximum compressive stress at the rear of the wedge becomes
\[
\tan 2 \delta = \frac{2q (V \sin \gamma_0 - v_y)}{k^2 - q^2 (V \sin \gamma_0 - v_y)^2}.
\] (84)

Equations (73) and (74) for the stress state at the wedge front remain the same. Equation (75) for the taper at the front of the wedge becomes
\[
\theta = \frac{p_0}{2k} \sqrt{\frac{\nu^2 \cos^2 \gamma_0 + 4 \nu v_y^2}{k^2 - q^2 (V \sin \gamma_0 - v_y)^2}}.
\] (85)

**COULOMB WEDGE**

**Rheology and Assumptions**

A Coulomb wedge is assumed to be capable of deforming internally by fracture and by frictional sliding on surfaces with a wide range of orientations distributed throughout it. In terms of the continuum approximation, the behavior of the wedge on a scale large than the individual discontinuities can then be described in terms of a Coulomb "rheology" analogous to that of a plastic rheology [Davis et al., 1983; Dahlen, 1984; Dahlen et al., 1984].

Following Davis et al. [1983], I use a noncohesive Coulomb criterion, which can be expressed in terms of the maximum shear stress \( \tau_m \) and the average of the maximum and minimum principal stresses:
\[
\pm \tau_m = \frac{1}{2} \left[ (\sigma_1 + \sigma_3) + P_f \right] \sin \phi.
\] (86)

where \( P_f \) is the pore fluid pressure and \( \tan \phi = \mu_i \), where \( \mu_i \) is the coefficient of internal friction. \( P_f \) can usefully be expressed in terms of \( \lambda \), the ratio of the pore fluid pressure to the vertical normal stress:
\[
\lambda = -\frac{P_f}{\sigma_{zz}} \cos \beta.
\] (87)

One of the problems with analyses of Coulomb wedges is that \( \lambda \) is likely to vary in unpredictable ways through the wedge. For discussions of this problem and possible approaches to understanding the constraints on \( \lambda \), see Davis et al. [1983] and Platt [1990]. \( \mu_i \) can also vary according to rock type. In order to achieve an analytical solution, however, \( \lambda \) and \( \phi \) are assumed to be constant throughout the wedge.

The Coulomb criterion is analogous to a plastic yield criterion but with the difference that the yield stress depends on the mean stress. If the criterion is not satisfied, the deformation rates are zero and the stresses are unconstrained. If the criterion is satisfied, the material deforms at rates dictated by the external boundary conditions. An initially homogeneous and isotropic material will extend instantaneously in the direction of \( \sigma_1 \) (the minimum compressive principal stress) and shorten in the direction of \( \sigma_3 \). This is the basis for a variety of methods of paleostress analysis. Natural materials are, of course, generally neither homogeneous nor isotropic, and even if they are so initially, the effects of brittle deformation will render them anisotropic, particularly if the deformation is noncoaxial. Nevertheless, for the purposes of this discussion, I assume homogeneous and isotropic behavior. The deformation of a Coulomb material is two-dimensional and lies in the plane of the maximum and minimum principal stresses: it is unaffected by the value of the intermediate principal stress. States in which a Coulomb wedge can exist may therefore be defined exactly as for a plastic wedge: state 1, a subcritical wedge in which the deformation rates are zero and the stresses are unconstrained (this will not be discussed further); state 2, a critical wedge in which the Coulomb criterion is satisfied everywhere but the deformation rates are zero; and state 3, a supercritical wedge in which the yield condition is satisfied everywhere, and which is deforming at rates that are dictated by the external boundary conditions, in the directions of the maximum and minimum principal stresses.

The condition \( D_\gamma = 0 \) can be applied because of the assumption of infinite extent in the \( y \) direction. Because the deformation is independent of the intermediate principal stress, the rates are not directly related to the components of the deviatoric stress tensor, as is the case for plastic materials. Hence \( \sigma_\tau^\gamma \) cannot be constrained from the condition \( D_\gamma = 0 \), as was done for viscous and plastic wedges. I argue that the condition \( \sigma_\tau^\gamma = 0 \) can still be applied, because on geological time scales, rock materials creep in such a way as to relax deviatoric stress.

In the general case, the principal stresses within a Coulomb wedge will be inclined to all three coordinate axes. In order to render the problem more tractable, I assume for the purposes of this analysis that the shear stress components \( \sigma_\tau^x \) and \( \sigma_\tau^y \) can be neglected on the scale of the wedge as a whole. This assumption is equivalent to assuming that the vertically averaged principal stresses lie parallel to the \( z \) axis or to the \( xy \) plane and is justified by the low taper of natural wedges, which suggests that they are strong relative to the resistance at the base [Davis et al., 1983]. Davis et al. [1983] and Dahlen [1984] present an analysis of Coulomb wedge in two dimensions that is more elaborate in that they allow for the effects of shear stresses parallel to the \( x \) direction and they determine the effect of the orientation of the maximum principal stress in the vertical plane on the geometry and internal mechanics of the wedge. The analysis presented here concentrates on the effect of the orientation of the principal stresses in the \( xy \) plane. At the end of the final section I compare the results with those obtained by Davis et al. [1983].

**Boundary Conditions**

The most suitable boundary conditions for a Coulomb wedge are frictional sliding laws, producing shear stresses that are
dependent on the normal stresses on those boundaries. On the base this condition gives
\[ \tau_b = \mu_b \left( \rho g h \cos \beta - P_f \right) \, , \tag{88} \]
and on the backstop it gives
\[ \left( \sigma_{xy} \right)_b = \mu_0 \left[ \left( \sigma_{xx} \right)_b + P_f \right] \, , \tag{89} \]
where \( \mu_b \) and \( \mu_0 \) are the coefficients of friction on the base and backstop boundaries, respectively. If \( \lambda \) is defined as in (87) and if at the base of the wedge \( \lambda_b \) is defined as
\[ \lambda_b = P_f / \rho g h \cos \beta \, , \tag{90} \]
then (assuming \( \beta \) is small)
\[ \left( \tau_b \right)_x = \mu_b \rho g h \cos \gamma \, , \tag{91a} \]
\[ \left( \tau_b \right)_y = \mu_b \rho g h \sin \gamma \, , \tag{91b} \]
\[ \left( \sigma_{xy} \right)_0 = \mu_0 \left[ 2 \left( \sigma_{xx} \right)_0 - \rho g (h_0 - z) (1 - \lambda) \right] \, , \tag{92} \]
where \( \mu_b^* = \mu_b (1 - \lambda_b) \), and \( \tan \gamma = \left( V_x \right)_b / V \cos \gamma_0 \) (Figure 1).

For naturally occurring wedges with low angles of taper, \( \mu_b \) will be significantly less than \( \mu_1 \).

The other possible boundary conditions are the constant shear stress conditions (55) and velocity-dependent conditions (56) and (57). Both these boundary conditions lead to potential problems because they can create shear stresses near the front of the wedge that are greater than the strength of the wedge itself, which for a noncohesive Coulomb rheology drops to zero as the wedge tapers to zero thickness. This problem does not arise for the frictional boundary conditions, which, like the wedge strength, depend on \( h \). Physically realistic wedge models could be constructed using a cohesive Coulomb rheology [Dahlen et al., 1984] and one of the other boundary conditions, but these are unlikely to lead to significantly different conclusions than those reached here, and they will not be investigated further.

**Deforming Wedges (State 3)**

As for plastic wedges, this section establishes that state 3 wedges can only have a transient existence and that the strike-parallel velocity \( v_y \) cannot therefore vary through a stable Coulomb wedge.

If two of the principal stresses lie in the \( xy \) plane (see above) and the wedge is deforming by shear parallel to the \( y \) direction but is not changing length in the \( y \) direction, then the maximum and minimum principal stresses must lie at 45° on either side of \( x \) in the \( xy \) plane. If \( \sigma_{xy}' = 0 \), then \( \sigma_{xx}' = \sigma_{zz}' = 0 \), and
\[ \sigma_2 = \left( \sigma_1 + \sigma_3 \right) / 2 = -\rho g (h - z) \, . \tag{93} \]

This stress configuration is illustrated in Figure 6. Note that in this situation the wedge would deform by slip on conjugate sets of strike-slip faults, neither of which would be parallel to the backstop. The acute dihedral angle between the sets, and hence the maximum rate of shortening, would be at 45° to the rear boundary; and one set of faults would be in the Riedel orientation to that boundary. In a natural situation the latter would probably tend to link up to form through-going sets of strike-slip faults approximately parallel to the boundary.

Note that because of the plane strain character of the deformation imposed by the Coulomb rheology and because of the assumption that two of the principal stresses lie in the \( xy \) plane, a zero rate of elongation in the \( y \) direction requires the same in the \( x \) direction. The wedge must therefore already have a critical geometry to allow shear parallel to \( y \).

From the Coulomb criterion (86) and from (93), the vertically averaged maximum shear stress \( \tau_m \) is
\[ \tau_m = \frac{1}{2} \rho g h (1 - \lambda) \sin \varphi \, . \tag{94} \]

From the force balance equation (7) and the basal shear stress condition (91), remembering that \( \sigma_{xx}' = 0 \),
\[ \alpha = \mu_b^* \cos \gamma \, . \tag{95} \]

Since \( \tau_m = -\sigma_{xy} \) (Figure 6), the force balance equation (8) gives
\[ \mu_b^* \rho g h \sin \gamma = \tau_m - h \frac{\partial \tau_m}{\partial x} \, . \tag{96} \]

Assuming that \( \lambda \) and \( \phi \) are constant and noting that \( \theta = -\partial h / \partial x \), substitution of (94) then gives
\[ \mu_b^* \sin \gamma = \theta (1 - \lambda) \sin \varphi \, . \tag{97} \]

Equation (95) predicts constant or increasing \( \alpha \) toward the front of the wedge, whereas (97) predicts constant or decreasing \( \theta \) toward the front of the wedge, for any reasonable velocity distribution. The only geometry that can reasonably satisfy both constraints is one in which \( \alpha \) and \( \theta \) are constant, which then requires that \( \gamma \) and hence \( v_y \) be constant also. Then \( v_y \) can be obtained from (95) and (97), bearing in mind that \( \tan \gamma = v_y / V \cos \gamma_0 \) (Figure 1):
\[ v_y = \frac{\theta}{\alpha} (1 - \lambda) V \cos \gamma_0 \sin \varphi \, . \tag{98} \]

In this case the wedge will not be deforming and will be in state 2. The constraints on the stress state described above do not apply if the velocity is constant, however, so that (98) does not generally apply; and it does not follow from this
argument that \( \alpha \) and \( \theta \) must be constant in the general case: that has to be demonstrated independently.

**Nondeforming Critical Coulomb Wedges (State 2)**

These wedges move coherently, with a constant strike-parallel velocity \( v_y \). The purpose of this section is to define the controls on the geometry, state of stress, and strike-parallel velocity of the wedge in terms of its mechanical properties, boundary stress conditions, and the obliquity of the plate motion. I start by demonstrating that an obliquely converging noncohesive state 2 Coulomb wedge has a constant angle of taper; that is, its geometry is scale independent, as has already been demonstrated in the two-dimensional case [Davis et al., 1983].

As discussed above, I assume that the principal stresses lie parallel to \( z \) or in the \( xy \) plane. Figure 5 illustrates the three possible stress configurations, together with the values of the vertically averaged principal stresses in terms of the stress terms \( \bar{\sigma}_{xy} \) and \( \sigma_z \), and the values of the maximum shear stress, in each case. The most likely configuration of the principal stresses for a critical compressive wedge has \( \sigma_z \) (the maximum tensile stress) parallel to \( z \). In this orientation, the following relations among the stress components apply:

\[
\bar{\tau}_m = \left( \sqrt{\bar{\sigma}_{xx}^2 + 4\bar{\sigma}_{xy}^2 - 3\bar{\sigma}_{xx}^2}/4 \right). \tag{99}
\]

The Coulomb criterion (86) then gives

\[
\bar{\tau}_m = \frac{\rho g h (1 - \lambda)}{2 (1 - \sin \varphi)}. \tag{100}
\]

Defining a dimensionless material constant \( K \) as

\[
K = \frac{\sin \varphi}{1 - \sin \varphi}, \tag{101}
\]

and substituting into (102),

\[
\bar{\tau}_m = \rho g h (1 - \lambda) K/2. \tag{102}
\]

In the case of an extending critical wedge, with \( \sigma_z \) (the maximum compressive stress) parallel to \( z \), \( K \) in (103) and (104) becomes

\[
K = \frac{\sin \varphi}{1 + \sin \varphi}. \tag{103}
\]

If \( \sigma_z \) is vertical, then from the Coulomb criterion (86) and the stress relationships shown in Figure 5,

\[
\bar{\tau}_m = \frac{1}{2} (\rho g h (1 - \lambda) \pm 3\bar{\sigma}_{xx}^2) \sin \varphi. \tag{104}
\]

The positive value of the \( \pm \) sign is valid if \( \sigma_z \) is at more than 45\(^\circ\) to the \( x \) direction, and \( \bar{\sigma}_z > (\bar{\sigma}_1 + \bar{\sigma}_3)/2 \); the negative value is valid if \( \sigma_z \) is at less than 45\(^\circ\) to the \( x \) direction, and \( \bar{\sigma}_z < (\bar{\sigma}_1 + \bar{\sigma}_3)/2 \).

In the transition states, when \( \bar{\sigma}_z = \bar{\sigma}_1 \) or \( \bar{\sigma}_3 \),

\[
\bar{\sigma}_{xy} = \sqrt{2} \bar{\sigma}_z, \tag{105}
\]

\[
\bar{\tau}_m = -3\bar{\sigma}_{xx}^2/2. \tag{106}
\]

The force balance equations (7) and (8) and the basal shear stress condition (91) give

\[
\rho g h (\mu_b \cos \gamma - \alpha) + 2\bar{\sigma}_{xx} \theta - 2h \frac{\partial \bar{\sigma}_{xx}}{\partial x} = 0, \tag{110}
\]

\[
\rho g h \mu_b^* \cos \gamma + \bar{\sigma}_{xy} \theta - h \frac{\partial \bar{\sigma}_{xy}}{\partial x} = 0. \tag{111}
\]

Integrating these with respect to \( x \), noting that \( \theta = \alpha + \beta \) and \( \theta = -dh/dx \) gives

\[
\rho g (\mu_b^* \cos \gamma + \beta) \int_0^x (hdx - A) + \rho g h^2/2 - 2\bar{\sigma}_{xx} h = 0, \tag{112}
\]

\[
\rho g \mu_b^* \sin \gamma \int_0^x (hdx - A) - \bar{\sigma}_{xy} h = 0, \tag{113}
\]

where

\[
A = \int_0^x hdx \tag{114}
\]

is the cross-sectional area of the wedge. The unknown integral \( \int hdx \) can be evaluated by substituting the expressions for \( \bar{\sigma}_{xy} \) and \( \bar{\sigma}_{xx} \) from (112) and (113) in (99) and using the Coulomb criterion (100). This gives

\[
\int_0^x hdx = \frac{h(3K^* + 1) + \sqrt{K^*^2 R^2 + 2M (8K^*^2 + 6K^* + 1)}}{4M^2 - 2R^2} + A. \tag{115}
\]

where

\[
R = \mu_b^* \cos \gamma + \beta, \tag{116}
\]

\[
K^* = K (1 - \lambda), \tag{117}
\]

\[
M = \mu_b^* \sin \gamma. \tag{118}
\]

Writing (115) as

\[
\int_0^x hdx - A = h^2 G \tag{119}
\]

and substituting in (112) gives

\[
\bar{\sigma}_{xx} = \rho g h (2RG + 1)/4. \tag{120}
\]

\( R \) and \( G \) are functions of \( \mu_b^* \), \( \phi \), \( \lambda \), \( \gamma \), and \( \beta \). The first three are all assumed to be independent of \( x \). \( \gamma \) was shown to be constant with \( x \) in the preceding section. \( \beta \) has to be independently specified, but if it is also constant with \( x \), then

\[
\frac{\partial \bar{\sigma}_{xx}}{\partial x} = -\rho g \theta (2RG + 1)/4, \tag{121}
\]

and hence from (110),

\[
\theta = -\frac{1}{2G}. \tag{122}
\]

On the assumption of constant \( \beta \), therefore, \( \theta \) is independent of \( x \) and the geometry of the wedge is independent of scale.

Having established that \( \theta \) is independent of \( x \), the integral \( \int hdx \) can be evaluated using the relation \( h = h_0 - \theta x \) to give

\[
\int_0^x hdx = h_0 x - \theta x^2/2 - h_0 L/2 + A. \tag{123}
\]

The force balance equations (110) and (111) can be then be arranged more conveniently to give expressions for \( \bar{\sigma}_{xy} \) and \( \bar{\sigma}_{xx} \) in terms of \( \theta \) and \( \alpha \):

\[
\bar{\sigma}_{xx} = \frac{\rho g (\mu_b^* \cos \gamma - \alpha) (h_0 x - \theta x^2/2 - h_0 L/2)}{2(h_0 - \theta L)}. \tag{124}
\]
The angle $\delta$ between the maximum compressive principal stress $\sigma_3$ and the x direction is given by $\tan 2\delta = 2\sigma_{s3}/\sigma_{xx}$ (Figure 5), so from (121) and (122)

$$\tan 2\delta = -\frac{4\mu_b^* \sin \gamma}{\mu_b^* \cos \gamma - a}.$$  

$\delta$ is therefore not a function of $x$, so the orientation of the principal stresses is constant across the wedge.

When $x=0$, at the rear of the wedge, (121) and (122) give

$$\left(\sigma_{x3}\right)_0 = -\frac{1}{4} \rho g L \left(\mu_b^* \cos \gamma - a\right).$$

As for a plastic wedge, (125) gives the angle of obliquity of relative motion between the wedge and the underthrust plate as the arcsine of the ratio of the resistances on the lower and rear boundaries.

The rear boundary condition on the wedge (89) gives, using (6),

$$\left(\sigma_{xy}\right)_0 = \mu_0 \left[ \frac{1}{2} \left(\sigma_{x3}\right)_0 - \frac{1}{2}(1-\lambda) \rho g \mu_0 \right].$$

Substituting (124) and (125) in (126), noting that $\theta = \mu_0 L$, gives

$$\sin \gamma = \mu_0 \cos \gamma + \frac{\mu_0}{\mu_b^*} \theta(1-\lambda) - \alpha.$$  

This equation can be recast to give an expression for $\tan \gamma$:

$$\tan \gamma = \mu_0 \left[ 1 \pm \frac{\theta(1-\lambda) - \alpha}{\mu_b^*} \sqrt{1 + \mu_b^*} \right].$$

Inspection of (127) shows that the positive value of the root should be taken in (128). Noting that $\tan \theta = V \cos \gamma_0$, (128) can be used to give a minimum value of the angle of obliquity of convergence for there to be slip along the backstop:

$$\tan \gamma_0 = \mu_0 \left[ 1 \pm \frac{\theta(1-\lambda) - \alpha}{\mu_b^*} \sqrt{1 + \mu_b^*} \right].$$

If $\tan \gamma_0$ is less than this, there will be no slip on the rear boundary and the wedge will move at the same velocity as the backstop. If $\tan \gamma_0$ is greater than this, there will be slip on the rear boundary and the wedge will move laterally at a velocity given by (129). As for plastic wedges, two types of state 2 Coulomb wedge can therefore be distinguished. State 2A wedges form at relatively low angles of obliquity of plate motion and move at the same velocity as the upper plate. State 2B wedges form at relatively high angles of obliquity and move as forearc slivers at a velocity intermediate between those of the two bounding plates. This behavior is illustrated schematically in Figure 4.

For compressive critical wedges, with $\theta$ (the maximum tensile stress) parallel to $z$, (99), (104), (124), and (125) can be combined to give an expression for $\theta$:

$$\theta = \frac{\left(\mu_b^* \cos \gamma - a\right)}{2A(1-\lambda)} \left( -B \pm \sqrt{B^2 - 4AC} \right).$$

where

$$A = 1 - 4K^2,$$

$$B = 2\mu_b^* + 3K,$$

$$C = \mu_b^* - \frac{1}{2}.$$  

$K$ will always be greater than 1 (103), so $A$ will be negative, $B$ will be positive, and $C$ is likely to be positive. The term under the square root sign in (131) will always be positive and greater than $B$, so the negative value of the root is required for a positive value of $\theta$. This expression can be recast for $\beta$, noting that $\theta = \alpha + \beta$, to give

$$\theta = \frac{\left(\mu_b^* \cos \gamma + \beta\right)}{2A(1-\lambda)} \left( -B + \sqrt{B^2 - 4AC} \right).$$

Let

$$G = \frac{B + \sqrt{B^2 - 4AC}}{B + \sqrt{B^2 - 4AC} - 2A(1-\lambda)}.$$  

Then, substituting (133) in (127) and reorganizing for $\tan \gamma$,

$$\tan \gamma = \mu_0 \cos \gamma \left[ 1 + \frac{B}{\mu_b^*} \sqrt{1 + \mu_b^*} \right].$$  

and hence:

$$\nu_x = V \cos ^\gamma_0 \mu_0 \left[ 1 + \frac{B}{\mu_b^*} \sqrt{1 + \mu_b^*} \right].$$

This expression for $\nu_x$ is entirely in terms of the component of plate velocity in the x direction, the material properties of the wedge, and $\beta$, all of which have to be specified.

These expressions have to be modified slightly for other possible orientations of the principal stresses. In the case of an extending critical wedge, $K$ is given by (105) and $B$ in (132) becomes

$$B = 2\mu_b^* - 3K.$$  

For a wedge in which $\theta$ is vertical, so that the deformation is entirely by strike-slip faulting,

$$K = \sin \phi.$$  

and

$$A = 1 - K^2,$$

$$B = 2\mu_b^* + 3K/2,$$

$$C = \frac{1}{4} + \mu_b^* - 9K^2/16.$$  

For zero obliquity of plate motion (i.e., the two-dimensional situation in which there is no strike-parallel component of motion), (133) reduces to

$$\theta = \frac{\mu_b^* + \beta}{1 + 2K(1-\lambda)}.$$
Allowing for the effect of immersion in water, this equation is directly comparable in form to equation (22) of Davis et al. [1983], although the value of $K$ (equivalent to $2K$ in equation (140)) derived by these authors is more complicated than that obtained here (103) and is a function of both $\mu_b$ and $\mu_i$. They show, however, that if $\mu_b$ is small compared with $\mu_i$, $K$ simplifies to the expression given in (103), allowing for the factor of 2 difference in definition.

**CONCLUSIONS**

The analyses presented in this paper provide a set of predictions about the geometry and velocity distribution in three-dimensional critical wedges that can in principle be tested against observation. In wedges with a linear viscous rheology there is a corner-flow circulation of material in the strike-normal plane at velocities that are functions of the coupling constants $p$ and $q$ on the boundaries (27). For wedges that are strong (high viscosity) relative to the coupling at the base, these velocities are related to the strike-normal component of plate motion by the factor $ph/3\mu_b$, where $p$ is the coupling constant on the base, $\mu$ is the viscosity of the wedge, and $h$ is the thickness of the wedge (28). The profile of the wedge is a function of the basal shear stress, the body forces within it, and hence its dimensions (30).

The strike-parallel component of motion within a viscous wedge decays exponentially away from the rear and is effectively concentrated in a shear zone with a width comparable to the thickness of the wedge at the rear (41). The velocity also increases from the corner upward along the rear boundary according to an exponential law, approaching the velocity of the backstop at the upper surface of the wedge (44). The velocity depends on $\mu$, $p$, and $q$. Convergence at the wedge front is normal to strike. A critically tapered wedge with a perfectly plastic rheology moves coherently with respect to the underlying slab: no geometrical configuration exists that allows the strike-parallel motion to be distributed through the wedge. For obliquities of the plate convergence vector $\gamma_0$ below a limiting value (64), the wedge moves at the same velocity as the backstop (upper plate). For angles of obliquity above this value, the wedge moves laterally at a limiting velocity dependent on its dimensions and the stress conditions on its boundaries (63), and it is separated from the upper plate by a strike-slip fault defining a forearc sliver. The limiting velocity of $\gamma_0$, and the obliquity $\gamma$ of relative motion between the wedge and the underthrust plate when $\gamma_0$ exceeds this value, are both equal to the arcsine of the ratio of the resistances on the lower and rear boundaries (62) [cf McCaffrey, 1992]. The latter is therefore independent of the obliquity of the plate convergence vector.

The stress state within a plastic wedge varies across it, but the obliquity of the maximum principal compressive stress in the rear is a function of the shear stress on that boundary and the strength of the wedge (70), and at the front it is a function of $\gamma$ (74).

A Coulomb wedge behaves in much the same way as a plastic wedge, but its geometry and velocity are independent of scale. The limiting value of $\gamma_0$, below which the wedge moves at the same velocity as the backstop, is given by (130); the limiting velocity of the wedge at obliquities higher than this is given by (129); and the resulting value of $\gamma$ is given by equation (127); all of these are functions of the mechanical properties of the boundaries and the geometry of the wedge. As for a plastic wedge, $\gamma$ is equal to the arcsine of the ratio of the resistances on the lower and rear boundaries (124). $\gamma_0$ and the limiting velocity of the wedge are also expressed in (135) and (136) exclusively in terms of the strike-normal component of motion, the mechanical properties of the wedge and its boundaries, and $\beta$, all of which are variables that have to be specified for a given accretionary wedge and cannot be calculated. The taper of the wedge is independent of scale and is given by (131) in terms of $\gamma$, $\beta$, and the mechanical properties of the wedge and its boundaries.

Focal plane mechanisms of earthquakes from obliquely converging margins clearly indicate that there is a degree of partitioning and that the mechanical behavior of the forearc varies from one location to another [McCaffrey, 1991, 1992; Yu et al., submitted manuscript, 1992]. McCaffrey [1991] suggests that there is a limiting value to the obliquity of plate motion, below which deformation is not partitioned and above which it is, giving a forearc sliver with a limiting velocity and hence limiting angle of obliquity relative to the underthrust plate. This behavior is most consistent with the predictions made here for critical plastic or Coulomb wedges. Yu et al. (submitted manuscript, 1992), on the other hand, maintain that the data from the Aleutian forearc indicate a constant ratio between the obliquity of forearc motion relative to the underthrust plate and the obliquity of plate motion. This behavior is most easily explained in terms of a bulk viscous rheology for the forearc, which would also predict a variation in the obliquity of motion across the forearc. More detailed measurements of the obliquity of relative motion both along and across obliquely converging forearcs are needed to test the predictions made in this paper adequately.
\( v_b, v_0 \) velocities of material points along the base of the wedge and along the backstop.

\( u \) dimensionless velocity component.

\( \gamma \) angle of obliquity of \( v \) to \( x \) in the \( xy \) plane.

\( \gamma_0 \) angle of obliquity of \( V \) to \( x \) in the \( xy \) plane.

\( v \) stream function.

\( \mu \) viscosity.

\( p, q \) coupling constants along the base of the wedge and the backstop.

\( k \) plastic yield strength.

\( \phi \) angle of internal friction.

\( \mu_i \) coefficient of internal friction, equal to \( \tan \phi \).

\( \mu_b, \mu_0 \) coefficients of friction along the base of the wedge and the backstop.

\( p_f \) fluid pressure.

\( \lambda \) fluid pressure ratio, equal to \( \lambda = -p_f / \sigma_{xx} \cos \beta \)

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