Calibrating the bulk rheology of active obliquely convergent thrust belts and forearc wedges from surface profiles and velocity distributions

J. P. Platt
Department of Geological Sciences, University College London, London

Abstract. The geometrical profiles and surface velocity distributions in active forearc wedges depend on the bulk rheology and the surface tractions. Analytical and numerical solutions of the force balance and corner flow circulation equations indicate that the mechanical parameters should, in principle, all be directly determinable from the observational variables. The analysis suggests the following preliminary conclusions. (1) The surface profiles of forearc wedges are consistent with either a bulk plastic rheology or a viscous rheology with a transition to plastic or Coulomb behavior in the upper few kilometers. (2) Corner flow circulation in the absence of accretion is diagnostic of wedges with a bulk viscous rheology. The surface strike-normal velocity and the surface profile both depend on the bulk viscosity and the viscous coupling at the base, allowing these mechanical variables to be determined: current estimates of this velocity in some wedges suggest the bulk viscosity is not less than $3 \times 10^9$ Pa s. (3) Obliquely converging nonaccreting plastic and Coulomb wedges show no distributed strike-parallel shear. If the obliquity is above a critical angle, the wedge is separated from the upper plate by a strike-slip fault, defining a forearc sliver. The critical angle of obliquity, the velocity of the forearc sliver, and the wedge geometry allow the boundary tractions to be determined. In obliquely convergent viscous wedges the strike-parallel velocity is distributed throughout the wedge, decaying exponentially from the rear to the front. Its value at the rear and the length scale for the decay allow the bulk viscosity and the coupling at the rear to be determined. Slip vectors in some forearc wedges and thrust belts suggest complete partitioning of the strike-parallel component of motion, characteristic of a bulk viscous rheology. Others show partitioning above a limiting angle of obliquity, suggesting bulk plastic behavior. There are no documented cases of active margins where there is an absence of partitioning for obliquity $\sim 45^\circ$, which appears to rule out Coulomb behavior as a general description for the bulk rheology of forearc wedges.

1. Introduction

A primary goal of geodynamics is to determine the nature and distribution of the forces that drive deformation within the Earth and the mechanical properties of the rocks involved. Convergent plate margins are commonly occupied by thrust or accretionary wedges, composed mainly of a body of deformed rock lying directly on the rigid underthrust plate. Our understanding of the internal dynamics of these systems is now sufficient to allow the possibility of calibrating their bulk mechanical properties directly from observations of their geometry and surface velocity distributions. The purpose of this paper is to summarize our present state of understanding, to present the results of a new analysis of the geometry and internal flow of obliquely converging wedges showing a bulk viscous rheology, and to draw some preliminary conclusions about the bulk mechanical properties of these wedges on the basis of the available data on their geometry, seismicity, and structure. Part of the material in sections 2-5 of this paper is summarized from previous work, particularly that of Platt [1993], but I have reformulated it here to emphasize ways in which observations on natural systems could be used to place constraints on their mechanical properties.

2. The Physical Basis of the Analysis

Mechanical analyses of the bulk behavior of thrust belts and accretionary wedges are all based on the assumption that the deforming material behaves as a medium without internal mechanical discontinuities, in which the stresses and rates of deformation vary continuously. In reality much of the deformation takes place by slip on faults, with displacements of up to several tens of kilometers. This does not invalidate the analyses, but it does mean that the results are only meaningful on a scale significantly larger than that of the discontinuities. The behavior of an individual thrust slice, for example, cannot be predicted by this approach.

Observation suggests that thrust belts and accretionary wedges generally adopt a tapered cross section (Figure 1), with a deforming wedge of material bounded below by a rigid underthrust slab [e.g., Davis et al., 1983]. The wedge is bounded to the rear by a region that is deforming less rapidly or not at all, known as the backstop [Byrne et al., 1993]. The backstop may be rigid, or the lack of deformation may be a result of high elevation, such that the loads generated by the topography roughly equal those exerted on its boundaries. The Tibetan plateau is an example of a nonrigid backstop to the Himalayan thrust wedge [England and Houseman, 1988]. Both the position and the orientation of the backstop boundary may be difficult to determine in practice; the boundary may be hidden beneath a forearc basin, and it may not correspond to the geologically defined rear of the accretionary wedge. Deformable forearc crust that lies directly on the underthrust slab will
behave mechanically as part of the wedge, whatever its geological origin. In extreme cases, such as the Mariana forearc, there is no accretionary wedge, but the mechanics of the deforming forearc region may still be describable in the terms adopted here. For this reason I refer to these systems as forearc wedges, whether or not they are accretionary in origin.

Two-dimensional analyses suggest that the state of stress in a thrust wedge is a function of its geometry and the shear traction on its base [e.g., Chapple, 1978; Davis et al., 1983; Dahlen, 1984]. If the deviatoric stresses are sufficient to cause the material to deform, the wedge will change shape in such a way as to relieve them. Deformation therefore continues until the wedge reaches a configuration such that the deviatoric stresses are no longer sufficient to cause any further change in its geometry. If the wedge is composed of a material with a finite strength, this will happen when the material is just on the point of failure or plastic yield. Such a configuration is known as a critical geometry [Davis et al., 1983]. Coulomb or plastic wedges have two critical geometries: they can be on the point of horizontal compressional deformation, in which case they have a critical geometry for compression; or they can be on the point of horizontal extension, in which case they have a critical geometry for extension [Dahlen, 1984]. A viscous wedge will deform until the bulk stretching rates (and hence deviatoric normal stresses) are zero. Such a configuration is known as a stable geometry, and there is only one such geometry [Platt, 1986].

These analyses predict a relationship between the geometry of the wedge and the state of stress within it. The geometry of active thrust wedges is continually disturbed by erosion, sedimentation, and accretion of new material at the front and along the base, and as a result they are commonly in a state of active deformation. The rates of deformation are a function of the mass fluxes into and out of the wedge and are not directly predictable from analysis of the critical state. The effects of accretion are discussed in section 6.

This type of analysis has now been extended into three dimensions [Platt, 1993]. Addition of the third dimension involves another mechanical variable (the shear traction on the rear of the wedge) and another set of observational variables (the distribution of the strike-parallel velocity within the wedge). Because of the way the mechanical variables are interrelated, these analyses provide additional routes for constraining the mechanical properties of the wedge as a whole.

The analyses have been carried out for three different isotropic bulk rheologies, namely, linear viscous, perfect plastic, and noncohesive Coulomb. None of these rheologies is likely to be a good approximation to the bulk rheology of a real forearc wedge. They do, however, roughly cover the extremes of possible rock behaviors, and analyses based on simple rheologies are essential before more complicated rheologies or combinations of rheologies can be attempted. These analyses provide predictions that can be tested against observation and provide a guide to the probable behavior of wedges with more complicated and possibly more realistic rheologies, such as power law creep [Liu and Ranalli, 1998].

Assumptions also have to be made about the mechanical properties of the boundaries of the wedge, which generate the tractions that control its behavior. For the analysis of a
viscous wedge I used a velocity-dependent shear stress boundary condition. This condition states that the shear stress on a boundary is a function of the velocity difference across it, and it is equivalent to assuming that there is a thin layer of fluid with viscous properties along the boundary. For a plastic wedge I used a constant shear stress boundary condition, equivalent to assuming that there is a thin layer of plastic material (assumed to be weaker than the wedge itself) along the boundaries. For a wedge with Coulomb behavior I used a normal-stress-dependent shear stress boundary condition, equivalent to assuming frictional behavior along the boundaries. The use of these boundary conditions is justified in more detail by Platt [1993].

A limitation of the three-dimensional analyses discussed here is that the wedge is assumed to be linear along strike and of infinite extent. The effect of arc curvature is to introduce unconstrained variations in the stresses and strain rates parallel to strike, which complicate or render intractable the analysis. If the scale of arc curvature is large compared to the width of the wedge, however, the analysis remains valid to a reasonable degree of approximation, and conclusions can therefore be drawn about the effects of the curvature. This is discussed further in section 7.

The reference frame used for the analyses is illustrated in Figure 1. The wedge has a surface slope $\alpha$, a basal slope $\beta$, an angle of taper $\theta$, and a local thickness $h$ measured in the $z$ direction. The angles $\theta$, $\alpha$, and $\beta$ are assumed to be small for the purposes of approximations and are all taken to be positive. The thickness of the wedge at the rear is $h_0$, and its width measured up the dip of the underthrust slab is $L$. The underthrust slab is taken as the reference frame for velocities. The backstop moves at velocity $V$ in the $xy$ plane at an angle of obliquity $\gamma_0$ to $x$. The wedge itself may be deforming, and material points within it have a velocity $v$ that is variable. The local obliquity of the velocity is given by $\gamma_0$, where $\tan \gamma_0 = v_y/v_x$ (Figure 1). For stresses I use the engineering sign convention (tensile stresses positive) throughout. The wedge is assumed to have an average density $\rho$, which has to be adjusted for the effects of immersion in water.

Much of the analysis derives from two force balance equations that relate the geometry of the wedge to the shear stress on its base and to the state of stress within it [Platt, 1993, equations 7 and 8]. The first is concerned with stress components in the $xz$ plane, normal to strike,

$$
(\tau_z)_x - \rho g h \alpha + 2 \bar{\sigma}_n \theta - \frac{2 h \bar{\sigma}_n}{\bar{x}} = 0,
$$

(1)

and the second with components in the $yz$ plane, parallel to strike,

$$
(\tau_y)_y + \bar{\sigma}_n \theta - \frac{h \bar{\sigma}_n}{\bar{x}} = 0.
$$

(2)

Shear stress components $(\tau_z)_x$ and $(\tau_y)_y$ act on the base of the wedge in the $x$ and $y$ directions, and $\bar{\sigma}_n$ and $\bar{\sigma}_m$ are vertically averaged deviatoric normal and shear stress components within the wedge. Equations (1) and (2) are independent of rheology and are given in their most general form. Substitution of appropriate rheologies in (1) and (2) yields the various forms of the two-dimensional force balance equations for stable viscous wedges [Platt, 1986], critical plastic wedges [Chappie, 1978], and Coulomb wedges [Davis et al., 1983]. The three-dimensional analyses for these rheologies are dealt with in turn in sections 3-5.

3. Viscous Rheology

A viscous material lacks a yield strength and deforms at rates that are related to the deviatoric stress components by the viscosity $\mu$. This type of behavior could result, for example, if pressure solution (diffusive mass transfer) [Rutter, 1983; den Brok, 1998] is the dominant mechanism of deformation. Feehan and Brandon [1999] and Ring and Brandon [2000] make the case for diffusive mass transfer as the dominant mechanism of internal deformation in two circum-Pacific accretionary wedges. I have assumed that the wedge has a constant viscosity $\mu$ and that the shear stresses on the boundaries are related to the local velocity of the material relative to the boundary by coupling constants $p$ and $q$ on the base and rear of the wedge, so that $\tau_x = p v_x$ along the base and $\tau_y = q [(v_y - V x) \cos \gamma_0]$ along the rear. The coupling constants have dimensions of viscosity divided by distance and in Figures 2-9 are cited in multiples of $10^{13}$ Pa s m$^{-1}$. The viscosity of the wedge is cited in multiples of $10^{14}$ Pa s.

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**Figure 2.** Profile of a viscous wedge, calculated for $\beta = 9^\circ$, $\rho = 2.5$, and $\mu = 100$. The elevation difference between the wedge front and a point at the surface of the wedge 100 km from the front is the variable $h$ shown in Figure 3.
3.1. Wedge Profile

A viscous wedge approaches a stable configuration with a convex-upward profile (Figure 2), in which the thickness and surface slope are related to the basal shear stress:

$$ \tau_b = \rho g h \alpha. \quad (3) $$

The surface profile and thickness of the wedge therefore provide a direct means of determining the shear stress at the base. If the shear stress obeys a velocity-dependent boundary condition, then the wedge profile will be a function of the coupling constant at the base and the convergence velocity. The velocity contrast at the base, and hence the shear stress, will be modified by corner flow circulation within the wedge, however (see below). As a result, the profile will also be dependent on the bulk viscosity of the wedge and has to be calculated numerically. A simple way to characterize the profile is by the elevation difference $e$ between the front of the wedge and the surface 100 km from the wedge front. Numerical solutions for $e$ calculated for a range of values of $p$ and $\mu$ are shown in Figure 3a (note that $e$ does not depend on the coupling constant $q$ at the rear of the wedge). For low values of $p/\mu$, $e$ is strongly dependent on $p$, but as $p/\mu$ increases, the material at the base of the wedge becomes increasingly entrained by the subducting slab. This means that the velocity contrast at the base, the resulting basal shear stress, and hence the value of $e$ become progressively less sensitive to increases in $p$. As a result, for $\mu = 10^{18}$ Pa s, $e$ never exceeds 3 km for any value of $p$.

The results of the calculation illustrated in Figure 3a would differ for different values of the convergence velocity, basal slope, and bulk density. The effect of $\beta$ is relatively small. For $\beta = 3^\circ$ the values of $e$ would be higher by roughly 1 km; for $\beta = 12^\circ$ they would be lower by $\sim$1 km (Figure 3b). The effect of the convergence velocity is large because of the assumed linear dependence of the basal shear stress on velocity. The value assumed for the calculations in Figure 3 is 60 mm yr$^{-1}$ for the normal component of the plate velocity, which is toward the upper end of the present-day range. Lower rates of convergence would result in lower values of $e$. Most active forearc wedges have values of $e$ of 5 km or more, so Figure 3a suggests that if they exhibit a bulk linear viscous rheology, $\mu$ is unlikely to be less than $3 \times 10^{18}$ Pa s. This compares well with the conclusions of Emerman and Turcotte [1983], based on a similar line of reasoning.

The calculations in Figure 3 were based on a density of 1500 kg m$^{-3}$, which is the effective bulk density of moderately consolidated sediment immersed in water. For an emergent thrust belt consisting of fully consolidated rock, a bulk density of 2600 kg m$^{-3}$ would be more appropriate. Values of $e$ using this density are reduced by 25-30% from those in Figure 3a. Active thrust belts on land tend to have lower differential relief between toe and rear (3-4 km over a 100-km cross-strike distance), consistent with this. The overall conclusion still holds, however: the bulk viscosity is unlikely to be less than $3 \times 10^{18}$ Pa s.

3.2. Corner Flow Circulation

The material within a viscous wedge is in a continuous state of flow, driven by the traction on the base and the gravitational forces within the wedge. This flow, known as corner flow, circulates the material around the wedge at a rate that depends on the viscosity and the boundary conditions [Emerman and Turcotte, 1983]. If this flow can be measured, therefore, it provides an independent means of calibrating the mechanical properties of the wedge. The rate of flow in the $x$ direction along the base, using the reference frame shown in Figure 1, is

$$ \left( v_x \right)_b = \frac{3 \mu V \cos \gamma_x}{3 \mu + \phi h}, \quad (4) $$

and along the surface it is

$$ \left( v_x \right)_s = \frac{3 \mu V \cos \gamma_x (2 \mu + \phi h)}{2(3 \mu + \phi h)}. \quad (5) $$

![Figure 3.](image-url) (a) Variation of $e$ (the elevation difference between the rear and front of the wedge) with the viscous coupling $p$ for various values of viscosity $\mu$ ($\beta = 6^\circ$). (b) Variation of $e$ with $p$ for various values of $\beta$ ($\mu = 100$).
It is more convenient to visualize these velocities by relating them to the upper plate, giving

\[ v_{x,h} = \frac{-ph \cos \gamma_0}{3\mu + ph} \]  

Equation (7) makes it clear that the material on the surface moves away from the upper plate. Strike-normal velocities predicted by (7) at the rear surface of the wedge, as a function of \( p, \mu, \) and \( h_0 \), are shown in Figure 4. These were calculated for a plate convergence velocity of 60 mm yr\(^{-1}\) normal to the margin, but can be scaled linearly for any other value of the plate velocity.

The calculations shown in Figure 4 are for the corner flow circulation that would take place in the absence of accretion of new material to the wedge. Large rates of frontal accretion will tend to decrease the rate of return flow; conversely, the rate will be considerably accelerated if new material is carried beneath the wedge and underplated [Platt, 1986]. Analogous forms of viscous circulation that take place in bivergent orogenic wedges have been investigated by Buck and Sokoutis [1994] and Willett [1999]: these may be more appropriate descriptions for some types of collisional setting.

As discussed in section 8, the limited evidence available suggests rates of corner flow circulation are, in general, unlikely to be much greater than 10 mm yr\(^{-1}\). Even this very rough constraint suggests that the ratio \( ph/\mu \) (shown as contours on Figure 4) is unlikely to be greater than 1. Since most accretionary wedges have a thickness \( h_0 \) of 20-30 km and an elevation contrast \( e \) of 5 km or more, Figure 3a suggests that the bulk viscosity is unlikely to be less than \( 3 \times 10^9 \) Pa s.

### 3.3. Strike-Parallel Velocity Distribution

The discussion of the strike-parallel velocity distribution presented by Platt [1993, equations (39)-(48)] is oversimplified and, in part, incorrect. A revised discussion is presented in the appendix and summarized here. For convenience, this part of the discussion is presented in terms of the vertically averaged dimensionless strike-parallel velocity \( \bar{u} \), where \( u = v_x/\sqrt{\sin \gamma_0} \). The distribution of strike-parallel velocity across the wedge is governed by the force balance equation (2), which for a viscous rheology can be expressed in terms of \( \bar{u} \) and its derivatives:

\[ \mu \frac{\partial^2 \bar{u}}{\partial x^2} - \mu e \frac{\partial \bar{u}}{\partial x} - \frac{\rho \bar{u}}{D} = 0, \]  

where \( D = \bar{u}/\bar{u}_0 \), the ratio of the vertically-averaged velocity to that at the base. Because both \( h, \theta, \) and \( D \) are functions of \( x \), (8) has to be integrated numerically, incorporating the results of a numerical solution for \( h \), based on (3) and (4).

If the taper of the wedge is neglected, and if the coupling at the base is sufficiently low in relation to the viscosity of the wedge that vertical gradients in velocity can be neglected, \( \bar{u} = \bar{u}_0 \) and (8) simplifies to

\[ h \mu \frac{\partial^2 \bar{u}}{\partial x^2} = \rho \bar{u}. \]  

Equation (9) can be integrated to give an expression for $\bar{u}$:

$$\bar{u} = \bar{u}_0 e^{-\frac{x}{B}}, \quad B = \sqrt{\frac{p}{\mu h}}, \quad (10)$$

$$\bar{u}_0 = \frac{q}{B\mu + q}, \quad (11)$$

where $\bar{u}_0$ is the velocity at the backstop.

Equation (10) provides us with a simple analytical expression for the strike-parallel velocity, which we can use to compare with the results of numerical solutions for (8). The distribution and magnitude of $\bar{u}$ across the wedge is sensitive to many variables, including the wedge taper $\theta$, thickness $h$, and the mechanical variables $p$, $q$, and $\mu$. In the next section I attempt to show how and why these variables affect $\bar{u}$.

The basal slope $\beta$ affects $\bar{u}$ through its effect on $\theta$ and $h$, but whereas $\theta$ and $h$ both vary with $x$, $\beta$ varies little across the wedge (and I have neglected such variations because they are not directly predictable from any other aspect of wedge geometry). It is therefore easiest to start by looking at the effect of $\beta$ on $\bar{u}$. The variation of $\bar{u}$ with $x$ is shown for $\beta = 1^\circ$ and $\beta = 12^\circ$ in Figure 5a. Higher values of $\beta$ give slightly higher values of $\bar{u}_0$ and $\bar{u}$ overall and a greater length scale for the decay of $\bar{u}$. The larger values of $\bar{u}$ result from the larger values of $h$, which causes a decrease in the basal shear stress and so via the force balance relation requires a decrease in the shear stress at the backstop. The latter depends on the velocity contrast $(1- \bar{u}_0)$; hence $\bar{u}_0$ must be lower. The increased length scale for the exponential decay of $\bar{u}$ away from the backstop results from its approximate dependence on $B = \sqrt{p/\mu h}$ (10); hence a larger $h$ favors a greater length scale.

The effect on $\bar{u}$ of the coupling constant $p$, which acts at the base of the wedge, is illustrated in Figure 5b. Increasing $p$ decreases $\bar{u}$ because of the increasing coupling of the fluid wedge to the base, and it also produces a decrease in the length scale for the decay of $\bar{u}$ because of the dependence of the exponent $B$ on $p$ (10).

The effect on $\bar{u}$ of the coupling constant $q$, which acts at the rear of the wedge, is profound, as illustrated in Figure 6. These calculations were carried out for a wedge with $\beta = 6^\circ$, $p = 1500$ kg m$^{-3}$, cross-strike width $L = 100$ km, $V = 120$ mm yr$^{-1}$, $\gamma_b = 60^\circ$, $p = 5$, and $\mu = 100$ ($\mu h/\mu = 1$ for a wedge 20 km thick). These values correspond to a moderate viscosity wedge with a fairly high basal traction ($\tau_b = 20$ MPa; $\tau_b = 10$ MPa) and an average taper. The coupling constant $q$ was varied over the range 0.1 (corresponding to weak rear coupling compared to the basal coupling) to 100 (corresponding to high coupling compared to the wedge viscosity, approaching the no-slip condition). The variation in $\bar{u}_0$ with $q$ is almost linear over a large range in $q$ but falls off as $qh$ approaches $\mu$. When $qh$ exceeds $\mu$, $\bar{u}_0$ approaches $1$ asymptotically.

Equation (10) suggests that variations in $q$ have no effect on the length scale for the decay of $\bar{u}$, and this is illustrated in Figure 7. The length scale is likely to be controlled mainly by $p$, $\mu$, and $h$, and this is illustrated by a plot of length scale against $1/B$, where $B = \sqrt{p/\mu h}$, in Figure 8. The length scale here is calculated as the distance across strike from the backstop over which $\bar{u}$ decays to half $\bar{u}_0$. Figure 8 shows a roughly linear increase in length scale as $B$ decreases, until the length scale approaches the width of the wedge, at which point it flattens off abruptly and then decreases slightly. Over most of its trajectory the slope of the curve increases gradually with $1/B$. 

Figure 5. (a) Variation of the vertically averaged strike-parallel velocity $\bar{u}$ with $x$ for $\beta = 1^\circ$ and $\beta = 12^\circ$ ($p = 2.5$, $q = 10$, and $\mu = 100$). (b) Variation of $\bar{u}$ with $x$ for $p = 0.5$ and $p = 10$ ($\mu = 100$, $q = 10$, and $\beta = 6^\circ$).
At very low values of $1/B$ the curve becomes roughly horizontal. This reflects the approach to an effective no-slip boundary condition at the base as $ph$ exceeds $\mu$. For the section of the curve between $1/B = 20$ (where $ph = \mu$) and $1/B = 80$ (at which point the length scale approaches the width of the wedge), the data can be fitted reasonably well by a straight line with the formula $\lambda = 0.855/B - 2.296$ where $\lambda$ is the length scale.

Because $\bar{u}$ is a function of $q$, $p$, $\mu$, and $h_0$, the most useful way to look at the results of the numerical calculations is to compare the results with those predicted by the simple analytical expressions (10) and (11) for a nontapered high-viscosity wedge. A plot of $\bar{u}$ (calculated numerically) against the analytical value for $\bar{u}$ from (11) is shown in Figure 9, based on 93 calculations covering the ranges $p = 0.1 - 10$, $q = 0.1 - 100$, $\mu = 1 - 1000$, and $h_0 = 11 - 24$ km for a wedge 100 km wide with a taper of 6°. The bulk of the numerical values lie very close to the analytical values (points in solid black lying close to a line with unit slope). Points lying above this line fall into two groups. For $\mu \leq 30$, $\bar{u}$ and $\bar{u}_0$ become insensitive to variations in $p$ once $ph$ exceeds $\mu$; and the points therefore lie above the line (open symbols in Figure 9). Secondly, if the length scale for the decay of $\bar{u}$ becomes comparable to the width of the wedge, the numerically calculated values for $\bar{u}_0$ lie above the line (asterisks in Figure 9).

Figure 9 suggests that over a wide range of viscosities, $\bar{u}_0$ can be determined fairly accurately from (11). The main exceptions are wedges with a very low viscosity ($\mu < 30$), which should be identifiable by their high return flow velocities, and
Figure 8. Length scale for the decay of against 1/B (both variables in km).

3.4. Viscous Wedge Behavior: Summary

The results of the calculations presented in this section lead to some useful conclusions about the behavior of obliquely convergent viscous wedges, which should allow them to be identified, if they exist, and which should allow their mechanical parameters to be determined from their geometry and internal velocity distributions (Figure 10a).

1. Viscous wedges are characterized by a convex-upward surface profile, with a surface slope that, in theory, increases to vertical at the wedge front. This point is discussed further in section 8.

2. Viscous wedges show a corner flow circulation, which results in a strike-normal velocity at the surface away from the backstop.

The velocity decreases to zero at the wedge front. This circulation in nonaccreting wedges is diagnostic of a viscous rheology.

3. The elevation contrast e between the front and rear of a viscous wedge and the magnitude of the return flow (v_y) in a nonaccreting wedge are both functions of p, q, t, and h. Since h can be measured, p and q can, in principle, be determined by measuring p and (v_y). To show how this works, consider a nonaccreting wedge 100 km wide with a strike-normal convergence velocity of 60 mm yr^{-1} and a basal slope of 6°. If (v_y) at the rear of the wedge is found to be 2.5 mm yr^{-1}, we can use Figure 4 to show that ph/µ = 0.2 (this implies that the shear stress at the base of the wedge is 0.2 of the value required were the imposed shear strain distributed throughout the wedge). If e is found to be 5 km, then h = 15.5 km, and from Figure 3 we can show fairly precisely that µ = 154 and p = 2, in the units adopted in this paper. The basal shear stress in this example would be 4 MPa at the wedge front, decreasing rearward.

4. The strike-parallel velocity distribution in a viscous wedge shows a distinctive exponential decay from the rear to the front. As discussed in the sections 4 and 5, plastic and Coulomb wedges do not show such distributions, so this pattern is diagnostic of viscous behavior. A real thrust belt or accretionary wedge, however, even if it has a bulk viscous viscosity (E18 Pa s)

- 1
- 2.5-5
- 10
- 30
- 100 - 1000
- high length scale

Figure 9. Numerical versus analytical solution for u as a function of p, q, µ, and h. The ordinate shows values for u calculated numerically; the abscissa shows values for u calculated using equation (11).
plastic or Coulomb wedges.

5. The strike-parallel velocity also increases upward from the base of the wedge (see appendix). This means that near the rear of the wedge there will be a significant amount of shear parallel to the base of the wedge in the strike-parallel direction, as well as a large component of strike-parallel shear. Both the vertical and horizontal variations in strike-parallel
velocity will be superimposed on the corner flow circulation and on any strike-normal components of motion caused by accretion of new material. The resulting variations in structural style within the deforming wedge are likely to be considerable (Figure 10b).

6. The length scale for the decay of the strike-parallel velocity is a function of \( p, \mu, \) and \( h, \) and it is unlikely to be strongly sensitive to accretion. In accreting wedges it can therefore be used instead of the return flow velocity to determine \( p \) and \( \mu, \) in conjunction with the elevation contrast \( e. \) If we take the previous example, a length scale of 28 km would, from Figure 8, give \( \frac{\sqrt{(\mu h/p)}}{35} \) and hence allow determination of \( \mu \) and \( p \) from Figure 3.

7. If \( \rho \mu < 1 \) and the length scale for the decay of \( \overline{u} \) is less than \(-0.8\) of the cross-strike width of the wedge, \( q \) can be determined simply from (11).

4. Plastic Rheology

I have based the analysis of a plastic wedge on the Levy-Mises perfectly plastic constitutive relation [Malvern, 1969]. A material with this property has the characteristic that if the second invariant of the deviatoric stress tensor is below a certain value, the deformation rates are zero and the stresses are unconstrained. Above this value the material deforms at rates that are proportional to the deviatoric stress components, but the constant of proportionality is dictated by the external boundary conditions. In the critical state a plastic wedge is on the verge of deformation at all points throughout the wedge, but the deformation rate normal to strike is zero. This means that the stress components are all related to each other and to the yield stress \( k. \) The yield stress is a property of the material and may vary through the wedge if the material changes. For simplicity I have assumed that it is constant. Plastic behavior might result if the dominant mechanism of deformation is intracrystalline slip controlled by the conservative motion of dislocations, and this rheology formed the basis for the two-dimensional analyses by Chapple [1978] and Stockmal [1983].

I have assumed that the shear stresses on the base (\( \tau_s \)) and rear (\( \tau_r \)) of the wedge are constant and independent of the velocity contrast across them. This type of behavior is the most compatible with the assumed rheology of the wedge, but if a velocity-dependent boundary condition is used, the results do not change in any fundamental respect.

The most important result of the analysis is that it is not normally possible for the strike-parallel component of motion to be distributed throughout a critical nonaccreting plastic wedge. If the wedge yields, so as to produce distributed motion, the plastic yield criterion requires the wedge to shorten horizontally at a rate dictated by \( \overline{\tau}_n. \) If the wedge is already in the critical state, there should be no horizontal shortening; hence \( \overline{\tau}_n \) would have to be zero. The force balance equations (1) and (2) then reduce to a very simple form, in which they predict continuously increasing \( \alpha \) and decreasing \( \theta \) toward the front of the wedge for any reasonable velocity distribution. The only wedge geometry that can satisfy both constraints requires the basal slope \( \beta \) to decrease toward the front at a greater rate than \( \alpha \) is increasing, so that the maximum value of \( \beta \) (at the rear of the wedge) must be larger than the maximum value of \( \alpha \) (at the wedge front). No natural wedges have such a geometry. In nature, therefore, a plastic wedge that has a variable strike-parallel velocity will fail to satisfy the force balance equations. The unbalanced forces will then cause parts of the system to accelerate in such a way as to remove the velocity variations. Distributed strike-parallel shear can therefore only occur if the wedge is accreting material sufficiently rapidly that some internal shortening is required to maintain the critical geometry. This situation is discussed further in section 6.

Because there is no variation of strike-parallel velocity in the wedge, it must either move together with the upper plate, or form a coherent forearc sliver that moves independently of upper and lower plates. Which will happen depends on the obliquity of the relative plate motion and on the geometry of the wedge. Put simply, there is a limiting value to the strike-parallel velocity of the wedge relative to the upper plate, and hence there is a limiting value to the obliquity of motion \( \gamma \) between the wedge and the underthrust plate, given by

\[
\gamma = \sin^{-1} \frac{\overline{\tau}_n}{LT},
\]

If \( \gamma \) (the obliquity of relative plate motion) is less than the limiting value \( \gamma \), the wedge will move at the same velocity as the upper plate does. For greater values of \( \gamma \) there will be slip on the rear boundary, and the wedge will move independently as a forearc sliver. In this case, the observed obliquity of motion at the front and along the base of the wedge will be less than the true obliquity and will be equal to the limiting value given by (12). This behavior is illustrated schematically in Figure 11.

The remaining component of strike-parallel motion will be taken up by slip along the backstop. The limiting angle of obliquity of relative motion between the wedge and the underthrust plate given by (12) is the arcsine of the ratio of the resistances on the lower and rear boundaries. This is essentially the same expression as that reached by McCaffrey [1992] on the basis of a force balance analysis applied to a forearc sliver of arbitrary geometry.

A plastic wedge will, in general, have a convex-upward profile, with a taper that increases toward the front. The profile is similar to that of a viscous wedge, but a plastic wedge can be distinguished by the fact that it can sustain oblique motion at the wedge front.

The low angles of taper of natural thrust wedges suggest that if they exhibit plastic or Coulomb rheology, the wedge is strong relative to the resistance to sliding at the base. If this is true, and if the wedge is moving independently of the forearc so that (12) applies, considerations of the wedge geometry and the stress state within it allow two more relationships to be derived between the geometry and deformation of the wedge on the one hand and its mechanical properties on the other [Platt, 1993]. These relationships are

\[
\tau_s = \frac{2\theta_j}{k \sqrt{1 + 3\sin^2 \gamma}}, \tag{13}
\]

where \( \theta_j \) is the taper at the wedge front, and

\[
\tau_s = \frac{\tan 2\theta_j}{k \sqrt{4 + \tan^2 \theta_j}}, \tag{14}
\]
where $\delta$ is the angle between the maximum principal compressive stress at the upper rear of the wedge and the strike-normal direction. All three mechanical variables $r_B$, $r_F$, and $k$ can therefore be related to the observational variables $\gamma$, $\theta$, and $\delta$. The obliquity $\gamma$ should, in principle, be determinable using geodetic methods or from seismic focal mechanism analysis near the wedge front, and $\theta$ can readily be measured from bathymetric or topographic data. If the material in the wedge is isotropic in the horizontal plane, then the stress orientation $\delta$ can be inferred from observational data that relate to the elastic strain field (such as borehole breakouts) or to small permanent strains (such as focal plane mechanisms of earthquakes or striations on active faults). The main limitations are the assumptions that the wedge has constant material properties across strike and that it is not accreting: if either of these are violated, there is likely to be some distributed shear and/or shortening within the wedge, which will affect the results (see section 6).

5. Noncohesive Coulomb Rheology

A Coulomb rheology is based on the assumption that the material is capable of deforming internally by fracture and by frictional sliding on surfaces with a wide range of orientations distributed throughout it. The behavior on a scale larger than the individual discontinuities can then be described in terms of a Coulomb "rheology", analogous to that of a plastic rheology [Davis et al., 1983; Dahlen, 1984]. Because of the abundance of faults within thrust and accretionary wedges as well as the evidence from seismicity for brittle failure, this is commonly assumed to be the most likely description of the bulk rheology of these systems. A Coulomb material can sustain shear stresses up to a limiting value $r_s$ which depends on the mean stress. If the maximum shear stress is below this value, the deformation rates are zero and the stresses in the material are unconstrained. If the maximum shear stress reaches this value, the material deforms at rates dictated by the external boundary conditions. A common assumption is that an initially homogeneous and isotropic Coulomb material will extend instantaneously in the direction of the minimum compressive principal stress, shorten in the direction of the maximum compressive principal stress, and undergo no length change in the intermediate principal stress direction. The deformation of a Coulomb material is therefore two dimensional and lies in the plane of the maximum and minimum principal stresses: it is unaffected by the value of the intermediate principal stress.

In a noncohesive Coulomb material the maximum shear stress $r_s$ depends on a material constant $\phi$ and on the pore fluid pressure, which is usually expressed in terms of $\lambda$, the ratio of the pore fluid pressure to the vertical normal stress. Both $\phi$ and $\lambda$ can vary through the wedge and are likely to do so. The boundary shear stresses on a Coulomb wedge are also likely to be frictional in nature and are governed by their own values of the coefficient of friction ($\mu_s$ on the base and backstop boundaries respectively) and by the local values of the pore fluid pressure. There are therefore at least five independent mechanical variables, all of which can vary with both position and time. Addition of a cohesion term to the Coulomb equation complicates matters still further [e.g., Dahlen et al., 1984]. A full analysis of the state of stress in a 3-D Coulomb wedge, including the effects of cohesion, has been presented by Enlow and Koons [1998]. In order to obtain a set of relatively straightforward relationships between the mechanical and observational variables, however, I have reduced the mechanical variables to the following four, which are assumed to be constant: $\mu_s^* = \mu_s(1 - \lambda)$, where $\lambda$ is the pore fluid pressure ratio along the basal décollement, $\mu_s$, $\phi$, and $\lambda$ (the average pore fluid pressure ratio in the body of the wedge).
Critical wedge theory deals with the geometry and mechanics of wedges that have an essentially static configuration, in which there is no bulk change of shape. It is possible, however, to make predictions about the effects of addition or removal of material from the wedge, because we know that the wedge will always tend to deform in such a way as to return to a critical geometry. For two-dimensional critical wedges the effects of accretion and erosion can be simply summarized as follows. Addition of new material at the front of the wedge (frontal accretion) and erosion of material from the rear of the wedge are likely to decrease the overall taper and surface slope, so that the wedge will tend to shorten internally to return to a critical geometry. Reactivation of thrusts, out-of-sequence thrusts, late back thrusts, and superimposed late upright folds are similar but with the additional complication that a component of shortening or extension across strike relaxes some of the conditions on which the conclusions about the distribution of the strike-parallel deformation were based. In a viscous wedge the cross-strike deformation required to maintain the stable configuration will simply be superimposed on the corner flow and strike-parallel velocity distributions discussed in section 3. Frontally accreting viscous wedges will therefore show orthogonal thrusting and shortening in the frontal region, and if the rate of accretion is large enough to overcome the effects of the corner flow circulation, they will show oblique out-of-sequence thrusting and en echelon folding at the
rear. Underplating viscous wedges, on the other hand, will show an increased rate of oblique extension in the upper rear of the wedge (Figure 10b). These effects may affect conclusions about the rheology based on the strike-normal velocity.

In obliquely converging plastic and Coulomb wedges the effects of accretion are very significant. If an isotropic plastic material starts to deform, the ratios of the different components of the strain rate tensor are the same as the equivalent components of the deviatoric stress tensor. Hence, for example, if the wedge shortens internally at a certain strain rate in response to frontal accretion, it will undergo strike-parallel shear at a rate in proportion to this by the factor \( \frac{\sigma_y}{\sigma_\alpha} \), where \( \sigma_y \) and \( \sigma_\alpha \) are the vertically averaged deviatoric stress components responsible for these components of deformation. The ratio of these two stress components in a Coulomb wedge is given by

\[
\frac{\sigma_y}{\sigma_\alpha} = \frac{2\mu_\alpha^* \sin \gamma}{\mu_\alpha^* \cos \gamma - \alpha^*}.
\]

In a plastic wedge this ratio can only be predicted in terms of the external variables at the front and rear of the wedge (see Platt [1993] for further discussion of this point). At the front of the wedge the relationship is

\[
\frac{\sigma_y}{\sigma_\alpha} = 2\tan \gamma,
\]

and at the rear of the wedge, if the wedge is strong relative to its basal décollement,

\[
\frac{\sigma_y}{\sigma_\alpha} = \frac{\tau_0}{\sqrt{\kappa_0^2 - \tau_0^2}}.
\]

Note that the stress ratio is related to \( \delta \) (the angle between the maximum principal compressive stress at the upper rear of the wedge and the strike-normal direction) by \( \tan 2\delta = 2\sigma_y/\sigma_\alpha \), and it can therefore be determined, in principle, by direct observation.

7. Implications for Arcuate Thrust Wedges

Curvature of a thrust belt or forearc wedge allows components of systematic shortening or extension parallel to strike, which cannot occur in a long straight system. These components of deformation invalidate the analysis presented here. If the radius of curvature is at least an order of magnitude larger than the width of the forearc, however, the analysis remains valid to a reasonable degree of approximation, and the effects of arc curvature can therefore be predicted to within that degree of approximation.

For the purposes of this discussion it is more convenient to take the upper plate as the reference frame for velocities. The change in orientation and hence in obliquity of convergence of the arc with distance \( s \) along strike is directly related to the radius of curvature \( R \) by

\[
\frac{dy}{ds} = \frac{1}{R}.
\]

In a viscous wedge the bulk of the wedge has zero strike-parallel velocity relative to the underthrust plate. The strike-parallel velocity \( v \), relative to the upper plate is therefore \( V \sin \gamma_\alpha \), where \( V \) is the velocity of the lower plate relative to the upper plate. If \( \gamma_\alpha \) changes around the arc, the velocity will change with \( \gamma_\alpha \) at a rate given by:

\[
\frac{dv}{d\gamma} = V \cos \gamma_\alpha.
\]

The strike-parallel velocity will increase with obliquity around the arc, as illustrated in Figure 12.

From (23) and (24) the variation in velocity will correspond to a strike-parallel stretching rate \( \dot{\varepsilon}_s = \frac{dv}{ds} \) [Jarrard, 1986; McCaffrey, 1991] given by

\[
\dot{\varepsilon}_s = \frac{V \cos \gamma_\alpha}{R}.
\]

At the front of the arc, where the convergence is normal to strike, the strike-parallel stretching will reach its maximum value, and it will decrease with increasing obliquity around the arc (Figure 12) according to

\[
\frac{d\dot{\varepsilon}_s}{d\gamma} = -\frac{V \sin \gamma_\alpha}{R}.
\]

![Figure 12: (a) Pattern and style of deformation in an arcuate viscous wedge (plan view). Horizontal stretching parallel to strike and horizontal shortening normal to strike are achieved mainly by strike-slip faulting. (b) Cross section before (light shading) and after (heavy shading) illustrating that strike-parallel horizontal stretching of a stable viscous wedge is accompanied mainly by strike-normal horizontal shortening without much thinning.](image)
Strike-parallel stretching will be accompanied by a strike-normal shortening to maintain the wedge in a critical or stable configuration. In the absence of accretion, therefore, the wedge will become narrower as well as thinner. The dominant style of surface deformation is therefore likely to be a combination of strike-slip faulting (possibly in conjugate sets) and normal faulting (Figure 12). Normal faulting will, however, be suppressed if there is frontal accretion.

The predicted pattern of velocities and strain rates in an arcuate viscous wedge will have the effect of progressively removing material from the front of the arc and transferring it to the obliquely converging limbs. In the absence of accretion, therefore, the wedge will be narrowest at the front of the arc and broaden on the limbs. This effect is likely to be largely or completely masked by accretion, however, because all other things being equal, the rate of accretion will be greater at the front than on the limbs because the rate of orthogonal convergence is larger.

Arcuate plastic wedges should behave very differently. In the frontal part of the arc, where the angle of obliquity of the convergence is below the critical value given by (12), the wedge remains attached to the upper plate, and there will be no strike-parallel extension. There may be two locations on either side, however, where the obliquity reaches the critical value, and at these two points the strike-parallel velocity of the wedge relative to the upper plate starts to increase with obliquity according to

$$v_s = V(\sin \gamma_o - C \cos \gamma_o),$$  \hspace{1cm} (27)

where $C$ is a function of the dimensions and material properties of the wedge given by

$$C = \frac{\tau_s h_b}{\sqrt{(L^2 \tau_s^2 - \tau_s^2 h_b^2)}}.$$  \hspace{1cm} (28)

Note that (28) is modified from Platt [1983, equation (63)], in which the sign of the expression inside the square root was accidentally reversed. There will therefore be an abrupt onset of strike-parallel stretching at these two locations, at a rate given by

$$\dot{\gamma}_s = \frac{V}{R}(\cos \gamma_o + C \sin \gamma_o).$$  \hspace{1cm} (29)

The stretching rate may then increase or decrease from this point on, depending on the value of $C$.

The predicted pattern of velocities and stretching rates for an arcuate plastic wedge is illustrated in Figure 13. The onset of strike-parallel extension at two locations on either side of the front of the arc is very distinctive and should serve to distinguish wedges with a plastic behavior from those with a viscous bulk rheology. As in viscous wedges, the predominant effect of the strike-parallel stretching will be to cause the wedge to become narrower across strike as well as thinner, as it will deform so as to maintain a critical geometry. An arcuate plastic wedge should therefore be widest at the front of the arc and become abruptly and significantly narrower at the two critical locations on either side.

An arcuate Coulomb wedge behaves in the same way as a plastic wedge does, except that the term $C$ is given by

$$C = \mu_s \left\{ 1 + \frac{\mu_s (1-\lambda) - \alpha}{\mu_s 1 + \mu_s} \right\}.$$  \hspace{1cm} (30)

A potential complication in this analysis is the presence of a rigid buttress or obstacle at some point within the arcuate structure, at a plate boundary intersection, for example. If the forearc is moving relative to the upper plate, it may experience arc-parallel shortening in the vicinity of the buttress, significantly modifying its geometry, pattern of internal deformation, and velocity distribution [Wang, 1996].

8. Constraints From Natural Examples

The geometry of many thrust and accretionary wedges is well constrained by surface topographic and seismic evidence. The surface profiles of active accretionary wedges are almost all convex up, steepening toward the toe, with average surface slopes in the frontal 10 km of up to 5°. This is most readily explained by a plastic bulk rheology: a viscous rheology would predict a vertical slope at the toe; a Coulomb rheology
The velocity distribution suggests arc-normal shortening and earthquake on the subduction zone interface, which appears to limit the region of high arc-normal velocity coincides with elastic strain buildup between major earthquakes. The northern part of the subduction zone has an arc-normal component of motion (toward Sumatra, on the upper plate) comparable to the motion between the Indian and Eurasian plates in this area. The velocity distribution suggests arc-normal shortening and hence thickening at about $10^{-7} \text{yr}^{-1}$. This could not be sustained over geological time (it implies a doubling in thickness of the forearc crust every 7 Myr), so it probably reflects transient elastic strain, it may prove difficult to use GPS methods to measure velocity distributions. Measurement over several seismic cycles may solve the problem, but in many forearcs the interval between major earthquakes on the subduction interface is of the order of 100-1000 years: somewhat longer than the budgetary cycles of funding agencies. Terrestrial geodetic measurements in the Hikurangi margin of the North Island of New Zealand represent the accumulation of strain over a period of several decades and may be representative of the long-term permanent deformation [Walcott, 1987], but the sign of the arc-normal horizontal deformation changed from contractional before the $M = 7.9$ Hawke Bay earthquake to extensional after it, indicating that interseismic elastic strain buildup has influenced the data.

In the absence of underplating, corner flow circulation is diagnostic of viscous bulk behavior, but there are few reliable measurements of the rate of this circulation, and it is difficult to demonstrate the absence of underplating over a sufficient time period (a few tens of millions of years). Extension in forearcs has been widely postulated on the basis of seismic and structural evidence [e.g., McIntosh et al., 1993; Wessel et al., 1994; Davey et al., 1997; Davis et al., 1998], and some form of corner flow has been postulated to explain the structural relationships of exhumed high-pressure metamorphic rocks in accretionary wedges [e.g., Cowan and Silling, 1978; Pavlis and Bruhn, 1983; Platt, 1986]. The timescale over which high-pressure rocks are exhumed from depth in orogenic wedges suggests rates of upflow varying from $0.5 \text{mm yr}^{-1}$ in the Franciscan Complex of California [e.g., Ring and Brandon, 2000], to as much as several tens of mm yr$^{-1}$ for parts of the Alps [e.g., Duchêne et al., 1997]. Underplating in wedges with any rheology may cause corner flow circulation if they become sufficiently supercritical [Platt, 1986], however, and a somewhat similar pattern of upflow of rock at the rear of thrust wedges may result from a combination of underplating and erosion [Platt, 1975; Brandon et al., 1998]. Geodetic observations in the Hikurangi margin suggest surface velocities locally as high as 30 mm yr$^{-1}$ away from the arc [Walcott, 1987; Darby and Meertens, 1995], associated with extension in the forearc [Cashman and Kelsey, 1990]. Walcott [1987] relates this deformation to substantial underplating of sediment beneath the wedge. Possible rates of corner flow circulation are also constrained by the presence on active forearc wedges of a surface veneer of sediment a few hundred to several thousand meters thick [e.g., Westbrook et al., 1988; Moore et al., 1990]. It seems unlikely that this could accumulate if the wedge were being resurfaced by corner flow on a timescale of less than 10 Myr, say. This implies that corner flow circulation velocities are, in general, unlikely to exceed $10 \text{mm yr}^{-1}$. As pointed out in section 3.2, this very approximate constraint suggests that the effective bulk viscosity of accretionary wedges is unlikely to be less than $3 \times 10^{19} \text{Pa s}$.

An important source of information comes from forearc seismicity. As pointed out by Jarrard [1986] and developed in detail by McCaffrey [1992, 1994] and Yu et al. [1993], mean slip vectors determined from seismicity are commonly deflected toward the arc normal relative to the motion vector between the upper and lower plates. Some of this effect is due to back arc spreading [Yu et al., 1993], but for the most part it reflects partitioning of part or all of the arc-parallel component of deformation onto strike-slip faults or zones of distributed shear in the rear of the forearc wedge or along the magmatic arc itself. Unfortunately, most earthquakes along the subduction interface come from depths of 20-40 km [Ruff and Tichelaar, 1996], toward the rear of a mature accretionary wedge, and the uncertainties on hypocentral locations are too large to constrain the velocity distribution within the wedge itself. Nevertheless, full partitioning is demonstrable on the Hikurangi margin of New Zealand from both seismic [Webb and Anderson, 1998] and structural [Cashman et al., 1992] data; and some arcuate systems, such as the Marianas, show seismic vectors normal to the trench right around the arc [Yu et al., 1993; McCaffrey, 1994]. There is at present no structural evidence to indicate where the strike-parallel component of motion in the Marianas is being taken up, and Yu et al. [1993] attribute the pattern of seismicity to back arc spreading. While back arc spreading will contribute to the divergence of slip vectors around the arc, it cannot by itself explain the absence of any strike-parallel component of motion. If this motion is being taken up at the rear of the wedge, the orthogonal pattern of seismic vectors suggests a bulk viscous rheology for the forearc (note that McCaffrey [1994] attributes the pattern in the Marianas arc to a "perfectly plastic" rheology, using a different line of reasoning).

Seismic and structural data from several on-land thrust belts also suggest almost perfect partitioning, characteristic of the predictions for a viscous bulk rheology. Seismic slip vectors for thrusting vectors around the Himalayan arc are consistently normal to strike [Mohar and Lyon-Caen, 1989]. These authors attribute the pattern to extension within the Tibetan plateau. As discussed above for the Marianas arc, this does not
provide a complete explanation for this pattern of seismicity, and strike-slip faults such as the Karakoram fault appear to be playing a role in partitioning oblique convergence [Searle et al., 1998]. Thrust vectors around the arc of the western Alps are also essentially radial [Platt et al., 1989]. In this case there is no evidence for back arc extension, and the pattern reflects partitioning of the overall WNW motion of the Alpine indentor relative to Eurasia into arc-normal thrusting and strike slip on faults such as the Tonale Line [Laubscher, 1988].

Seismic data from many forearcs suggests that there is a limiting angle of plate obliquity below which there is little partitioning [McCaffrey, 1994]. The Aleutian arc, for example, shows little evidence for partitioning where the plate obliquity is less than 30° (although there is a lot of scatter in the data). Above this angle the observed obliquity of the seismic slip vector remains at around 30° until the plate convergence vector reaches very high obliquities. This pattern is broadly consistent with a plastic rheology for the wedge (see section 4). A somewhat similar pattern is shown by the Java-Sumatra wedge, but the limiting angle in this case is lower, ~20°. These conclusions have to be taken with some caution, however, because of the lack of resolution in the location of the seismic data.

A striking observation is that there are no documented cases of active margins where there is an absence of partitioning for angles of plate motion obliquity of ~45°. This appears to rule out Coulomb behavior as a general description for the bulk rheology of forearc wedges (see section 5).

As pointed out by Jarrard [1986] and McCaffrey [1991], partitioning of the strike-parallel component of motion in obliquely convergent arcs requires arc-parallel stretching, and the distribution of this stretching may be diagnostic of the rheology (see section 7). Arc-parallel extension has been documented structurally and seismically in a number of forearcs, including the Aleutian [Ryan and Scholl, 1989; Avé Lallemant, 1996], Venezuela [Avé Lallemant, 1997], Mariana [Wessel et al., 1994], and Sumatra [McCaffrey, 1991], though the last is disputed by Bellier and Sebrier [1995]. Variations in the rates of stretching are as yet insufficiently well defined to constrain the bulk rheology.

9. Conclusions

Mechanical analysis of obliquely convergent forearc wedges allows predictions of the wedge geometry and surface velocity distributions, such that the bulk mechanical properties of natural wedges and their boundaries can be determined from measurements of the thickness and surface slope of the wedge and the strike-normal and strike-parallel velocity distributions. The means of making the necessary velocity measurements may not yet exist: short-term geodetic measurements using GPS tend to be dominated by interseismic elastic strain buildup; and while seismic and structural observations indicate along-strike variations in the bulk slip vector, they do not provide sufficient information on rates or on the cross-strike velocity variation. Nevertheless, some statements can be made, and refinements in technique can be envisaged that would allow sufficiently precise measurements to be made in the near future. The following preliminary conclusions can be drawn.

(1) The surface profiles of accretionary wedges are consistent with a bulk plastic rheology or a viscous rheology with a transition to plastic or Coulomb behavior in the upper few kilometers of the wedge. (2) If wedges do exhibit bulk viscous behavior, the slow rates of corner flow circulation (≤ 10 mm yr$^{-1}$), taken together with the observed surface profiles, suggest that the effective bulk viscosity is unlikely to be less than ~3x10$^{19}$ Pa s. (3) Seismic and structural data on slip vectors in forearc wedges and thrust belts suggest that some (Marianas, Himalayas, and the Alps) show complete partitioning of the strike-parallel component of motion, characteristic of a bulk viscous rheology. Others (Aleutians and Sumatra) show partitioning above a limiting angle of obliquity, suggesting bulk plastic behavior. There are no documented cases of active margins where there is an absence of partitioning for angles of plate motion obliquity of ~45°, which appears to rule out Coulomb behavior as a general description for the bulk rheology of forearc wedges.

Appendix: Strike-Parallel Velocity Distribution in a Viscous Wedge

A1. Introduction

The discussion in this appendix is presented in terms of the dimensionless strike-parallel velocity $u$, where $u = v_y / V \sin \gamma$. Assuming a wedge-shaped body of viscous material lying on a rigid slab below and with a rigid backstop behind it, the overall controls on $u$ are as follows. The shear traction $\tau_y$ along the backstop, created by the velocity difference between the material in the wedge and the backstop, produces a velocity gradient $\partial u / \partial x$ adjacent to it and a stress gradient $\partial \sigma_y / \partial x$ with respect to more distant material in the wedge. Since there is no topographic gradient in the $y$ direction, this stress gradient will cause material to accelerate in the $y$ direction unless it is balanced by a corresponding vertical stress gradient $\partial \sigma_y / \partial z$ [Platt, 1993, equation (2b)]. A velocity distribution therefore evolves in the wedge such that the stress gradients balance, and hence the restricted Stokes equation applies ($\partial^2 u / \partial x^2 = -\partial^2 u / \partial z^2$) [Platt, 1993, equation (35)]. If we can place constraints on $\partial u / \partial x$, we would then be in a position to define the variation of $u$ with $x$.

The existence of the velocity $u$ in the wedge creates a shear traction ($\tau_y$), at the base and hence a vertical velocity gradient $\partial u / \partial z$ (this is negative in the reference frame used in this paper). Because the shear traction $\tau_y$ must be zero at the free upper surface of the wedge, there is a vertical stress gradient $\partial \sigma_y / \partial z$ and hence a finite (positive) value to $\partial u / \partial z$. If the shear traction at the base is velocity dependent, the stress gradient and hence $\partial u / \partial z$ are related to the velocity at the base.

The force balance equations (1) and (2) depend on vertical integration of the stresses and velocities. The problem is therefore to determine a relationship between the vertically averaged strike-parallel velocity $\bar{u}$ and the velocity at the base $u_y$.

A2. Wedge with Zero Taper

It is easiest to start the analysis neglecting the taper of the wedge. The force balance in the $y$-direction [Platt, 1993, equation (8)],
\((\tau_s) + \bar{\sigma}_x \theta - \frac{\partial \bar{u}}{\partial x} = 0, \) \hspace{1cm} (A1)

then simplifies to
\[ \frac{h \partial \bar{u}}{\partial x} = (\tau_s), \] \hspace{1cm} (A2)

Assuming a velocity-dependent boundary condition \((\tau_s)_b = \rho(v_\alpha)_b\) and noting that \(\sigma_y = \partial v/\partial x\), and \(u = v/Vsin\gamma\), then
\[ h \mu \frac{\partial^2 \bar{u}}{\partial x^2} = \rho u_s, \] \hspace{1cm} (A3)

where \(\bar{u}\) is the vertically averaged velocity and \(u_s\) is the velocity at the base. If \(\phi << \mu\), so that vertical velocity gradients in the wedge are negligible, \(\bar{u} = u_s\), then
\[ h \mu \frac{\partial^2 \bar{u}}{\partial x^2} = \rho \bar{u}. \] \hspace{1cm} (A4)

For a zero-taper wedge the boundary conditions for integrating this equation are
\[ \begin{align*}
  x = 0, \quad & \bar{u} = \bar{u}_0, \quad (A5a) \\
  x = \infty, \quad & \bar{u} = 0, \quad (A5b) \\
  x = 0, \quad & \frac{\partial \bar{u}}{\partial x} = \frac{q}{\mu} (\bar{u}_0 - 1), \quad (A5c) \\
  x = \infty, \quad & \frac{\partial \bar{u}}{\partial x} = 0, \quad (A5d) \\
  x = \infty, \quad & \frac{\partial^2 \bar{u}}{\partial x^2} = 0. \quad (A5e)
\end{align*} \]

Hence
\[ \bar{u} = \bar{u}_0 e^{-Bx}, \] \hspace{1cm} (A6)

where \(B = \sqrt{\rho/\mu h}\).

Also, from boundary condition \((A5c),\)
\[ B\bar{u}_0 = \frac{q}{\mu} (1 - \bar{u}_0), \]
\[ \therefore \bar{u}_0 = \frac{q}{B\mu + q}. \]

Equation \((A6)\) for \(\bar{u}\) satisfies the external force balance equation for the wedge, i.e., the requirement that the forces on the base and the rear should balance:
\[ \int_0^h \rho dz = \int (\tau_s)_s dx, \] \hspace{1cm} (A8)

because \((A4)\) can be integrated horizontally to give:
\[ \left( \frac{\partial \bar{u}}{\partial x} \right)_s = \frac{P}{\mu h} \int_0^h \bar{u} dx, \]
\[ \therefore \left( \frac{\partial \bar{u}}{\partial x} \right)_0 = \frac{P}{\mu h} \bar{u}_0 \]
\[ \therefore h\tau_0 = \int_0^h (\tau_s) dx, \]

which is equivalent to \((A8)\).

As \(\phi\) approaches \(\mu\), vertical velocity gradients in the wedge become significant, and we need an expression for the relationship between \(u_s\) and \(\bar{u}\), in order to integrate \((A3)\). I suspect that this relationship will vary with \(x\), as well as with \(p, \mu\) and \(u_s\) and I know of no way to derive a precise expression. The following is an attempt at an approximation valid under certain conditions. Note that the following argument does not depend on the force balance equations and hence is not subject to the approximations and assumptions that underlie those equations.

The shear stress, and hence \(\partial u/\partial x\) drop to zero at the free upper surface, so \(\partial u/\partial x^2\) is positive, but its variation with \(z\) is unknown. If we assume as a first approximation that \(\partial u/\partial z\) is constant and equal to its value at the base \((\rho u_s)\) we can get a first-order idea of the relation between \(u_s\) and \(u_s\):
\[ \int_0^h \frac{\partial u}{\partial z} dz = u_s - u_s, \]
\[ \therefore \frac{u_s h}{2\mu} = u_s - u_s, \]
\[ \therefore \frac{u_s}{u_s} = 1 + \frac{p h}{2 \mu}. \]

The likely rates of corner flow in real wedges (discussed in this paper) suggest that the ratio \(\phi h/\mu\) is likely to be significantly less than one, so according to this approximation, \(u_s/u_s\) is unlikely to exceed 1.5.

We can obtain a more precise approximation if we make a more realistic assumption about \(\partial u/\partial x\). Since \(\bar{u}\) decreases with \(x\) according to \((A6)\), it is reasonable to assume that \(u_s\) decreases with \(x\) in the same way and with the same length scale. If it did not, the situation could arise in which either \(\bar{u}\) or \(u_s\) were effectively zero and the other finite, which would not be realistic. Hence we assume that \(\bar{u}/u_s = D\), where \(D\) is constant, and hence from \((A6)\),
\[ u_s = \frac{\bar{u}}{D} \]

If \(u_s\) is greater than \(u_s\), but is decaying exponentially and on the same length scale, then \(\partial u/\partial x^2\) must be greater at the surface than at the base. So to obtain a more precise approximation to the vertical velocity distribution, we assume that \(\partial u/\partial x^2\) varies linearly with depth, with
\[ (\partial^2 u/\partial x^2) = B^2 u_s, \quad (\partial^2 u/\partial x^2)_s = G^2 u_s, \] \hspace{1cm} (A10)

where \(G^2/B^2 = u_s/u_s\).

Then
\[ \frac{\partial^2 u}{\partial x^2} = -B^2 u_s - \frac{u_s z (G^2 - B^2)}{h}, \]
\[ \therefore \frac{u_s}{\partial x} = \frac{B u_s - B^2 u_s z + u_s z^2} {2h} (B^2 - G^2). \]

Since at the surface of the wedge \((\partial u/\partial x)_s = 0,\)
\[ \frac{p}{\mu} = \frac{B^2 h + G^2 h}{2}, \]
\[ \therefore G^2 = \frac{2p}{\mu h} - B^2. \] \hspace{1cm} (A11)
\[
\dot{\partial u} = \frac{p}{\mu} u_s - B^2 u_s z - \frac{u_s^2}{2h} \left( \frac{2p}{\mu h} - B^2 \right), \quad (A12) \\
u = u_s \left[ 1 + \frac{p z}{\mu} \frac{B^2 z^2}{2} - \frac{z^3}{3h} \left( \frac{p}{\mu h} - B^2 \right) \right], \quad (A13) \\
u = \frac{u_s}{1 + \frac{2ph}{3\mu} - \frac{B^2 h^2}{6}}, \quad (A14) \\
u = 1 + \frac{2ph}{3\mu} - \frac{B^2 h^2}{6}. \quad (A15)
\]

From (A13),
\[
\int u dz = u_s \left[ \frac{h^2}{12\mu} + \frac{6p h^2}{12} \right], \\
\vdash u = u_s \left[ \frac{h^2}{12\mu} + \frac{6p h^2}{12} \right], \quad (A16)
\]

From (A10), (A11), and (A15) we can obtain an expression for \( B \):
\[
B^2 \left[ 1 + \frac{2ph}{3\mu} - \frac{B^2 h^2}{6} \right] = \frac{2p}{\mu h} - B^2, \\
\vdash B^2 = \frac{2ph + 6\mu \pm 2\sqrt{p^2 h^2 + 9\mu^2 + 3p\mu h}}{\mu h^2}. \quad (A17)
\]

Equation (A17) has two roots. The larger gives negative values for \( u / u_s \) and \( \bar{u} / u_s \), so we take the smaller, using the negative value of the square root. For small \( ph/\mu \), (A17) reduces to \( B^2 = p/\mu h \). For the general case, \( B^2 < p/\mu h \). When \( ph = \mu \), \( B^2 = 0.8p/\mu h \). For the no-slip condition, when \( p = 0 \), \( B = 0.8 \).

Substituting (A17) into (A16) allows \( D \) to be determined, giving a relationship between \( \bar{u} \) and \( u_s \).

A3. Tapered Wedge: High Viscosity

For a tapered wedge we have to integrate the complete force balance equation (A1). This can be rewritten in terms of \( u \) and \( u_s \), assuming a velocity-dependent boundary condition \( (\tau_0) = \rho(\nu_s) \) and noting that \( \sigma_{\nu} = \partial \nu / \partial x \), and \( u = \nu / V \sin \gamma \):
\[
\mu h \frac{\partial^2 \bar{u}}{\partial x^2} - \mu^2 \frac{\partial \bar{u}}{\partial x} - p u_s = 0, \quad (A18)
\]

(note that \( \theta \) is taken as positive). If \( ph/\mu < 1 \), \( u_s = \bar{u} \), so
\[
\mu h \frac{\partial^2 \bar{u}}{\partial x^2} + \mu^2 \frac{\partial \bar{u}}{\partial x} - p \bar{u} = 0. \quad (A19)
\]

Equation (A19) can only be integrated numerically. The boundary conditions are
\[
x = 0, \quad \bar{u} = \bar{u}_0, \quad \frac{\partial \bar{u}}{\partial x} = \frac{q}{\mu} (\bar{u}_0 - 1). \quad (A20)
\]

The value of \( \bar{u}_0 \) has to satisfy (A8), the overall force balance equation, so
\[
qh_0 (1 - \bar{u}_0) = p \int_0^\infty u_s dx, \\
\vdash \bar{u}_0 = 1 - \frac{p}{qh_0} \int_0^\infty u_s dx. \quad (A21)
\]

If \( ph/\mu < 1 \), the integral can be evaluated assuming \( u_s = \bar{u} \); otherwise, \( \bar{u}_0 \) has to be determined by trial and error.

A4. Tapered Wedge: General Case

First, we have to reexamine the derivation of \( B \) in (A17), because \( D \) is a function of \( h \), and hence in a tapered wedge it will be a function of \( x \). This potentially undermines the basis of (A9) and (A10) and hence of the whole analysis. If we assume that \( B^2 = ph/\mu h \), treat \( \theta \) as constant, and assume that (A16) is approximately true,
\[
D = 1 + ph/2\mu, \\
\vdash \frac{\partial^2 D}{\partial x^2} = \frac{p\theta}{2\mu}, \\
\vdash \frac{\partial^2 D}{\partial x^2} = D \left( \frac{\partial^2 u}{\partial x^2} \right)_0 + \frac{p\theta}{\mu} \left( \frac{\partial u}{\partial x} \right)_0. \quad (A22)
\]

If \( ph/\mu < 1 \), the second term in (A22) will be small compared to the first and can be neglected, so
\[
\left( \frac{\partial^2 u}{\partial x^2} \right)_0 = D \left( \frac{\partial^2 u}{\partial x^2} \right)_0. \quad (A23)
\]

Hence (A9) and (A10) and the rest of the analysis remain valid to within a reasonable level of approximation. Equation (A18) becomes
\[
\mu h \frac{\partial^2 \bar{u}}{\partial x^2} - \mu^2 \frac{\partial \bar{u}}{\partial x} - \frac{p \bar{u}}{D} = 0, \quad (A24)
\]

where \( \bar{u} = Du_s \) is the vertically averaged value of \( u \). Note that \( \theta \) is taken as positive.

Equation (A24) can only be solved numerically, but it is useful to consider its implications in general terms. It can be recast as follows:
\[
\frac{p \bar{u}}{\mu D} = h \frac{\partial^2 \bar{u}}{\partial x^2} - \theta \frac{\partial \bar{u}}{\partial x}. \quad (A25)
\]

A viable solution has to have a broadly inverse exponential form, with \( u \) decreasing away from the backstop. If the length scale for the decay is small compared to the width of the wedge, then \( u, u', \) and \( u'' \) (where \( u' \) and \( u'' \) are the first and second derivatives, respectively) must all decay to zero. Note that the
term involving $u''$ is positive, and since $u'$ is negative, the term involving $u'$ also has a positive value. Hence as $u$ decreases, the absolute values of $u'$ and $u''$ must decrease.

For larger values of taper ($\Theta$), $u$ will, in general, be slightly larger (to counter the effect of a larger $h_0$ on the force on the backstop (A21), and the absolute values of $u'$ and $u''$ will be less (note that $u''_d$ decreases as $u_0$ increases according to (A20), thereby increasing the length scale for the decay).

The effect of the decrease in $h$ toward the front of the wedge is to lower the term involving $u''$. This has to be compensated by a decrease in the decay of $u'$, which implies a decrease in $u''$. If the length scale for the decay of $u$ is large, so that $u$ still has a significant value near the front of the wedge, this effect will become very marked: as $h$ decreases to zero, $u''_d$ may change sign, causing the absolute value of $u'$ to increase and $u$ to decrease or change the sign of $u''$, which means that $u'$ decays less rapidly or even increases, and hence $u$ decreases more rapidly. Further decreases in $p$ do not cause any further increase in the length scale.

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References


Bellier, O., and M. Sebrier, Is the slip rate variation due to a change in the length scale with q. Increasing q changes the force balance relation for the wedge as a whole (A21) and hence requires lower values of $u$ and larger values of $u'$ and $u''$, thereby decreasing the length scale for the decay of $u$. Increasing $q$ also changes the force balance, requiring higher values of $u$, and $u'$ also increases with $q$ (A20), commensurately with $u$, and so $u''$ does too. As a result, there is no change in the length scale with $q$. Increasing $\mu$ does not change the force balance relation, so it has little effect on $u$, but it does require a decrease in $u''$ (A20) and $u''$, thereby increasing the length scale for the decay of $u$.

The control on the length scale by $p$ and $\mu$ is limited by the effect of small $h$ and large $\Theta$ in the toe zone, as discussed above. Once the length scale becomes comparable to the width of the wedge, the effect of decreasing $h$ and increasing $\Theta$ is to decrease or change the sign of $u''$, which means that $u'$ decays less rapidly or even increases, and hence $u$ decreases more rapidly. Further decreases in $p$ do not cause any further increase in the length scale.


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J.P. Platt, Department of Geological Sciences, University College London, Gower Street, London WC1E 6BT, England, U.K. (john.platt@ucl.ac.uk)

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