Numerical and theoretical analyses of in-plane dynamic rupture on a frictional interface and off-fault yielding patterns at different scales

Shiqing Xu$^1$ and Yehuda Ben-Zion$^1$

$^1$Department of Earth Sciences, University of Southern California, Los Angeles, California, USA, 90089-0740
Email: shiqingx@usc.edu

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Summary

We perform numerical simulations of in-plane ruptures with spontaneous Mohr-Coulomb yielding in the bulk and analyze properties of the ruptures and yielding zones at different scales. Using a polar coordinate system, we show that the overall shapes and patterns of the simulated yielding zones can be well explained by combining the slip-induced Coulomb stress change and the background stress. While there is no apparent mechanism for preferring synthetic vs. antithetic shearing at a scale much smaller than the yielding zone size, this is not the case at larger scales. For shallow angles $\Psi$ between the maximum background compressive stress and the fault, representing thrust faulting, large-scale off-fault synthetic fractures are dominant but there are two conjugate sets of fractures with a typical size comparable to the yielding zone thickness. For smooth rupture propagation with moderate-to-high $\Psi$ values representing large strike-slip faults, most of the off-fault fractures that grow across the entire yielding zone are of the synthetic type. The less preferred antithetic set may become more pronounced for rupture propagation encountering fault heterogeneities. In particular, a strong fault barrier promotes antithetic fractures with a comparable size to those of the synthetic type around the barrier, where very high permanent strain is also observed. A consideration of non-local properties of the stress field in space or time can explain the above differences. Our results provide an alternative way of understanding Riedel shear structures and the potentially preferred synthetic shear fractures suggested in previous studies. The examined dynamic processes may be distinguished from quasi-static patterns by the timing, location, and inclination angle of characteristic fracture elements. In agreement with other studies, we propose that backward or orthogonally inclined antithetic shear fractures on strike-slip faults and very high permanent strain could be used as signals that reflect abrupt rupture deceleration. On the other hand, relative lack of off-fault yielding at given locations may indicate abrupt rupture acceleration.
Introduction

Various studies have used plasticity in recent years to model off-fault yielding during propagation of dynamic ruptures on frictional faults (e.g. Andrews 2005; Ben-Zion & Shi 2005; Templeton & Rice 2008; Dunham et al. 2011a). There is no length scale or specific orientation in plasticity described by stress invariants in a continuum, so the smoothly distributed plastic strain can only reflect certain yielding zone properties. These include the overall shape of the yielding zone and local orientations of two possible conjugate microfractures. However, plasticity allowing for shear localization (Templeton & Rice 2008) or direct modeling of discrete shear branches (Ando & Yamashita 2007) can provide some insights on the potential large-scale fractures inside the yielding zone. The properties of the generated yielding zones reflect, in addition to the assumed yielding rheology, the background stress operating on the fault and the dynamic stress generated during rupture propagation. Improved understanding of the conditions leading to different sets of yielding properties at different scales can help inferring from field observations information on the occurrence and properties of dynamic ruptures.

Examinations of the dominant stress field at various scales and comparisons with expectations from Linear Elastic Fracture Mechanics (LEFM) or modified models in the near field, and the background stress in the far field, provide fundamental tools for understanding the yielding zone properties (e.g. Poliakov et al. 2002; Rice et al. 2005). However, previous analyses of yielding features either did not consider fully effects resulting from superposition of space-time varying transient failure zone lobes (Poliakov et al. 2002), or considered effects of changing rupture front configuration but presumed the type of shear branches from two possibilities (Ando & Yamashita 2007). In addition, most previous studies on the topic did not explore the relation between the evolving stress field and the smoothness of rupture process.

In this paper we consider the above mentioned effects explicitly and show that analyses of non-local properties of the stress field, in space at fixed time and in time with changing rupture front configuration, along with the smoothness of rupture propagation, lead to a refined understanding of off-fault yielding characteristics on several scales. The results highlight the roles of incremental yielding zones, the existence of outer and possible internal envelopes of cumulative yielding, and competition between two possible sets of conjugate shear fractures during smooth and non-smooth rupture processes. The presented results, together with those of previous studies, help to develop better connections between failure processes on a fault and
yielding zone properties.

2. Model setup

We consider 2D in-plane ruptures and off-fault yielding under plane strain conditions. The relevant stress components are illustrated in Figure 1. A right-lateral rupture is initiated in a prescribed zone (red bar in Figure 1) and is then allowed to propagate spontaneously along the frictional fault (solid black line). The first motion of the radiated $P$-waves defines four quadrants relative to the hypocenter with “C” and “T” denoting compressional and extensional quadrants, respectively. The initial normal and shear stresses on the fault are $\sigma_0 = \sigma_{0y}$ and $\tau_0 = \sigma_{0y}^0$, respectively, and $\Psi$ represents the acute angle between the background maximum compressive stress $\sigma_{\text{max}}$ and the fault. A relative strength parameter $S = (\tau_s - \tau_0) / (\tau_0 - \tau_d)$ represents the relation between the initial shear stress, static strength $\tau_s = f_s(-\sigma_0)$ and dynamic shear strength $\tau_d = f_d(-\sigma_0)$, with $f_s$ and $f_d$ being the static and dynamic friction coefficient. It is well known that increasing the initial shear stress on a frictional fault toward the static strength leads to a transition from subshear to supershear rupture propagation (Burridge 1973; Andrews 1976; Das & Aki 1977; Day 1982). In this study we set the value of $S$ to produce subshear ruptures, which is the typical situation for most earthquakes (e.g. Ben-Zion 2003, and references therein).

2.1 Slip-weakening friction

We adopt a linear slip-weakening friction (SWF) to describe the breakdown process along the fault outside the nucleation zone. Specifically, the frictional strength $\tau$ has the following dependence (e.g. Ida 1972; Palmer & Rice 1973; Andrews 1976) on slip $\Delta u$

$$\tau = \begin{cases} 
\tau_s - (\tau_s - \tau_d)\Delta u / D_c & \text{if } \Delta u \leq D_c \\
\tau_d & \text{if } \Delta u > D_c
\end{cases}$$

(1)

where $D_c$ is a characteristic slip distance over which shear strength reduces from $\tau_s$ to $\tau_d$. When the background shear stress is only slightly higher than $\tau_d$, the size of the spatial region associated with the strength reduction, referred to as the process zone, can be estimated (e.g.
Rice 1980) by
\[ R = \frac{R_0}{f_{\|}(v_r)}, \]  
(2a)

where \( R_0 \) is the static value of \( R \) at zero rupture speed. For Poissonian solids this is given by
\[ R_0 = \frac{3\pi \mu D_c}{8 (\tau_s - \tau_d)}, \]  
(2b)

where \( \mu \) is shear modulus and \( f_{\|}(v_r) \) is a monotonic function of rupture speed \( v_r \) that increases from unity at \( v_r = 0 \) to infinity at the limiting Rayleigh wave speed. For proper resolution in numerical simulations, we discretize \( R_0 \) with multiple numerical cells (19 or more) and check the simulation results to ensure that the process zone \( R \) is well resolved.

### 2.2 Nucleation procedure

We follow the procedure of Xu et al. (2012a) with a time-weakening friction (TWF) to artificially trigger the rupture within the nucleation zone. The rupture front during the nucleation stage is enforced to propagate outward with a constant subshear speed and the frictional strength at locations reached by the rupture front linearly weakens with time up to the dynamic level \( \tau_d \):
\[ \tau = \min \left\{ \max \left\{ \tau_0 - \frac{(\tau_s - \tau_d)(v_r t - r)}{L_0}, \tau_d \right\}, \tau_s \right\}, \]  
(3)

where \( r \) is the along-strike distance from the hypocenter and \( L_0 \) is a spatial scale for the strength reduction (similar to the length scale \( R \) for SWF). We usually use \( L_0 \) as a reference for other length scales since its value is fixed. The size of the nucleation zone is determined by the prescribed \( v_r \) and a desired time span. In practice, we choose the effective frictional strength within the nucleation zone to be the minimum of those determined by the TWF and SWF, and make the nucleation zone large enough to produce a sustained rupture outside the nucleation zone under SWF.

### 2.3 Off-fault material response

The off-fault yielding during dynamic ruptures is governed in most cases by the Mohr-Coulomb plasticity. The onset of yielding is described by a yield function of stress
invariants:

\[ F = \tau_{\max} - \sigma_y, \]

(4)

where \( \tau_{\max} = \sqrt{(\sigma_{xx} - \sigma_{yy})^2 / 4 + \sigma_{xy}^2} \) is the maximum shear stress and \( \sigma_y = -1/2(\sigma_{xx} + \sigma_{yy})\sin(\phi) + c\cos(\phi) \) is the yielding strength, with \( \phi \) being the internal friction angle and \( c \) being the rock cohesion. Yielding occurs when \( F \geq 0 \).

After the onset of yielding, plastic flow is partitioned into different components through a plastic flow potential \( M = \tau_{\max} \) assuming zero inelastic volumetric deformation over a characteristic time scale \( T_v \):

\[
\dot{\varepsilon}_i^p = \frac{\langle F \rangle}{\mu T_v} \frac{\partial M}{\partial \sigma_{ij}},
\]

(5)

where \( \dot{\varepsilon}_i^p \) is the rate of plastic strain and \( \langle x \rangle = (x + |x|) / 2 \) is the ramp function. An appropriate finite value of \( T_v \) for stress relaxation (rate-dependent visco-plasticity) can help reducing shear localization that usually has a strong dependence on the numerical mesh. A rate-dependent (rather than instantaneous) response may also reflect the actual physical process of microcracking growth under high strain rates (see Simo & Ju 1987, and references therein). In some of the simulations we adopt this strategy to obtain smoothly distributed plastic strain. In other cases we use a perfect rate-independent plasticity by setting \( T_v = 0 \) (e.g. Loret & Prevost 1990; Prevost & Loret 1990) to focus on physical implications of the shear localization features. Relevant theoretical background for shear localization is reviewed when appropriate. To facilitate our theoretical stress analysis, we do not consider post-failure weakening or hardening mechanisms in this study. Various weakening or hardening mechanisms provide more realistic yielding behaviors for given circumstances (e.g. de Borst 1988; Leroy & Ortiz 1989; Ando & Yamashita 2007; Shi et al. 2010; Lyakhovsky et al. 2011) and should be explored in future studies. In all cases we use the scalar seismic potency defined as \( \varepsilon_0^p = \sqrt{2\varepsilon_i^p \varepsilon_i^p} \) to measure the intensity of the plastic strain (e.g. Ben-Zion 2008). More details on the adopted plasticity and related energy balance verification can be found in Xu et al. (2012a).

### 2.4 Numerical methods and parameters

The simulations are done with the 2D spectral element code developed by J. -P. Ampuero
(SEM2DPACK-2.3.8, http://sourceforge.net/projects/sem2d/). The calculation domain is typically discretized into square elements with 5 Gauss-Lobatto-Legendre nodes non-uniformly distributed per element edge. Occasionally we use obliquely oriented spectral elements to investigate the mesh alignment effect on the pattern of dynamic shear bands (see Appendix C). A typical traction-at-split-node technique has been implemented in the code to solve dynamic rupture problems (Andrews 1999; Kaneko et al. 2008). Absorbing boundary conditions are assumed around the calculation domain, which is set large enough (much larger than that shown in figures) to ensure that the rupture and generated off-fault yielding patterns do not interact with the absorbing boundaries. A visco-elastic layer of the Kelvin-Voigt type is added near the fault to damp the high frequency numerical oscillations near the fault. Additional details on the code can be found in the SEM2DPACK-2.3.8 user’s guide.

Similar to Xu et al. (2012a), the simulated quantities are normalized by reference parameters. The values of some quantities corresponding to conditions for natural faults are estimated in some cases. Table 1 lists the values of various parameters that are fixed in this work. The quantities most relevant for the focus of our study are the length scale and intensity of the off-fault yielding. We therefore use \( L_o \) and \( \sigma_c / \mu \) (Table 1) to provide reference length and strain values, respectively. Table 2 summarizes the conversions between the physical and normalized quantities. We typically use an average grid size (between two adjacent nodes) of \( \Delta x \approx L_o / 16 \) for the numerical simulations; refined meshing is used when studying the mesh size effect on the pattern of dynamic shear bands (see Appendix C). The relation between \( \Delta x \) and the estimated value of \( R_o \) for ruptures under various stress and strength conditions are also presented.

3. Results

3.1 Basic properties of the yielding zones

Previous works have shown that the distribution of off-fault plastic yielding generated by in-plane ruptures depends strongly on the angle \( \Psi \) (e.g. Poliakov et al. 2002 and later studies). Yielding occurs primarily on the compressional side when \( \Psi \) is less than about 15º and progressively switches to the extensional side when \( \Psi \) is larger than about 30º. The strong dependence on \( \Psi \) holds regardless of whether the rupture mode is crack-like (e.g. Andrews...
2005; Templeton & Rice 2008; Xu et al. 2012a) or pulse-like (e.g. Ben-Zion & Shi 2005; Rice et al. 2005; Dunham et al. 2011a). This is illustrated in Figure 2 for crack-like ruptures with 2 values of $\Psi$ characterizing faults stressed with low ($\Psi = 10^\circ$) and high ($\Psi = 45^\circ$) angels. The assumed angels represent approximately 2D configurations corresponding, respectively, to thrust faults (in a cross-section view) and large strike-slip faults (in a map view), although other factors may also influence the faulting style.

Poliakov et al. (2002) analyzed the conditions favoring large-scale synthetic shear (i.e. with the same sense of shear as the main fault) along off-fault paths and showed that the optimal orientation is located on the compressional and extensional sides when $\Psi$ is low and high, respectively. They also suggested where to search for the potential failure plane by analyzing through Mohr circles the stress field including the background stress and slip-induced stress change in front of the rupture tip. Dunham et al. (2011a) calculated based on the non-singular model proposed by Poliakov et al. (2002) the stress evolution with respect to the yielding level to estimate where off-fault yielding is likely to occur under relatively low and high values of $\Psi$. These studies explain the general location dependence of the off-fault yielding zone on $\Psi$. However, recent results of Xu et al. (2012a) show a characteristic angular distribution pattern of the transient stress field around the rupture tip that may lead to several distinct failure zone lobes. The orientation and relative size of different lobes provide additional information about the dynamic stress field that can be used to improve the understanding of off-fault yielding. In the following we first discuss briefly key results of Xu et al. (2012a) on the angular failure zone pattern around the rupture tip (section 3.1.1). Then we provide a simple theoretical analysis to explain the simulated angular pattern (section 3.1.2), and describe how the cumulative yielding zone forms in the wake of a propagating rupture (section 3.1.3).

### 3.1.1 Incremental yielding zone around the rupture tip

We follow the procedure described by Xu et al. (2012a) to show the equivalent incremental plastic yielding zone around the rupture tip. To achieve the goal, we plot the normalized Coulomb stress $(\tau_{\text{max}} - \sigma_T) / |\sigma_m^0|$ around the rupture tip in Figure 3 at the same time step used for Figure 2, where $\sigma_m^0$ is the initial mean stress. The thick and thin black bars represent, respectively, the local orientation of expected right-lateral (synthetic) and left-lateral (antithetic) shear fractures inside the current yielding zone at the examined time step. According to the
employed visco-plasticity in Eq. 5 and the assumed non-zero value for $T_v$ (Table 1), if the stress field exceeds the yielding strength it will be relaxed gradually while staying above the yielding level within the same time step. Therefore the stress-based plots of Figure 3 have similar patterns to the incremental plastic strain. As seen, in addition to the overall location-dependence of the plastic yielding zone on $\Psi$, the expected plastic strain increment displays an angular distribution pattern around the rupture tip with several distinct lobes (marked by letters). Some lobes are oriented forward while others are oriented backward and may dominate the contribution to off-fault yielding (e.g. lobe-A in Fig. 3b). The angular distribution of yielding zone increment, and especially the emergence of the backward-oriented lobes, has not been explicitly explained by previous studies (e.g. Poliakov et al. 2002) and is analyzed in more detail below.

3.1.2 Stress analysis based on LEFM

With the assumed constant values for rock cohesion and internal friction angle, the distribution of off-fault plastic yielding could be determined by the total stress field (Eq. 4). In our study, this is the sum of the background stress field $\sigma_{ij}^0$, which is given as a model input, and the slip-induced incremental stress field $\Delta \sigma_{ij}$. For basic analysis, we adopt the singular crack model (Freund 1990) to investigate the angular variation of $\Delta \sigma_{ij}$ with respect to the crack tip. In this case,

$$\Delta \sigma_{ij} = \frac{K_{II}^d}{\sqrt{2\pi r}} \Sigma_{ij}^{II}(\theta, v_r) + O(1),$$

(6)

where $K_{II}^d$ is the instantaneous dynamic stress intensity factor, $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$ are the transformed polar coordinates with origin at the crack tip, $\Sigma_{ij}^{II}$ are dimensionless functions of $\theta$ and rupture speed $v_r$, and $O(1)$ denotes higher order terms that are bounded by constants as $r \to 0^+$. The expressions for the different components of $\Sigma_{ij}^{II}$ and the dynamic stress intensity factor $K_{II}^d$ are given in Appendix A.

With the expressed quantities in a Cartesian coordinate system (Eq. A1), the normal and shear stress change along arbitrarily oriented planes around the crack tip can be written as
\[
\Delta \sigma_{\theta \theta} = \Delta \sigma_{xx} \sin^2 \theta + \Delta \sigma_{yy} \cos^2 \theta - \Delta \sigma_{xy} \sin 2\theta \\
\Delta \sigma_{\theta \phi} = [(\Delta \sigma_{yy} - \Delta \sigma_{xx}) \sin 2\theta]/2 + \Delta \sigma_{xy} \cos 2\theta
\]

(7)

where \( \theta \) is the inclination angle of the chosen plane with respect to the fault (see inset in Figure 4b). The angular variations of the leading terms of \( \Delta \sigma_{\theta \theta} \) and \( \Delta \sigma_{\theta \phi} \) normalized by \( K_{II}^d / \sqrt{2\pi r} \) are plotted in Figure 4, using \( v_r = 0.87c_s \) for \( \Psi = 10^\circ \) and \( v_r = 0.83c_s \) for \( \Psi = 45^\circ \), with \( c_s \) being the S-wave speed. The different assumed \( v_r \) values are based on the simulations results of Figure 2. As seen, \( \Delta \sigma_{\theta \phi} \) is an even function of \( \theta \) (solid black curve) and \( \Delta \sigma_{\theta \theta} \) is an odd function of \( \theta \) (dashed black curve). Two sets of the Coulomb Failure Stress change \( \Delta \text{CFS}^- = -\Delta \sigma_{\phi \theta} + 0.6\Delta \sigma_{\theta \theta} \) (blue) and \( \Delta \text{CFS}^+ = \Delta \sigma_{\phi \theta} + 0.6\Delta \sigma_{\theta \theta} \) (red) are plotted to help evaluating the efficiency of \( \Delta \sigma_{ij} \) in promoting non-local (i.e. over some distance \( r \) from the crack tip) left-lateral and right-lateral shear fractures. Local maxima of \( \Delta \text{CFS} \) are marked with numbers and letters to indicate the preferred sense of shear with respect to the rupture tip. We have checked that each local maximum is associated with a local peak value of \( \Delta \text{CFS} \) and a non-zero shear component \( \Delta \sigma_{ij} \) of a proper sign (e.g. a local maxima of \( \Delta \text{CFS} \) with very low \( \Delta \sigma_{ij} \) will not be identified as a favored shear fracture; rather it indicates a potential tensile fracture).

The incremental stress field induced by slip is expected to determine whether off-fault yielding can be activated near the rupture tip (e.g. Poliakov et al. 2002). Determining the ability of activated yielding zone to extend to larger scales requires consideration of the stress field at those scales. We follow these ideas to examine the orientations and extent where the two stress fields \( \sigma^0_{ij} \) and \( \Delta \sigma_{ij} \) interact positively to promote Coulomb-type yielding. As pointed out by Poliakov et al. (2002), there are two (constant) stress fields \( \sigma^1_{ij} \) and \( \sigma^0_{ij} \), differing only by the shear component, that affect the total stress at intermediate and far distance ranges from the rupture tip, respectively. In our study the shear stress components in \( \sigma^1_{ij} \) and \( \sigma^0_{ij} \) are a small fraction of \( \tau_s \). We therefore do not distinguish between \( \sigma^1_{ij} \) and \( \sigma^0_{ij} \), and adopt \( \sigma^0_{ij} \) as the (constant) dominating stress field at large scales. The resolved maximum compressive stress orientation obtained using \( \sigma^1_{ij} \) or \( \sigma^0_{ij} \) differs only by about 5\(^\circ\), which does not affect significantly our analysis.
Figure 5 illustrates the spatial interaction between $\sigma^0_{ij}$ and $\Delta \sigma_{ij}$ in a conventional polar coordinate system. Four dashed quarter circles represent the quadrants favoring left-lateral shear (dashed blue) or right-lateral shear (dashed red) by the background stress field $\sigma^0_{ij}$. The polar diagrams in the center display the angular variation of the positive part of $\Delta \text{CFS}^+$ (defined in Figure 4), showing where left-lateral (solid blue) or right-lateral (solid red) shear is encouraged by the leading term of the incremental stress field $\Delta \sigma_{ij}$ near the rupture tip. Where off-fault yielding can be activated near the rupture tip depends on the location of local maxima of $\Delta \text{CFS}^+$ (in terms of $\Delta \sigma_{ij}$). On the other hand, whether the activated yielding zone can extend to larger scales depends on the spatial (angular) interaction of $\Delta \text{CFS}^+$ with $\sigma^0_{ij}$. This is indicated conveniently in Figure 5 by whether or not the solid lobes are located within a dashed quadrant with the same color.

As shown in Figure 5a for $\Psi = 10^\circ$, two $\Delta \sigma_{ij}$-promoted local maxima in the upper half plane $\theta \in [0^\circ, 180^\circ]$, corresponding to “#3-R” and “#4-L” in Figure 4a, are located in the right-lateral $\theta \in [-10^\circ, 80^\circ]$ and left-lateral $\theta \in [80^\circ, 170^\circ]$ quadrants, respectively. Therefore, the two stress fields are positively interacting in these orientations, consistent with the well-developed “lobe-B” and “lobe-C” in Figure 3a. However, in the lower half plane $\theta \in [-180^\circ, 0^\circ]$, two $\Delta \sigma_{ij}$-promoted local maxima, corresponding to “#1-L” and “#2-R” in Figure 4a, are located in the right-lateral and left-lateral quadrants, indicating a competition between the two stress fields. The only exception is near $\theta = -100^\circ$, where either right-lateral or left-lateral shear is encouraged by both $\sigma^0_{ij}$ and $\Delta \sigma_{ij}$ (probably due to the maximum tensile stress change along this orientation; see Figure 4a). However, as this yielding zone extends further, the background maximum compressive stress $\sigma_{\text{max}}$, operating as the normal stress along the inclined plane $\theta = -100^\circ$, starts to dominate the stress field and is expected to suppress its further development. This prediction is consistent with the activated but only weakly developed “lobe-A” in Figure 3a.

Similarly, as shown in Figure 5b for $\Psi = 45^\circ$, among all the three $\Delta \sigma_{ij}$-promoted local maxima of Figure 4b, only “#1-L” oriented to $\theta \approx -125^\circ$ is located in a stress quadrant with the same sense of shear promoted by $\sigma^0_{ij}$. Therefore only in the vicinity of this orientation can the
activated yielding zone extend to a larger scale from the rupture tip, in agreement with the well-developed “lobe-A” in Figure 3b. Although none of the $\Delta \sigma_y$-promoted local maxima is located right ahead of the rupture tip (e.g. $\theta \in [-45^\circ, 45^\circ]$), the two stress fields $\Delta \sigma_y$ and $\sigma_0^0$ are still positively interacting and promote together right-lateral shear. This can produce a weakly-developed yielding zone or at least a close-to-failure stress zone right ahead of the rupture tip, corresponding to the weak “lobe-B” in Figure 3b. For such a case where the examined feature has a size comparable to or smaller than the process zone, the singular crack model may not work very well.

We note that considering the interaction between the remote and slip-induced stress fields in the employed polar coordinate system is merely a convenient way of illustrating a tendency for promoting left-lateral or right-lateral shear with respect to the rupture tip. On the other hand, the constitutive law for accumulating plastic strain at each point depends on the local stress field only through Eq. 5, with no connection to the overall non-local pattern of the stress field and no preference to a particular coordinate system for describing the stress field.

### 3.1.3 Formation of cumulative yielding zones

We can now describe the process producing cumulative plastic yielding zones of the type simulated in Figure 2. As the rupture propagates, a series of incremental yielding zone lobes are successively produced along strike. Each of these is expected to have fixed shape soon after their activation due to the short timescale $T_r$ for stress relaxation and the low residual stress level left behind the rupture tip. This implies that the envelope(s) of the cumulative yielding zone will be constructed by the farthest point(s) of each set of the incremental yielding lobes, and that the overall partition of the final yielding zone onto different sides of the fault will generally follow the pattern of the incremental yielding lobes. Considering the yielding zones generated during the rupture propagation stage along these lines leads to several expectations on observable properties of the cumulative final yielding zone.

One observable is the shape of the final yielding zone. Although an exact prediction is not possible due to the non-linearity of the problem, several end-member cases with relatively simple rupture history can be addressed. For example, a self-similarly expanding rupture is expected to produce a triangular-shaped yielding zone, whose thickness linearly increases in the rupture
propagation direction (Figure 2). Another end-member case is a slip pulse in a quasi-steady state, which is expected to generate a yielding zone with approximately constant thickness along fault strike. These end-member cases have been demonstrated earlier by Andrews (2005) and Ben-Zion & Shi (2005), respectively, and were explored further by Xu et al. (2012a, 2012b).

A second possible observable is an overprinting feature in the envelope of the yielding zone. As shown in Fig. 2a for $\Psi = 10^\circ$, in addition to the outermost yielding zone envelope (blue) that extends on the compressional (top) side beyond the rupture tip, there is another internal envelope with more intense yielding (red to green) that only approaches the rupture tip from behind. This is not surprising if we recognize that the former and latter envelopes form by the progression of the forward- and backward-oriented incremental yielding zone lobes, respectively (Fig. 3a). The observation that the more intense internal yielding zone (red to green in Fig. 2a) approaches the rupture tip from the behind can be explained by the fact that this region is swept commonly by both the backward- and forward-oriented incremental yielding zone lobes. Naturally, we expect a prominent overprinting feature if there are several distinct yielding zone lobes on the same side of the fault and each lobe is with a considerable size. For comparison, the case $\Psi = 45^\circ$ in Fig. 2b does not have an internal envelope with a prominent overprinting feature on the extensional side, since there is only one dominant yielding zone lobe on that side (Fig. 3b). As a result, the maximum plastic strain for $\Psi = 45^\circ$ is lower than that for $\Psi = 10^\circ$. This indicates that in addition to influencing the off-fault yielding zone location, the angle $\Psi$ also affects the maximum level of the permanent off-fault strain.

### 3.2 Generation of large-scale off-fault shear fractures

Here we explore how large-scale shear fractures, whose size may be comparable to or larger than the thickness of the yielding zone, may be induced by dynamic ruptures. The interaction between large-scale off-fault shear fractures and the stress failure zone associated with the moving rupture tip is shown to produce, on a fault with uniform properties, a preference for generating synthetic off-fault fractures over antithetic ones.

#### 3.2.1 A conceptual model

The analysis done so far has no mechanism for preferring either left-lateral or right-lateral microfractures with scale much smaller than the size of the yielding zone. Therefore, we assume
that the generated yielding zones are filled with many conjugate sets of microfractures (Figure 6). In contrast, the formation of large-scale shear fractures requires that the large-scale stress field should be consistently constructed in some non-local way in space or time. A non-locality in space means that the stress field should coherently promote the activation and growth of potential shear fractures along certain orientations over a considerable distance range. A non-locality in time means that the potential large-scale fractures should continuously be subjected over a considerable time span, with changing configuration of the rupture tip, to a stress field of similar sign.

Following the above reasoning, we may expect that the slip-induced incremental stress field near the rupture-tip is responsible for the activation and early growth of potential large-scale fractures, approximately along the orientations given by the local maxima of the “driving force” (e.g. $\Delta CFS$ in Figure 4). We note that there may be different criteria to predict the initiation orientation of such fractures with respect to the rupture tip. After the initial activation, whether or not the fractures can continue to grow will depend on the stress field at large scales and/or the time period of being loaded coherently from the moving rupture tip.

Figure 6a and Figure 6b illustrates schematically two scenarios corresponding to the compressional side of Figure 2a and the extensional side of Figure 2b. The grey lobes and their interiors represent the current stress failure zone defined by the Mohr-Coulomb criterion. The overall orientation and relative size of each lobe depend on the angular pattern of $\Delta CFS$ near the rupture tip (Figure 4) and its spatial interaction with $\sigma^0_{ij}$ at large scales (Figure 5). The orientation of the transient maximum compressive stress $\sigma^\prime_{\text{max}}$ is indicated by a pair of arrows based on Fig. 3.

As mentioned, the seed of each potential large-scale shear fracture is likely activated by the stress field near the rupture tip, presumably following the orientation given by one of the local maxima of $\Delta CFS$. As the rupture front moves forward the activated seeds will be located behind the rupture tip. The seeds associated with antithetic shear have growing direction opposite to the rupture propagation direction. Therefore, they spend relatively short time within any failure zone lobe associated with the moving rupture tip and are likely to stop growing, unless other mechanisms can promote their further growth. In contrast, the seeds associated with a synthetic shear tend to grow in the rupture direction so they spend longer time inside the moving rupture-tip failure zones. As illustrated in Fig. 6, the synthetic shear fractures are promoted over
some ranges of space and time by the stress field inside the updated failure zone lobes (see the relative orientation of $\sigma_{\text{max}}'$), so they can continue to grow in the forward direction. As the rupture tip moves sufficiently forward, so that the synthetic shear fractures are no longer promoted by any failure zone lobe, they will be arrested unless other mechanisms exist.

Since we are using the concept of failure zone lobes around the propagating rupture tip to construct the formation and extension of discrete large-scale shear fractures, the overall distribution of the large-scale fractures is expected to display a self-similar pattern along strike as indicated by the dashed black lines in Figure 6. However, we note that the above discussion assumes that the synthetic and antithetic fractures are equally nucleated at a series of locations along the fault and their subsequent growth is not affected by overlapping with other fractures. We also have not considered the feedback between the generated off-fault fractures and the main rupture, and the interaction between neighboring off-fault fractures. Moreover, when a rupture reaches a critical length that depends on the ambient stress environment, it is likely to continue to grow spontaneously further over some distance (see e.g. Fig. 12 of Ando & Yamashita 2007 and Figure 13 of Ben-Zion 2008). This provides a possibility for the occurrence of dynamic instabilities at the tips of shear fractures that are long enough, which may generate a hierarchical structure of off-fault shear fractures with branches and bifurcations over several length scales. These complexities are partly discussed in the following section.

### 3.2.2 Numerical simulations with off-fault shear localization

Many observational and theoretical studies analyzed shear localization features under various loading conditions (e.g. see the review by Hobbs et al. 1990 and Rosakis & Ravichandran 2000). While numerically simulated localization could have a strong dependence on the employed mesh (e.g. Needleman 1989; McKinnon & Garrido de la Barra 1998), they have been carefully used to successfully explain observed localization features in the field and in laboratory experiments (e.g. Poliakov & Herrmann 1994; Lecomte et al. 2012; Li et al. 2002). Templeton & Rice (2008) showed numerically that analysis based on the bifurcation theory for localization under quasi-static deformation could also be applied for dynamic rupture problems. Here we follow and extend their work to investigate deformation localization onto shear bands for yielding characterized by the Mohr-Coulomb type plasticity. We use perfect rate-independent plasticity ($T_v = 0$), with no hardening or softening and no volumetric change. The relevant
background material is summarized in Appendix B. Issues related to the reliability of numerically simulated localization during dynamic ruptures are discussed in Appendix C.

From the results in Appendix B, it can be shown that for our numerical implementation with the above plasticity, the following double-side inequality is satisfied: \( h_{\min} < h = 0 < h_{\max} \), where \( h_{\min} \) and \( h_{\max} \) are the critical minimum and maximum hardening moduli related to localization, and \( h \) is the specified hardening modulus. Therefore, shear localization could occur and the critical angle corresponding to \( h_{\max} \) is given by \( \theta_c = \frac{1}{2} \arccos \left( \frac{1}{2} \sin \phi \right) \approx 37.5^\circ \). This value is larger than the Mohr-Coulomb angle \( \theta_M = \frac{\pi}{4} - \frac{\phi}{2} \approx 29.5^\circ \) (Jaeger et al. 2007), smaller than the approximated Roscoe angle \( \theta_R = \frac{\pi}{4} - \frac{\psi}{2} = 45^\circ \) where \( \psi \) is the dilatancy angle assuming that elastic strains are negligible compared to plastic strains (Roscoe 1970), and is approximately equal to the angle suggested by Arthur et al. (1977) \( \theta_\lambda = \frac{\pi}{4} - \frac{\phi}{4} - \frac{\psi}{4} \approx 37.3^\circ \).

Figure 7a shows simulation results for \( \Psi = 10^\circ \) characterizing low angle thrust faulting. As seen, the overall pattern of the generated shear bands is similar to the conceptual model illustrated in Figure 6a. The off-fault yielding zone filled by shear bands displays a triangular shape as expected for self-similar growing pattern. There are clearly two sets of shear bands, with the longer (synthetic) and shorter (antithetic) sets oriented forward and backward, respectively. The relatively high angles of these off-fault shear bands imply that some amount of the total potency is partitioned onto the fault normal direction. The envelope formed by the longer shear bands characterizes the outmost boundary of the entire yielding zone with an overall self-similar pattern, and the internal envelope formed by the shorter shear bands also shows a general self-similarity. Although the two band sets are mainly developed in different locations characterized by the forward- and backward- oriented lobes (see Fig. 6a and Fig. 7a), the acute angle between the two sets is generally in a good agreement with the theoretical prediction \( 2\theta_c \), implying that the transient stress orientations within the two lobes are similar. Additional details may be seen in close-up views of the results at various locations. In location-I in Fig. 7a with a limited rupture distance, shear bands are hardly generated at the very beginning and the developed shear bands are primarily of the synthetic type. The overprinting feature as discussed
in section 3.1.3 mainly involves at this stage re-activation of the synthetic shear bands. In
location-II with an intermediate propagation distance, both synthetic and antithetic shear bands
are clearly observed and develop in a regular way, reflected by the quasi-linear envelope and
almost uniform spacing between adjacent bands. In contrast, the results in location-III with long
propagation distance show strong non-linear features such as fluctuations in shear band length
and spacing. The apparent slope of the envelope formed by the longest shear bands also deviate
from the previous slope where self-similarity is well retained.

Figure 7b shows corresponding results for $\Psi = 45^\circ$ characterizing large strike-slip faults.
In contrast to the conceptual model in Figure 6b with two sets of shear bands, the simulations
show only one predominant set of synthetic shear bands, similar to previous results of Templeton
& Rice (2008). Additional numerical simulations with various meshing strategies (Appendix C)
and investigation of the nucleation procedure confirm that a single dominant set of shear bands
as shown in Figure 7b is a robust feature at relatively large scales comparable to the rupture
propagation distance (for the examined cases with homogeneous fault properties). The predicted
antithetic shear bands may be observed in the very beginning of the rupture, near the edge of
nucleation zone (location-I), as well as at a local small scale close to the fault (location-II).
However, the most likely situation when rupture entered the spontaneous propagation stage is
that the antithetic shear bands are not activated at all, or soon stop growing, due to the extension
of earlier activated synthetic shear bands. The acute angle between two conjugate shear bands at
locations where they co-exist is about $75^\circ$, also in a good agreement with the theoretical
prediction $2\vartheta_c$. At larger propagation distance (location-III), clear second-order bifurcation
feature indicates that the generated synthetic bands develop their own branches, with strong
fluctuation in band orientation and spacing between adjacent bands. The bifurcation feature may
be explained as resulting from increasing stress field between two adjacent well-developed
synthetic bands with increasing rupture distance, so secondary branches need to be triggered to
relax the high stresses.

It is interesting to ask why the simulation for $\Psi = 45^\circ$ shows only one dominant set of
shear bands, in contrast to the results for $\Psi = 10^\circ$ with two well-developed sets. A careful
examination of the stress field surrounding the moving rupture front suggests that this may be
explained by a combination of several effects. The first is related to the off-fault locations where
the two sets of shear bands tend to grow. For $\Psi = 10^\circ$, the activated conjugate shear bands tend
to grow in different locations associated with distinct stress lobes (Figs. 6a and 7a). Although later activated antithetic bands may have to pass through earlier formed synthetic sets, the antithetic bands usually do not intersect at the tip region of a currently growing synthetic band thus may still extend outward. For $\Psi = 45^\circ$, however, the two sets attempt to grow in the same location at the same time (inside the backward-oriented stress lobe), so there is a clear competition between the antithetic and synthetic bands (Figs. 6b and 7b). Therefore, the earlier development of the synthetic set could suppress the growth of the antithetic set at the same location.

Figure 8 illustrates schematically a second effect related to competition between generations of the two sets of shear bands. The grey stress lobes are constructed in a similar way to Fig. 6, with solid lobes corresponding to a reference state at time $t$ and dashed lobes to an updated state at time $t + \Delta t$. The rupture advance $\Delta x$ is assumed to be small enough (compared to the size of the backward-inclined lobe) such that the transient stress fields around the rupture front at $t$ and $t + \Delta t$ are similar (they will be exactly the same for a propagating steady-state pulse regardless of the value of $\Delta x$). The total Coulomb stress at a given time generally decays with distance from the rupture front inside each characteristic lobe (Fig. 3b) in agreement with the $1/\sqrt{r}$ term in $\Delta \sigma_{ij}$ (Eq. 6). This is also generally true for a Lagrangian description (e.g. the stress field ahead of a virtually growing shear band along $\tilde{r}$ or $\tilde{r}'$) if the change of rupture configuration is small compared to the characteristic size of the stress lobe (as assumed above). In contrast, the Lagrangian-type stress variation along the transverse direction of the lobe (e.g. $\tilde{\theta}$ or $\tilde{\theta}'$) is probably small because the examined path more or less follows the translation direction from the reference state to the updated state.

Another effect is related to the orientation of the stress field. As shown in Figure 3b and examined more quantitatively by Xu et al. (2012b), there is an anti-clockwise rotation of the principal stress orientation with increasing distance from the fault. This can result in bending of the shear band growing path, moving initially backward-inclined antithetic bands towards the fault-normal direction and initially forward-inclined synthetic bands towards the fault-parallel direction. Combining the two effects, the antithetic set tends to grow in a direction with relative large stress decay gradient and/or relatively strong rotation of stress orientation, while the corresponding stress variations along the growing path of the synthetic set are likely to be relatively small. Therefore, the synthetic shear bands are expected to be more promoted and their
subsequent development may further suppress the growth of the less promoted antithetic set. The above analysis assumes that the rupture-induced stress field is dominant and the expected yielding zone size is relatively large. Therefore, it may not be appropriate for explaining the situation in the very beginning of a rupture. As mentioned in section 3.2.1, the synthetic bands also benefit from the moving stress lobes over longer duration than the antithetic shear bands even. All these effects explain the dominant generation of large-scale synthetic shear bands (or fractures) by dynamic ruptures along larger strike-slip faults.

3.3 Cases with fault heterogeneities

So far the main fault was assumed to have homogeneous initial stress and frictional properties. However, natural faults are associated with various geometrical and rheological heterogeneities (e.g. Ben-Zion & Rice 1993). In this section we analyze two simple end-member cases of fault heterogeneities.

3.3.1 Rupture behavior around fault junction

We consider a configuration where multiple piece-wise planar fault segments are connected at a junction point. In particular, we show how the ideas presented in section 3.2 can be applied to explain the experimental results by Rousseau & Rosakis (2003, 2009) on fault bend and branch problems. To be consistent with the impact loading and the material properties used in their laboratory experiments, we only consider the stress field induced by an incoming mode-II crack and assume an equivalent internal friction coefficient of 0.577 for the surrounding bulk. The model configuration is described by a fault branch or bend path that is oriented at \( \theta = -100^\circ \) with respect to the main fault segment (Figure 9). For the fault branch problem an extended horizontal path remains beyond the junction point (Figure 9a), while for the fault bend problem it is excluded (Figure 9b).

Based on the stress analysis of section 3.1, we plot the normalized stress field (the leading term) in the form of \( \Delta CFS \) ("+" and "−" sign for synthetic and antithetic rupture triggering, respectively) around the rupture tip (currently located at the junction point), with an instantaneous rupture speed \( v_r = 0.85c_R \) where \( c_R \approx 0.92c_s \) denotes the Rayleigh wave speed. We do not include a finite cohesion in our consideration of rupture triggering, but refer to the clarification by Rousseau & Rosakis (2003) that in the experiments fault interfaces are bonded.
with an ability to sustain shear ruptures even under a transient tensile stress. For the fault branch problem (Figure 9a), since the backward branch is unfavorably oriented for a direct rupture jump (because of the large change in segment direction), the incoming rupture is likely to stay on the horizontal path, although part of its induced stress field (solid blue lobe) attempts to trigger an antithetic shear rupture along the branch path. As the rupture tip rapidly moves forward along the horizontal path, the induced stress field is also removed quickly from the backward-oriented fault branch. Unless a critical state has been reached along the branch within this short time period, the potential antithetic shear rupture will not be triggered. Because some energy may have been consumed in the triggering attempt, the main rupture is expected to experience deceleration or even get arrested after some distance beyond the junction point.

For the fault bend problem (Figure 9b), the expectation may be different. Unless some breakouts are created in the bulk around the junction point, most of the energy of the incoming rupture will be transferred more efficiently to the weak bend path. Still, due to the unfavorable orientation of the bend path for a rupture jump, one may expect that the incoming rupture and its induced stress field can be temporarily halted (with a strong deceleration) around the junction point. Since the increased transient dynamic stress field associated with the reduced rupture speed tends to stay around the junction point and in particular along the bend path for longer time duration than for the fault branch problem, a new rupture will have more opportunities to be triggered following the stress distribution along the bend path. This may most likely be an antithetic shear rupture with a reversed sense of slip compared to the incoming rupture. If the bend path is not favorably oriented for a rupture transfer (e.g. $\theta \in [60^\circ, 120^\circ]$), the energy of the incoming rupture will probably be absorbed by breaking into the surrounding bulk through off-fault damage process.

We note that in contrast to the theoretical analysis of Rousseau & Rosakis (2003, 2009) on the absolute “driving stress”, our analysis with separate considerations of $\Delta CFS$ for synthetic and antithetic shears, and the time duration effect, has more explicit connections to observations. We also note that it is more realistic to consider in general the triggered secondary rupture along a kink path to be mixed mode-I and mode-II, in particular without remote compressive stress. Indeed, an “opening mode” was presumably observed by Rousseau & Rosakis (2003, Figure 13c) for a continuing intersonic rupture along a fault bend path with $\theta = -56^\circ$ relative to the previous fault segment.
3.3.2 Off-fault shear fractures around fault strength heterogeneities

The result in section 3.2.2 that there is a relatively clear antithetic shear around the edge of the nucleation zone (location-I in Fig. 7b), suggests that abrupt changes of rupture behavior can promote the generation of off-fault antithetic shear fractures for moderate-to-high \( \Psi \) values. This is examined here with simulations assuming heterogeneous static friction coefficient along the fault. We assume that the pattern of co-seismically generated shear bands indicate triggering potential of subsequent rupture if secondary faults exist along the optimal orientation of the shear bands, with the efficiency of triggering more or less proportional to the length of the shear bands.

Figure 10 shows a fault configuration with periodic spatial variations of \( f_s \) between a reference level and a slightly perturbed level, along with the evolution of rupture speed for this case. The generated distribution of off-fault shear bands is given in Figure 11. Within each piece-wise domain with constant \( f_s \), the evolution of rupture speed is relatively smooth and there are no prominent antithetic shear bands, similar to the previous results. However, at locations with abrupt jumps of \( f_s \), the rupture speed changes rapidly (Figure 10) with opposite sign to that of the strength change (\( \Delta f_s \)). This results in locally pronounced antithetic shear bands near places with rapid rupture deceleration and a seeming vacancy of antithetic band near places with rapid rupture acceleration (Figure 11). Since the rupture can still propagate through the assumed weak fault heterogeneities, the size of the activated antithetic shear bands is small.

Figure 12 shows the distribution of shear bands with a strong fault barrier located at \( X = 60L_0 \) where \( f_s \) abruptly jumps from 0.55 to a very high level that arrests the rupture. The rapid deceleration and eventual rupture arrest produces considerable off-fault yielding around the strong barrier (Fig. 12a). In this case there are several distinct shear band sets, including a well-developed antithetic band that is decoupled from the forward-oriented synthetic bands (i.e. the synthetic and antithetic bands do not overlap around the strong barrier, so they can grow approximately independently of each other). A corresponding simulation with a finite relaxation timescale produces similar shear band patterns around the strong barrier (Fig. 12b), suggesting that the decoupled shear bands are general features.

Another general feature is the very high plastic strain around the strong barrier (see the estimated values in the caption of Fig. 12). The generated large-scale off-fault synthetic and
antithetic shear bands around the strong barrier, together with the arrested rupture, form a pattern similar to the proposed diagram for non-conservative barriers of King & Nábělek (1985, Fig. 2). For our case, the finite motion resulted by the arrested rupture along the main fault, if constrained as a plane strain problem, will be partly compensated by two conjugate sets of off-fault shear-type yielding around the barrier (see insets in Fig. 12a and explanation in the caption). A permanent volumetric strain in the form of topographic features related to abrupt rupture arrest may also be generated around strong barriers (e.g. Ben-Zion et al. 2012).

We note that some off-fault yielding also occurs between the two dominant shear band sets around the strong barrier (on the extensional side). This is likely triggered by the radiated waves associated with the strong stopping phase and is presumably related to a potential tensile stress at that location. The relatively large size of the antithetic shear band in Figure 12 suggests that if a secondary fault path exists along the orientation $\theta \in [100^\circ,120^\circ]$, a potential antithetic shear rupture would have a high probability to be triggered, similar to the fault bend problem (Figure 9b) but with the presence of background stress.

4 Discussion

The main goal of the paper is to develop improved connections between characteristics of off-fault yielding at different scales (microfractures, intense inner yielding zone, outer yielding zone) and dynamic ruptures on earthquake faults. Previous works on the topic focused on properties of stationary stress fields with respect to given rupture tip locations (e.g. Poliakov et al. 2002), or considered a changing rupture tip configuration but ignored the possible generation of antithetic type fractures in non-smooth rupture processes (e.g. Ando & Yamashita 2007). Here we show that explicit considerations of the cumulative effects of transient features of the dynamic stress field (e.g. Figs. 4-6, 8-9), stemming from interactions between successive positions of the crack tip and the background stress field, provide refined understanding of several yielding features. These include the competition between large-scale synthetic and antithetic shear bands off the main fault, the existence of well-developed (decoupled) conjugate sets with both synthetic and antithetic bands in some circumstances, and the recognition of an internal intense yielding zone with overprinting of deformation fractures (produced by different yielding lobes) and an outer yielding zone generated by one dominant yielding lobe.

Our considerations of stress over extended regions around the rupture tip at given times and
extended time intervals at given locations, plus the degree of smoothness of the evolving stress field, help to clarify the type of large-scale shear bands generated during the initiation, propagation and arrest of ruptures. As shown in sections 3.2-3.3, these ideas explain consistently the relative generation of synthetic and antithetic shear bands for faults with homogeneous frictional properties under different background stress fields ($\Psi = 10^\circ$ and $\Psi = 45^\circ$), as well as examples with heterogeneous fault friction. We also explain results on rupture along a conjugate backward-oriented fault in the laboratory experiments of Rousseau & Rosakis (2003, 2009), and illustrate the generation of pronounced antithetic fractures by abrupt rupture arrest.

The results imply that both the stress amplitude (average or maximum level) and duration of a “favorable” stress field (e.g. positive Coulomb stress change) are important for rupture branching or damage generation, in particular at high angle or backward orientation to the main fault. The duration factor helps to explain (better than a mere consideration of “stress shadow effects”) frustrated failure attempts during rupture propagation with rapidly changing stress field, and the more encouraged attempt during strong rupture deceleration or arrest with longer operation of the stress field. Positive contributions from both the stress amplitude and duration are consistent with favored failure scenarios in laboratory experiments (Rousseau & Rosakis 2003), numerical simulations (Kase & Kuge 2001; this study) and natural earthquake examples (e.g. Hudnut et al. 1989).

The generation of microfractures by a single rupture at a scale much smaller than the yielding zone size is likely to be accomplished rapidly, and hence depends essentially only on the local stress field. Therefore, unless other mechanisms exist, there is no preference for either synthetic or antithetic shear microfractures. On the other hand, repeated earthquakes typically occur along the same fault segment, so distributed microfractures generated by earlier events could be reactivated by later events that can also increase the microfracture density. If a zone with high density of microfractures of a certain type (synthetic or antithetic) is aligned favorably with the stress generated by a new event, the microfractures can coalesce and form large-scale fractures. Since the length scale separating dense microfractures is small, the coalescence of microfractures into large scales could occur much faster than developing large-scale fractures in intact rocks. In such cases, well-developed antithetic large-scale fractures may develop during rupture propagation along a strike-slip fault (i.e., the relative importance of changing rupture configuration is reduced for rapid coalescence of pre-existing microfractures).
Another important issue is how to distinguish between deformation features generated by quasi-static faulting vs. dynamic rupture. A comparison between classic Riedel shear structure and the simulated results with off-fault shear bands for typical strike-slip faults suggests several distinguishable features which may be used jointly. The classic Riedel shear system (Fig. 13) is formed in a quasi-static process associated with a transition from distributed deformation to localized deformation (e.g. Tchalenko 1970; Ben-Zion & Sammis 2003, and references therein). The initially developed characteristic fracture elements (R, R’, etc) are generally quasi-symmetrically distributed with respect to the later formed Principal Displacement Zone (PDZ). Each set of fracture elements are usually regularly spaced along the general strike (referred to as en échelon), but may show more complex patterns towards increasing displacement direction in a single system (Ahlgren et al. 2001). The average angle with respect to the PDZ, under small background shear strain, is $\sim 15^\circ$ for the synthetic R-shear and $\sim 75^\circ$ for the antithetic R’-shear (Davis et al. 1999). Cases with very high-angle or backward-oriented R’-shear, produced by large background shear strain (Wilcox et al. 1973), significant volumetric compaction (Ahlgren 2001), or rigid block rotation (Nicholson et al. 1986) are not treated as representative quasi-static results in this study, but can be otherwise excluded by the recognition of sigmoidal form (Ramsay & Huber 1983), systematically re-oriented fracture elements of various types (Lecomte et al. 2012, Fig. 1), or severe fragmentations (Brosch & Kurz 2008).

On the other hand, the dynamic counterpart of Riedel shear usually requires a pre-existing localized fault to accommodate the rupture, and the off-fault fractures are produced by the stress concentration around the propagating rupture tip (e.g. Ando & Yamashita 2007). In typical strike-slip faults, the rupture-induced off-fault shear fractures or bands are mainly distributed on one (extensional) side of the fault, with likely increasing fracture length and/or complexity (e.g. hierarchical patterns and variations in fracture separation and length) towards the rupture propagation direction. In cases of expanding ruptures this is the direction of decreasing slip, leading to opposite expectation on the correlation between slip and complexity compared to quasi-static deformation (Ahlgren et al. 2001). Also, the average angle between the dynamically generated synthetic or antithetic shear bands and the fault plane is typically steeper than the corresponding quasi-static value, due to the re-orientation of the maximum compressive stress in the dynamic process (Xu et al. 2012b). In particular, if the antithetic shear bands emerge, they are usually oriented backward (with an obtuse angle) or almost orthogonally to the background
shear direction.

Despite the above differences, our results for smoothly propagating rupture (e.g. Fig. 7b) agree with the quasi-static model that there may be a statistical preference for the synthetic Riedel shear over the antithetic shear (e.g. Schmocker et al. 2003; Katz et al. 2006; Misra et al. 2009). In cases with strong fault heterogeneities, there could be strong fluctuations of rupture speed or even rupture arrest. When rupture experiences a sudden deceleration, either due to encountering a fault strength barrier (Figs. 11-12 of this study) or due to un-favored changing direction of fault path (Figs. 7 and 13 of Duan & Day 2008), prominent signals of antithetic shear fractures (or bands) associated with high permanent strain are observed, with the intensity of the signal generally proportional to the abruptness of rupture deceleration. This suggests that backward or almost-orthogonally inclined antithetic shear fractures, primarily on one (the extensional) side of a large strike-slip fault, may be used as a signal that reflects abrupt deceleration of rupture process. On the other hand, instances of abrupt rupture acceleration produce relative lack of off-fault yielding in the vicinity of the corresponding locations. Examples include the locations with reduced frictional strength (marked blue locations in Fig. 11), fault releasing bends (Fig. 3 of Dunham et al. 2011b), fault kink where the continued path is favorably oriented (Fig. 13 of Duan & Day 2008), and locations where a subshear rupture rapidly jumps to supershear regime associated with a low $S$ value (Fig. 13 of Templeton & Rice 2008).

Although the listed examples include both rupture acceleration within the subshear regime (this study; Dunham et al. 2011b) and more abrupt rupture transition from subshear to supershear (Duan & Day 2008; Templeton & Rice 2008), the similarity in producing relative lack of damage in these diverse examples likely stems from the fact that a fault portion over which the rupture accelerates has less chance to be subjected to high (transient) stress concentration.

A comparison between our model predictions and observations is best done in the context of damage zones well below the surface, since our 2D calculations with the adopted Mohr-Coulomb criterion do not account for free surface effects (Ma & Andrews 2010) or the competition between tensile and shear type failures under low confining pressures (Bourne & Willemse 2001). The predicted high-angle antithetic shear pattern by abrupt rupture deceleration is well supported by the indirect but strong evidence of aftershock distribution and/or conjugate earthquake triggering around known rupture termination ends. Numerous examples show clearly two groups of aftershock clusters around mainshocks termination ends: one is located in front of
earthquake terminus while the other forms a high angle (roughly along the conjugate direction) to the main fault plane mainly on the extensional side (King et al. 1994, and references therein). The overall lineaments of two distinct aftershock clusters around the same earthquake terminus, and sometimes the emergence of one dominant aftershock cluster, or triggered big event(s) along the orthogonal or even backward direction near the termination end(s) of a prior event (e.g. Aktar et al. 2007; Chi & Hauksson 2006; Das 1992; Hudnut et al. 1989) are in good agreement with our expectation without or with (preferred) off-fault plane of weakness (see section 3.3). More examples of conjugate earthquake triggering consistent (or at least partially consistent) with our expectations can be found in Das & Henry (2003), Jones & Hough (1995), and the recent off-Sumatra earthquake sequence (Meng et al. 2012; Yue et al. 2012). The above discussion on triggered antithetic shear near the tip of a master fault or earthquake terminus enriches the list of mechanisms for producing near orthogonal conjugate faulting, which also includes post-rotation of two conjugate faults or weak strength-dependence on pressure (Thatcher & Hill 1991, and references therein), block rotation of antithetic shear between two master faults (Kilb & Rubin 2002), and re-activation of a rifting system in a contemporary compressive stress field (Hamburger et al. 2011, and references therein).

Several important aspects of natural brittle deformation have not been considered in this work. These include effects associated with rough faults (e.g. Dunham et al. 2011b), free-surface effects (e.g. Ma and Andrews, 2010), volumetric deformation (e.g. Rudnicki & Rice 1975; Viesca et al. 2008), post-failure frictional weakening (Ando & Yamashita 2007), and changes of elastic moduli in the yielding zones (e.g. Lyakhovsky et al. 1997, 2011). These effects should be considered in future studies.

Appendix A: LEFM stress field near a propagating crack tip

The expressions for the different components of $\Sigma_{ij}^H$ of Eq. (6) in a Cartesian coordinate system, subjected to $\Sigma_{xy}^H(0,v_r)=1$, are given by (e.g. Freund 1990)
\[
\Sigma_{xx}^\Pi = -\frac{2\alpha_s}{D} \left( (1 + 2\alpha_s^2 - \alpha_s^2) \sin \frac{1}{2} \theta_d \sin \frac{1}{2} \theta_s \frac{1}{\sqrt{\gamma_d}} - (1 + \alpha_s^2) \sin \frac{1}{2} \theta_d \frac{1}{\sqrt{\gamma_s}} \right)
\]
\[
\Sigma_{xy}^\Pi = \frac{1}{D} \left( 4\alpha_s \alpha_d \cos \frac{1}{2} \theta_d \cos \frac{1}{2} \theta_s - (1 + \alpha_s^2) \cos \frac{1}{2} \theta_d \frac{1}{\sqrt{\gamma_s}} \right), \quad \text{(A1)}
\]
\[
\Sigma_{yy}^\Pi = \frac{2\alpha_s (1 + \alpha_s^2)}{D} \left( \sin \frac{1}{2} \theta_d - \sin \frac{1}{2} \theta_s \frac{1}{\sqrt{\gamma_s}} \right)
\]

where \( \alpha_d = \sqrt{1 - v_r^2 / c_d^2} \) with \( c_d \) being the dilatational \((P-)\) wave speed, \( \alpha_s = \sqrt{1 - v_r^2 / c_s^2} \) with \( c_s \) being the shear \((S-)\) wave speed, \( \gamma_d = \sqrt{1 - (v_r \sin \theta / c_d)^2} \), \( \gamma_s = \sqrt{1 - (v_r \sin \theta / c_s)^2} \), \( \theta_d = \tan^{-1}(\alpha_d \tan \theta) \), \( \theta_s = \tan^{-1}(\alpha_s \tan \theta) \), and \( D = 4\alpha_d \alpha_s - (1 + \alpha_s^2)^2 \).

The dynamic stress intensity factor \( K_{II}^d \) can be further expressed as a product of the static stress intensity \( K_{II}^s \) and a universal function of rupture speed \( k_{II}(v_r) \):
\[
K_{II}^d = k_{II}(v_r) K_{II}^s, \quad \text{(A2)}
\]
where \( K_{II}^s \propto \Delta \tau \sqrt{\pi L} \) with \( \Delta \tau \) being the stress drop and \( L \) being the half length of the crack, and \( k_{II}(v_r) \approx (1 - v_r / c_R) / \sqrt{1 - v_r / c_s} \) that monotonically decreases from 1 at \( v_r \approx 0^\circ \) towards 0 as \( v_r \) approaches the Rayleigh wave speed \( c_R \) (Freund 1990).

**Appendix B: Critical conditions for the onset of shear localization**

Rice (1976) showed that for elastically isotropic materials and deformation processes where rotational effects on stress rate can be neglected, the critical value of plastic hardening modulus \( h_c \) can be described as
\[
\frac{h_c}{2\mu} = 2n \cdot P \cdot Q - (n \cdot P \cdot n)(n \cdot Q \cdot n) - P : Q - \frac{v}{1 - v}(n \cdot P \cdot n - trP)(n \cdot Q \cdot n - trQ), \quad \text{(B1)}
\]
where \( n \) is a unit vector representing the normal direction of shear band, \( P \propto \nabla_\sigma M \) and \( Q \propto \nabla_\sigma F \) are the normalized gradients with respect to the stress tensor \( \sigma_{ij} \) of the plastic flow potential \( M \) and the yield function \( F \), and \( v \) is the Poisson's ratio. To make Eq. (B1) applicable, \( h_c \) must be larger than a minimum admissible value \( h_{\min} \), which corresponds to a loading requirement that the scalar plastic strain rate (sometime also called the rate of equivalent...
plastic strain) should never be negative (e.g. Rice & Rudnicki 1980). From Bardet (1990), this condition can be more explicitly expressed as:

\[
\frac{h_{\min}}{2\mu} = -\frac{\nu}{1 - 2\nu} \text{tr} \mathbf{P} \mathbf{Q} - \mathbf{P} : \mathbf{Q}.
\]  

(B2)

For our employed Mohr-Coulomb type yielding criterion and non-associated flow rule without volumetric change, we can follow the procedure of Bardet (1990) to re-express the entries on the right side of (B1) and (B2) in the principal stress coordinate system as:

\[
P_{ij} = \begin{bmatrix} -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix},
\]

\[
Q_{ij} = \begin{bmatrix} -1 + \sin \phi & 0 \\ \sqrt{2(1+\sin^2 \phi)} & 0 \\ \frac{1 + \sin \phi}{\sqrt{2(1+\sin^2 \phi)}} & \frac{2}{\sqrt{2(1+\sin^2 \phi)}} \end{bmatrix},
\]

(B3)

\[n_i = [\sin \theta \quad \cos \theta]^T\]

where \( \theta \) is the angle between shear band and the maximum compressive stress. Compared to the original expressions in Bardet (1990), each component of \( P_{ij} \) or \( Q_{ij} \) in Eq. (B3) is associated with an opposite sign due to the treatment of compressive normal stress with negative values in our study. However, this difference in sign-convention does not affect the evaluation of Eq. (B1). Using the expressions in Eq. (B3) into (B1), we have:

\[
h_c = \frac{\sin^2 \phi - (2 \cos 2\theta - \sin \phi)^2}{8(1-\nu)\sqrt{1+\sin^2 \phi}}.
\]  

(B4)

The maximum value of \( h_c \) that satisfies Eq. (B4) is given by

\[
h_{\max} = \frac{\sin^2 \phi}{8(1-\nu)\sqrt{1+\sin^2 \phi}},
\]  

(B5)

which is usually treated as the criterion for predicting the occurrence of shear bands for the first time. And the corresponding angle \( \theta \) satisfies:

\[\cos 2\theta = \frac{1}{2} \sin \phi.\]  

(B6)

Appendix C: Mesh dependence of shear localization
It is well documented that certain features of numerically modeled shear bands (the emergence, width, two possible conjugate sets, etc) can have a strong dependence on the adopted numerical meshing in both quasi-static and dynamic processes (e.g. Needleman 1989; McKinnon & Garrido de la Barra 1998; Templeton & Rice 2008). To get a confidence on the physical implications of the modeled shear bands, one should try different meshing strategies to ensure that the key discussed features are robust. If a particular mesh structure is designed to mimic material properties with some pre-existing preference (anisotropy, internal texture, etc), which is also believed to be the dominant mechanism leading to localization, the potentially-biased results may still be considered acceptable. Here we mainly discuss effects involving the size and orientation of numerical meshes on the generated shear bands.

Figure C1 shows generated shear bands with square elements of two different sizes. Consistent with the study by Templeton & Rice (2008), the average shear band width decreases with refining the mesh size (e.g. see the zoom-in view in box II). This mesh-size dependence has been explained by the expectation that, in the absence of physical parameters specifying shear band width, the mesh size provides a minimum length scale that is numerically allowable to optimize shear localization (e.g. Prevost & Loret 1990). In the spectral method used in our study the internal nodes within each element are not uniformly distributed, but have narrower and wider spacing towards the end and the center of an element, respectively (see the structure of the inset square). Therefore, the minimum possible width of the simulated shear bands in our study is not only controlled by the element size, but also depends on the node partition within the element. Moreover, the introduced non-uniformity by the internal nodes may contribute to shear band triggering near the fault.

Despite the above size-dependent features, some properties of the generated shear bands are preserved with varying the mesh size. (1) Within a limited distance range, there is a transition from antithetic shear bands to synthetic ones, and the angle between the two sets in the transition region has a good agreement with the theoretical prediction (see box I). (2) After the initial transition, the synthetic shear bands in the uniform fault problem always dominate the off-fault yielding zone and can develop their own branches at larger propagation distances (see box II). (3) The intensity distribution of plastic strain within each well-developed shear band is quite uniform (ignoring the edge and tip effects), although the absolute value depends on the mesh size. (4) The average area density not occupied by shear bands (but still within the overall yielding
zone) seems to increase with off-fault distance. It is interesting to note that the combination of (3) and (4), when compared to Fig. 2, may imply some equivalence of patterns between distributed and localized deformation (with a similar overall yielding zone shape) during dynamic ruptures. For distributed deformation, the geometric measure is macroscopically continuous (uniform) but the potency density decays with off-fault distance, while for localized deformation the potency density within each discrete band is uniform but the geometric measure of band density decays (on average) with off-fault distance. It may be interesting to explore in a future work whether a generalized deformation density, accounting for both yielding intensity in a failed element and geometric measure of failed elements can be used to describe uniformly distributed and localized deformation patterns.

Templeton & Rice (2008) suggested based on numerical simulations with small propagation distance (~10\(R_0\)) a characteristic shear band spacing that may physically scale with the current size of process zone (e.g. 0.2–0.4\(R_0\)). According to our results and Duan & Day (2008), these suggested features may hold in the very beginning of ruptures but appear to gradually lose their validity at larger propagation distance (e.g. the spacing between adjacent well-developed shear bands fluctuates and sometimes is even larger than the static size of process zone). This implies that a statistical description of shear band spacing is needed and the possible scaling coefficient with process zone may evolve with propagation distance.

The numerically simulated shear bands may also have a strong dependence on the mesh orientation. The effective width of shear bands can vary with the relative angle between shear bands and mesh edges. The minimum possible width can only be obtained when the corresponding mesh edges are aligned parallel to the band orientation (this is referred to as the mesh-alignment effect). For our study of two possible conjugate shear band sets, it is important to check the effect of mesh orientation on the relation between the generated shear bands. If quadrilateral meshes are used, the two pairs of mesh edges may be aligned parallel to the two principal stress axes such that any potential conjugate shear bands will be equally biased by the mesh-alignment effect (McKinnon & Garrido de la Barra 1998). For our dynamic problem, instead of performing a rigid rotation of the entire array of previously used square meshes, we keep a pair of mesh edges parallel to the fault but vary the angle between the other pair and the fault plane (i.e. changing the square mesh to a more general quadrilateral mesh). Other approaches for reducing the mesh-alignment and/or mesh-size effect can be found in related
works (e.g. McKinnon & Garrido de la Barra 1998; Li et al. 2001; Bažant & Jirásek 2002).

Figure C2 shows the generated shear bands with a variable angle $\delta$ between the inclined mesh edges and the fault plane. Clear difference can be found in the early stage of ruptures right after the emergence of shear bands; the dominant set is of the synthetic type in Fig. C2a while it is of the antithetic type in Figs. C2b and C2c (see box I). This difference may be explained by the contrast in resolvable shear band widths (see the conventionally defined $W_s$ and $W_a$ in the dashed box), of which the set with a narrower width is numerically more encouraged. However, with increasing rupture propagation distance the synthetic shear bands continue to be dominant in Fig. C2a, while there is a clear transition of the dominant set from antithetic to synthetic in Figs. C2b and C2c. The critical rupture propagation distance leading to this transition seems to increase with the value of $W_s/W_a$, and may never be reached in the explored distance range if $W_s/W_a$ is too high (not shown here but confirmed by a test with inclined mesh edges almost parallel to the expected antithetic shear bands). Finally, at relatively large propagation distances, the synthetic shear bands always dominates in all three test cases, while the possible existence of minor antithetic bands near the fault depends strongly on the mesh alignment (see box II). These results imply that some physical mechanism (like the one in section 3.2.2), rather than a numerical artifact, is involved in the generation of significant synthetic shear bands (in the examined cases without pre-existing favorable failure orientations off the main fault).

Our examined test cases with approximately equal values for $W_s$ and $W_a$, along with the implied features insensitive to mesh size, support our conclusion on large-scale shear-type yielding generated by earthquake ruptures in large strike-slip faults (see section 3.2.2). End-member cases with $W_s/W_a$ strongly deviating from one are severely affected by the mesh-alignment effect, and may not produce representative results that are useful in the context of this study.

Acknowledgements

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Cooperative Agreement 07HQAC0026).

References


**Figure captions**

**Figure 1.** Model configuration for dynamic in-plane ruptures along a frictional interface (thick black line at the center) with off-fault plastic yielding. The red portion on the fault indicates the imposed nucleation zone with size $L_{nuc}$. The medium is loaded by a uniform right-lateral background stress with angle $\Psi$ between the background maximum compressive stress and the fault plane. Symbols “C” and “T” represent, respectively, the compressional and extensional quadrants in relation to the first motion of P-waves from the nucleation zone. Due to symmetry, results in subsequent plots will be shown only for the right half.

**Figure 2.** Distribution of off-fault plastic strain generated by crack-like ruptures with (a) $\Psi = 10^\circ$ and (b) $\Psi = 45^\circ$. The scale in the $Y$ direction is exaggerated by a factor of 3.75 $(dx : dy = 1 : 3.75)$.

**Figure 3.** Distribution of equivalent plastic strain increment near the rupture tip for the cases of Figure 2 with (a) $\Psi = 10^\circ$ and (b) $\Psi = 45^\circ$. Black bars represent local orientations of expected shear microfractures, with “thick” and “thin” bars used for right-lateral and left-lateral, respectively.

**Figure 4.** Calculated angular variation of the slip-induced incremental stress field $\Delta\sigma_{\alpha\beta}$ in a polar coordinate (based on LEFM) for the cases of Fig. 2 with (a) $\Psi = 10^\circ$ and (b)
Ψ = 45°. The Coulomb failure stress change promoting left-lateral and right-lateral shear are denoted $\Delta CFS^-$ (blue) and $\Delta CFS^+$ (red), respectively. Locations of local peak values of $\Delta CFS^\pm$ are marked with numbers and initial letters; “L” for left-lateral and “R” for right-lateral.

**Figure 5.** Spatial representation in a polar coordinate system of the interaction between the background stress $\sigma_{ij}^0$ and the slip-induced incremental stress $\Delta \sigma_{ij}$ for the cases of Fig. 2 with (a) $\Psi = 10^\circ$ and (b) $\Psi = 45^\circ$. The results are based on the calculations shown in Fig. 4.

**Figure 6.** Schematic diagram illustrating the generation of off-fault large-scale shear fractures by propagating ruptures along a frictional fault. Panels (a) and (b) correspond, respectively, to the compressional side of Fig. 2a and the extensional side of Fig. 2b. The grey lobes show the current failure zone around the rupture tip. The orientation of the transient maximum compressive $\sigma'_{\text{max}}$ near the rupture tip is indicated by a pair of arrows.

**Figure 7.** Distribution of off-fault shear bands with rate-independent plasticity generated by ruptures with (a) $\Psi = 10^\circ$ and (b) $\Psi = 45^\circ$. The listed values of frictional, stress and rupture parameters in this and other figures provide a balance between achieving numerical stability and producing prominent off-fault shear bands. The main features discussed in the text have been tested by a parameter space study and confirmed to be robust. Regions I-III highlight local features of the simulated shear bands at various locations. The scale in the Y direction is exaggerated by a factor 2 ($dx : dy = 1 : 2$) for the large-scale plot but is kept the same as that in the X direction for the inset plots.

**Figure 8.** Schematic diagram showing possible competition between synthetic (red) and antithetic (blue) shear bands. The solid and dashed lobes are constructed similar to Fig. 6, but with finite small shift and slight size increase corresponding to changing rupture front configuration. Long solid curves represent already-developed paths of two conjugate shear bands ($l_s$ for synthetic and $l_a$ for antithetic), while short dashed curves indicate likely extension paths for the two bands.

**Figure 9.** Schematic diagram showing slip-induced $\Delta CFS$ lobes around a junction with (a) fault branch and (b) fault bend. The likely rupture evolutions are compared with laboratory experiments of Rousseau & Rosakis (2003, 2009). See text for more explanation.
Figure 10. Assumed space variations of the static friction coefficient $f_s$ (lower line) and corresponding evolution of normalized rupture speed averaged over $4L_0$ (upper line) along the fault.

Figure 11. (a) Off-fault shear bands generated for the heterogeneous case of Fig. 10. The inset plot near the bottom shows (with scale at the right vertical axis) the plastic strain distribution along the fault. (b-d) close-up views of (a) in different locations. The red and blue vertical bars indicate locations associated with small upward and downward jumps of $f_s$ along the fault (Fig. 10).

Figure 12. Distribution of off-fault plastic yielding with a strong barrier ($f_s >> 1$) located at $X = 60L_0$ (a) with rate-independent and (b) rate-dependent plasticity. The inset plot in (a) schematically shows how the high slip gradient near the barrier is partly compensated by the conjugate off-fault shear bands. Following King & Nábělek (1985), letters A, B, and C denote different blocks separated by the main fault and shear bands, while the vectors on the right show the relative shear motion from a starting side to ending side. The maximum normalized value of $\varepsilon_0^p$ in (a) is $\sim 8000$ and may correspond to a permanent strain of order 1 assuming $\sigma_c / \mu \sim 10^{-3}$. The maximum value in (b) is about one order of magnitude less than that in (a).

Figure 13. An idealized Riedel shear structure containing various fracture elements, modified from Sylvester (1988) and Davis et al. (1999, Fig. 2). The Y shear is parallel to the Principal Displacement Zone (PDZ), while other fracture elements are inclined with characteristic acute or obtuse angles to the PDZ in relation to $\sigma_{\max}$ and/or the internal friction angle.

Figure C1. Off-fault shear bands in simulations with square spectral elements with relatively large (a) and small (b) grid size. The inset sketches illustrate the employed non-uniform distribution of internal nodes. Boxes I and II provide zoom-in views of features at relatively small and large propagation distances, respectively. The maximum plastic strain is rescaled in box I but remains unchanged in box II.

Figure C2. Off-fault shear bands in simulations with quadrilateral spectral elements with variable angle between the non-horizontal mesh edges and fault plane. The critical rupture
length for transition from a possible antithetic to synthetic shear is very small in (a), intermediate in (b) and relatively large in (c). The mesh size is the same for all three cases. The ratio \( W_s / W_a \) in the dashed boxes roughly quantifies the mesh alignment effect (see text for more explanation). Boxes I and II provide zoom-in views of local features at different rupture propagation distances.

**Table 1.** Parameter values of material properties used in the simulations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamé parameters ( \lambda, \mu )</td>
<td>1, 1</td>
</tr>
<tr>
<td>( P )- and ( S )- wave speeds ( c_p, c_s )</td>
<td>1.732, 1</td>
</tr>
<tr>
<td>Mass density ( \rho )</td>
<td>1</td>
</tr>
<tr>
<td>Poisson’s ratio ( \nu )</td>
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</tr>
<tr>
<td>Characteristic slip distance (SWF) ( D_c )</td>
<td>1</td>
</tr>
<tr>
<td>Characteristic length scale (TWF) ( L_0 )</td>
<td>1</td>
</tr>
<tr>
<td>Internal friction angle ( \phi )</td>
<td>30.9638°</td>
</tr>
<tr>
<td>Rock cohesion ( c )</td>
<td>0</td>
</tr>
<tr>
<td>Time scale for stress relaxation ( T_v )</td>
<td>0.075( L_0 / c_s ) = 0.075 (rate-dependent)</td>
</tr>
<tr>
<td></td>
<td>0 (rate-independent)</td>
</tr>
<tr>
<td>Reference stress ( \sigma_c )</td>
<td>( \mu D_c / L_0 = 1 )</td>
</tr>
</tbody>
</table>

**Table 2** Conversions between physical \((x)\) and normalized dimensionless \((x')\) quantities.

<table>
<thead>
<tr>
<th>Length</th>
<th>Time</th>
<th>Stress</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l' = \frac{l}{L_0} )</td>
<td>( t' = \frac{t c_s}{L_0} )</td>
<td>( \sigma' = \frac{\sigma}{\sigma_c} )</td>
<td>( \varepsilon' = \frac{\varepsilon \mu}{\sigma_c} )</td>
</tr>
</tbody>
</table>
Figure 1.
Figure 2.

(a) 

(b)

\[ \frac{dx}{dy} = 1 : 3.75 \]

\[ \Psi = 10^\circ \]

\[ \Psi = 45^\circ \]

\[ f_s = 0.6, f_d = 0.1, S = 2.571 \]

\[ |\sigma_0| = \sigma_c, \Delta x \approx R_0/38 \]

\[ \frac{\sigma_0^0}{(\sigma_c/\mu)} \]

\[ 0 \leq 1 \]
Figure 3.
Figure 4.
Figure 5.
Figure 6.
Figure 7.

\[ S = 1.5, |\sigma_0| = \sigma_c, \Delta x \approx R_0/38 \]
Figure 8.
Figure 9.
Figure 10.

- Normalized rupture speed ($v_r / c_s$)
- Static friction coefficient ($f_s$)
- $\Delta f_s > 0$ and $\Delta f_s < 0$ at $X / L_0 = 0.55$ and 0.58.
Figure 11.
Figure 12.

\[ \Delta x \approx R_0 / 19 \]

\[ T_v = 0 \]

\[ T_v = 0.075 \]

\[ \Psi = 60^\circ, |\sigma_0| = 2\sigma_c \]

\[ |x| \leq 60 : f_s = 0.55, f_d = 0.05, S = 1.5 \]

\[ |x| > 60 : f_s \gg 1 \]
Figure 13.
Figure C1.

\[
\Psi = 60^\circ, \quad S = 1.5, \quad |\sigma_0| = 2\sigma_c, \\
f_x = 0.55, \quad f_d = 0.05
\]
Figure C2.