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Observational analysis of correlations between aftershock productivities and regional conditions in the context of a damage rheology model

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Accepted 2009 February 11. Received 2008 October 23; in original form 2008 February 10

SUMMARY
Aftershock sequences are commonly observed but their properties vary from region to region. Ben-Zion and Lyakhovsky developed a solution for aftershocks decay in a damage rheology model. The solution indicates that the productivity of aftershocks decreases with increasing value of a non-dimensional material parameter $R$, given by the ratio of timescale for brittle deformation to timescale for viscous relaxation. The parameter $R$ is inversely proportional to the degree of seismic coupling and is expected to increase primarily with increasing temperature and also with existence of sedimentary rocks at seismogenic depth. To test these predictions, we use aftershock sequences from several southern California regions. We first analyse properties of individual aftershock sequences generated by the 1992 Landers and 1987 Superstition Hills earthquakes. The results show that the ratio of aftershock productivities in these sequences spanning four orders of event magnitudes is similar to the ratio of the average heat flow in the two regions. To perform stronger statistical tests, we systematically analyse the average properties of stacked aftershock sequences in five regions. In each region, we consider events with magnitudes between 4.0 and 6.0 to be main shocks. For each main shock, we consider events to be aftershocks if they occur in the subsequent 50 d, within a circular region that scales with the magnitude of the main shock and in the magnitude range between that of the main shock and 2 units lower. This procedure produces 28–196 aftershock sequences in each of the five regions. We stack the aftershock sequences in each region and analyse the properties of the stacked data. The results indicate that the productivities of the stacked sequences are inversely correlated with the heat flow and existence of deep sedimentary covers, in agreement with the damage model predictions. Using the observed ratios of aftershock productivities, along with simple expressions based on the damage model, we estimate the relative values of the material parameter $R$ and seismic coupling coefficient in the different regions. The employed methodology for estimating the seismic coupling in different regions can be useful for seismic hazard studies.

Key words: Earthquake dynamics; Earthquake interaction, forecasting, and prediction; Seismicity and tectonics; Statistical seismology; Rheology and friction of fault zones.

1 INTRODUCTION
Aftershocks are a form of brittle relaxation of the crust to rapid stress loading generated by previous earthquakes that are referred to as main shocks. It may, thus, be expected that aftershock properties would depend on certain regional conditions such as rock type, ambient temperature and fluid content. Such correlations have been found in previous observational studies but mechanical models that make these connections explicit, and related observational analyses that employ the connections to derive additional aspects of earthquake behaviour, have been generally lacking (e.g. Ben-Zion 2008).

The decay rates of aftershock sequences are usually fitted by the Omori–Utsu law:

$$\frac{\Delta N}{\Delta t} = K(t + c)^{-p},$$

(1)

where $N$ is the cumulative number of events, $t$ is the time after the main shock, $K$ is the productivity of the sequence, the exponent $p$
has a value close to 1 and \( c \) is usually a small fraction of a day (e.g. Omori 1894; Utsu et al. 1995). However, aftershock decay rates can also be fitted with exponential and other functions (e.g. Mogi 1967; Otsuka 1985; Gross & Kisslinger 1994; Narteau et al. 2002). Observational studies indicate that cold continental regions have high aftershock productivity and long sequences associated with low effective power-law decay, whereas hot continental regions and oceanic lithosphere have low aftershock productivity and short sequences with fast decay or exponential fall-off (Mogi 1967; Davis & Frohlich 1991; Kisslinger & Jones 1991; Utsu et al. 1995; McGuire et al. 2005). However, as far as we know, a systematic analysis of the connections between aftershock properties and heat flow within a formalism associated with a physical model and specific quantitative expectations has not yet been done. In this paper, we perform such an analysis in the context of a damage rheology model (Ben-Zion & Lyakhovsky 2006) that makes explicit connections between the ambient temperature field and other conditions that may modify the effective viscosity of a region, the productivity of aftershock sequences and the degree of seismic coupling in a region.

Ben-Zion & Lyakhovsky (2006) derived an analytical solution for aftershock decay rates in a viscoelastic continuum damage model for evolving elastic properties of rocks sustaining irreversible brittle deformation. The damage model generalizes the strain energy function of a solid to account for first-order macroscopic effects of existing cracks (damage) and makes the elastic moduli functions of an evolving damage state variable \( \alpha \) representing the local crack density (e.g. Lyakhovsky et al. 1997; Hamiel et al. 2004). An undamaged material with \( \alpha = 0 \) is the ideal solid governed by 3-D linear elasticity, whereas a material with \( \alpha = 1 \) cannot support any load. Lyakhovsky & Ben-Zion (2008) and Ben-Zion (2008) provide detailed reviews of the damage model. The analytical solution of Ben-Zion & Lyakhovsky (2006) follows an exponential form that depends fundamentally on a non-dimensional material parameter \( R \), given by the ratio of timescale for brittle damage to timescale for viscous relaxation. The material parameter \( R \) also plays a fundamental role in the degree of seismic coupling \( \chi \), which quantifies the fraction of the stored elastic strain energy that is released in brittle deformation (i.e. earthquakes). Ben-Zion & Lyakhovsky (2006) showed that \( \chi \) is inversely proportional to \( R \) and is given by

\[
\chi = 1/(1 + R). \tag{2}
\]

Since the degree of seismic coupling has direct consequences for the seismic potential of a region, an ability to estimate the value of \( \chi \) from properties of aftershock sequences can have important practical applications.

Ben-Zion & Lyakhovsky (2006) showed that for simple assumptions that are expected to hold over relatively short time intervals (e.g. 100 d), the analytical solution for aftershock decay rates follows the power-law relation

\[
\frac{dN}{dt} = \frac{1}{2\phi R (1 - \alpha_s)} \left[ \frac{1}{t + 1/[2\phi R (1 - \alpha_s) N_0]} \right] \tag{3a}
\]

where \( N_0 \) is the initial aftershocks decay rate, \( \phi \) is a small scaling constant (e.g. \( 10^{-5} \)) between the number of events and average change of rock damage and \( \alpha_s \) is the initial rock damage. Eq. (3a) provides the following mapping between parameters of the Omori–Utsu relation (1) and parameters of the damage model:

\[
K = 1/[2\phi R (1 - \alpha_s)],
\]
\[
c = 1/[2\phi R (1 - \alpha_s) N_0] = K/N_0,
\]
\[
p = 1. \tag{3b}
\]

In particular, the ratios of the productivity values \( K \) of aftershock sequences in different regions, derived by fitting the aftershock decay rates to the Omori–Utsu law with an assumed \( p = 1 \), are expected to be inversely proportional to the \( R \) values that characterize the different regions. Since \( R \) is inversely proportional to the effective viscosity in a region, the productivity of aftershock sequences should decrease for lithological and ambient conditions that reduce the viscosity. These include increasing temperature, increasing fluid content and existence of sediments (or other rock units that have relatively low viscosity) at seismogenic depths.

In the next section, we test the forgoing expectations using observed aftershock sequences in several subregions of southern California. We adopt a simple approach associated with (a) selecting \textit{a priori} several subregions, using available information on the heat flow and crystalline versus sedimentary rocks at seismogenic depth, and (b) performing a comparative analysis of aftershock productivities in those regions, using identical values for all other parameters that characterize the entire southern California region. Other possible controlling variables, such as fluid content, are not examined because of the lack of systematic information that is needed for our comparative study. The obtained productivities of aftershock sequences are found to be inversely correlated with the heat flow and existence of deep sedimentary rocks, in agreement with the damage model predictions. Using eqs (2) and (3) and the derived aftershock productivities, we estimate the relative degrees of seismic coupling in the different regions. Since the regional conditions assumed by the damage model to affect the seismic coupling and aftershock productivities evolve only over geological time, the employed approach (and related more sophisticated analysis procedures) may be useful for estimates of seismic hazard.

2 ANALYSIS

We analyse aftershock properties in different regions of southern California using two separate approaches. In the first approach, we compare individual aftershock sequences of large main shocks that occurred in regions with known strong contrast of heat flow. To perform a more systematic analysis based on larger amount of data, we compare in the second approach properties of stacked aftershock sequences of moderate main shocks in five regions that have clear differences in heat flow and sedimentary thickness. We distinguish primarily between areas with thin sediments (e.g. the Mojave Desert), where essentially all the seismicity occurs in dense cohesive crystalline rock, and areas with deep sedimentary covers (e.g. Ventura Basin), where much of the seismicity occurs within more compliant sediments. Intermediate levels of sedimentary covers are referred to as regular or moderate and are assumed to have small effects on the results. More detailed information on the rocks types can be obtained from the SCEL velocity model (http://www.data.scec.org/3Dvelocity/), but this is not needed for the present analysis level. Fig. 1 shows a map view of the employed seismicity and heat flow data sets. The seismic data are taken from the relocated 1984–2002 southern California earthquake catalogue of Shearer et al. (2005). The heat flow data are taken from the USGS online heat flow database (http://earthquake.usgs.gov/heatflow/index.html).

2.1 Individual aftershock sequences of large main shocks

To compare properties of aftershocks of large individual main shocks, we choose the aftershock sequences of the 1992 \( M 7.3 \) Landers earthquake and 1987 \( M 6.6 \) Superstition Hills event. As
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Figure 1. A map view showing epicentre locations of \( M \geq 2.0 \) earthquakes (black dots) and heat flow data (coloured squares with values indicated in the legend) used in this work. The earthquakes are from the relocated catalogue of Shearer et al. (2005), and the heat flow values are from the USGS online heat flow database.

shown in Fig. 1, the area of the Landers sequence (region B) has a relatively low heat flow, whereas the area of the Superstition Hills sequence (region A) has a relatively high heat flow. We assume that the aftershock zone of each event consists of a \( 1^\circ \times 1^\circ \) box centred on the main shock epicentre (Figs 2a and d). We also consider a situation where the Landers aftershocks are limited to the polygon region in Fig. 2(a). We consider events inside the employed areas to be aftershocks if they occurred in the subsequent 50 d after the occurrence of the main shock. This somewhat arbitrary choice of time interval includes the most significant portions of the aftershock sequences. We performed similar analyses with time windows ranging from 30 to 60 d and our main results remain essentially unchanged.

To isolate the effect of rheological properties on the productivity of aftershock sequences, we have to account for the different magnitudes of the main shocks. Helmsstetter et al. (2005) showed that the average productivity \( K \) of aftershocks in southern California depends on the main shock magnitude \( M \) as

\[
K \propto 10^{\lambda(M-M_{\text{min}})},
\]

where \( \lambda = 1.05 \pm 0.05 \) and \( M_{\text{min}} \) is the magnitude cut-off. Felzer et al. (2004) obtained an average \( \lambda \) value for California seismicity of about 1. To account for the size difference of main shocks on the aftershock productivities, we assume that \( \lambda = 1 \) and use in each sequence aftershocks that are within the same magnitude range below that of the main shock. Specifically, we analyse properties of aftershock sequences that are within 4 magnitude units below those of the main shocks. This choice allows us to include the largest number of events that are above the completeness level of the catalogue for the two sequences. The latter is tested simply by fitting the frequency-magnitude statistics of the data with the Gutenberg–Richter relation (e.g. Wiemer & Wyss 2000).

The temporal evolution of the employed aftershock sequences of the Landers and Superstition Hills main shocks are shown in Figs 2(b) and (e), respectively. To estimate aftershock productivities that are associated with the Omori–Utsu law, we use an integral form of the Omori–Utsu law, with \( p = 1 \), given by

\[
N(t) - N(t_0) = K[\ln(t + c) - \ln(t_0 + c)],
\]

where \( t_0 \) is the initial time. As noted earlier, the constant \( c \) is typically a small fraction of a day. The determination of \( c \) is strongly affected by the early part of the aftershock sequence, which is associated with incomplete recording (e.g. Utsu et al. 1995; Peng et al. 2006;
Figure 2. (a) Epicentres of analysed aftershocks (circles) of the 1992 M 7.3 Landers California earthquake (red star). Events inside and outside the dashed polygon around the main rupture zone are marked with blue and green colours, respectively. (b) Magnitude versus time of the Landers aftershocks in (a) occurring in the first 50 d after the main shock. (c) Numbers of Landers aftershocks per day (circles) and fitted curves based on an integral form of the Omori–Utsu relation (eq. 5b) with \( t_0 = 5 \) d. The black and blue symbols correspond to events in the whole region and the polygon area, respectively. For presentation purpose, we use \( \log_{10}(t) \) instead of \( \ln(t) \) as the scale of the horizontal axis. The fitted \( K \) values of the aftershock productivities are indicated below the lines. (d) Same as (a) for the aftershock sequence of the 1987 M 6.6 Superstition Hills earthquake. (e) Magnitude versus time of the analysed aftershocks of the Superstition Hills earthquake. (f) Same as (c) for aftershocks of the Superstition Hills earthquake.

Enescu et al. 2007). To reduce artefacts associated with \( c \), we choose a relatively large \( t_0 \) value, so that \( c \) can be effectively omitted. We thus use in the analysis

\[
N(t) \approx K[\ln(t) - \ln(t_0)] + N(t_0), \tag{5b}
\]

where \( t \geq t_0 \gg c \). We discretize the time into successive 1 d intervals, sum the number of aftershocks that occur inside each interval and use a non-linear least-squares method (Bates & Watts 1988) to estimate the productivity parameter \( K \).

Figs 2(c) and (f) show the fitted results (solid lines) and obtained \( K \) values for the Landers and Superstition Hills aftershock sequences, using \( t_0 = 5 \) d. The estimated productivity values remain essentially the same with \( t_0 \) values of 4, 3, 2 and 1 d. The deviations of the data from the fitted straight lines may be related to the fact that we fix the exponent \( p \) to be 1, ignore possible effects of the \( c \) parameter by using large \( t_0 \) value and other simplifications of our analysis. However, the overall trends of the cumulative number of aftershocks follow the fitted curves, and the employed procedure is not expected to produce a bias that would affect our comparative study of aftershock productivities in the different regions. Since Fig. 2(a) includes aftershocks of the 1992 M 6.4 Big Bear event, which may or may not be considered as belonging to the Landers aftershock sequence, we analyse with identical procedure only events that occurred in the polygon region of Fig. 2(a) around the main Landers rupture zone. The resulting numbers of aftershocks (plus symbols) and associated productivity are indicated in Fig. 2(c). The productivity value of the aftershocks in the polygon region is smaller than that of the entire box in Fig. 2(a), but it is still larger than the productivity value of the Superstition Hills aftershock sequence.

Fig. 3(a) displays the observed aftershock sequences of the Landers and Superstition Hills main shocks, along with lines corresponding to the discrete form of the Omori–Utsu law (eq. 1) with the parameters associated with the fits of Figs 2(c) and (f) \( (c = 0, p = 1, t_0 = 5) \). The aftershock productivity in the Landers area is about 2.4 (or 1.5, using only the polygon region) times higher than it is in the Superstition Hills area. To compare the obtained aftershock productivities with the background heat flow, we use the USGS heat flow data (Fig. 1) inside the aftershock zones. We take the mean of the data in each region as a representative value of the heat flow and the 95 per cent confidence interval of the data to be the error range. The relationship between heat flow values and aftershock productivities for the different aftershock zones is displayed in Fig. 3(b). The results show inverse correlation between the ratios of heat flow and aftershock productivity values in the Landers and Superstition Hills aftershock zones. It is clear, however, that analysis associated with a small number of sequences (here only two) may be affected strongly by statistical fluctuations rather than giving an appropriate representation of the relations between aftershocks and environmental properties. We therefore expand our analysis in the next section to include additional regions, with tens of aftershock sequences in each region and consistent definition of the aftershocks area of each main shock.
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2.2 Stacked aftershock sequences of moderate main shocks

To reach stable estimations of aftershock productivities that are based on a larger amount of data, we analyse properties of stacked aftershock sequences in five representative southern California regions (Fig. 1). The regions are selected, as before, based on well-known environmental conditions that are assumed by the damage model of Ben-Zion & Lyakhovsky (2006) to affect the aftershock properties. The stacking of data smoothes out fluctuations associated with individual aftershock sequences and highlights the common features of all sequences inside each region.

The analysis is done as follows. We select aftershock sequences of main shock magnitudes between 4.0 and 6.0 in each of the five regions of Fig. 1. The areas of the regions are listed in Table 1 and decrease in the following order: A; B; D; C and E. The Imperial Valley and Coso regions (A and E) have relatively high heat flow levels, the largest and the smallest areas and regular-to-moderate sedimentary covers. The Landers and Hector-Mine area (B) has low heat flow, relatively large area and very thin sedimentary cover (order 100–200 m) that is above the seismicity. The San Bernardino region (C), which includes both the San Bernardino valley and mountains, has low average heat flow, moderate area and moderate average sedimentary cover. Finally, the Ventura Basin region (D) has low heat flow, moderate area and thick sedimentary cover (order 10 km) that extends nearly to the bottom of the seismogenic zone.

The left-hand panels in Fig. 4 show magnitude versus time of earthquakes with $2.0 \leq M \leq 6.0$ that occurred in each region between 1984 and 2002. We consider events to be aftershocks of moderate main shocks with $4.0 \leq M \leq 6.0$ if they occur in the subsequent 50 d within a circular region that scales with the magnitude of the main shock and in the magnitude range between that of the main shock and 2 units below. The use of circular regions to represent aftershock areas is appropriate for moderate main shocks with $M \leq 6.0$, which generally do not saturate the seismogenic

<table>
<thead>
<tr>
<th>Region</th>
<th>Area (km$^2$)</th>
<th>K/L</th>
<th>$\chi$</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10428</td>
<td>1.8</td>
<td>0.70, 0.66</td>
<td>0.42, 0.51</td>
</tr>
<tr>
<td>B</td>
<td>7274</td>
<td>4.3</td>
<td>0.85$^\circ$, 0.82</td>
<td>0.18, 0.21</td>
</tr>
<tr>
<td>C</td>
<td>5548</td>
<td>2.9</td>
<td>0.79, 0.76</td>
<td>0.26, 0.31</td>
</tr>
<tr>
<td>D</td>
<td>5740</td>
<td>2.3</td>
<td>0.75, 0.71</td>
<td>0.33, 0.40</td>
</tr>
<tr>
<td>E</td>
<td>3510</td>
<td>0.54</td>
<td>0.42, 0.37</td>
<td>1.4, 1.7</td>
</tr>
<tr>
<td>Green box in A</td>
<td>233</td>
<td>0.23</td>
<td>0.23, 0.20$^\ast$</td>
<td>3.3, 4.0</td>
</tr>
</tbody>
</table>

Notes: The $\chi$ value denoted by $^\circ$ is an educated guess for the Landers and Hector-Mine area (region B). The other left-hand entries of $\chi$ are based on this value. The $\chi$ value denoted by $^\ast$ is from Lohman & McGuire (2007). All other right-hand entries of $\chi$ and R are based on this value. See text for more explanation.

Table 1. Estimated $\chi$ and R values for the different regions of Fig. 1, based on aftershock productivities $K$ in areas that scale with the main shock magnitude and numerical factor $L = 20$. 

Figure 3. (a) Numbers of aftershocks per day (circles) and fitted curves (lines) based on the discrete Omori–Utsu relation (eq. 1), with $c = 0$, $p = 1$ and $t_0 = 5$ d for the Landers aftershock sequence (blue) and the Superstition Hills aftershock sequence (red). The blue circles and solid lines are for all the events in Fig. 2(a), whereas the plus symbols and dashed line are for the events that occur only inside the polygon area. (b) Relations between productivity and heat flow values for aftershocks of the Landers and Superstition Hills main shocks. The horizontal error bars represent 95 per cent confidence intervals of the heat flow data. The vertical error bars represent the standard deviations of the productivities.
Figure 4. Results associated with analysis of stacked aftershock sequences in the five regions of Fig. 1. Each row gives results for the region with the indicated name. The left-hand panels show magnitude versus time of aftershocks with $2.0 \leq M \leq 6.0$. The middle panels display the number of events per 0.5 d for all aftershock sequences in a given region (coloured curves with plus symbols) and the corresponding stacked aftershock sequences (solid black circles). The number of employed sequences in each region is shown at the top of the panels. The right-hand panels show the stacked number of aftershocks per 0.5 d (circles) and fitted curves (lines) based on the integral Omori–Utsu relation (eq. 5b), with $t_0 = 5$ d. The fitted $K$ value of the aftershocks productivity in each region is indicated at the bottom of the panels.
zone (e.g. Ben-Zion 2008), and it allows us to perform a systematic analysis with aftershock areas that scale with the sizes of the main shocks.

We estimate the rupture radius \( r \) of each main shock using the following scaling relation (e.g. Ben-Zion 2003) for a circular crack with a uniform stress drop:

\[
r = (P_0/c \Delta \varepsilon_s)^{1/2},
\]

where \( P_0 \) is the scalar seismic potency of the event, \( \Delta \varepsilon_s \) is the static strain drop assumed here to be \( 10^{-4} \) and \( c = 2.283 \). The \( P_0 \) value of each event is calculated from the empirical potency-magnitude scaling relation of Ben-Zion & Zhu (2002):

\[
\log_{10} P_0 = 0.06 M_L^2 + 0.98 M_L - 4.87,
\]

with \( M_L \) being the local magnitude of the event and \( P_0 \) is in km\(^2\) times cm. Fig. 6 of Ben-Zion (2008) shows \( r \) values as a function of \( M_L \), calculated with the same procedure. Since aftershocks extend beyond the main shock rupture area, we multiply the estimated rupture radius by a factor \( L \) that varies between 1 and 50 and use the result to be the radius of a circular aftershock zone. In the analysis below, we divide the productivities of the stacked aftershock sequences by the assumed \( L \) factor. The results show that \( L = 20 \) provides normalized productivities that remain similar when larger \( L \) values are used (Fig. 5). Dynamically triggered aftershocks may extend to larger distances (e.g. Felzer & Brodsky 2006; Hill & Prejean 2007), but the considered areas with \( L = 20 \) include the bulk of classical aftershocks and reduce mixing aftershocks with background seismicity. Since the stacked data have more events than individual aftershock sequences, we use here discrete time intervals of 0.5 d (half the size that was used in Section 2.1). This procedure produces 28–196 aftershock sequences in each of the five regions (middle panels in Fig. 4). The stacked data associated with all sequences in a given region (solid circles in the middle panels) highlight the common characteristics of the aftershock properties in the region.

We fit the stacked aftershock sequences with the Omori–Utsu law, using the same method as discussed in Section 2.1 with \( c = 0, p = 1 \) and \( t_0 = 5 \). The fitted results are shown in the right-hand panels of Fig. 4. The regional aftershock productivities decrease in the following order: the Landers and Hector-Mine area; the San Bernardino region; the Ventura Basin; the Imperial Valley and the Coso region. This is consistent with the facts that the largest earthquake in the analysed 1984–2002 catalogue (the M 7.3 1992 Landers earthquake) is in region B, and the smallest main shocks are in regions A and E (although the relative short duration of the catalogue compared with recurrence times of large earthquakes precludes drawing conclusions from this alone).

Fig. 6(a) shows the stacked aftershock sequences and lines corresponding to the Omori–Utsu curves in discrete form for the five different regions, using the parameters associated with the fits obtained in Fig. 4. The relations between the estimated heat flow values and normalized aftershock productivities (\( K \) values from Fig. 4 divided by \( L = 20 \)) for the five aftershock regions are shown in Fig. 6(b). The observed correlations between aftershock productivity, heat flow and seismogenic zone that includes sedimentary rocks, are compatible overall with the damage model predictions of Ben-Zion & Lyakhovsky (2006).

### 3 DISCUSSION

The results from both the analysis of large aftershock sequences in two different regions and analysis of many stacked sequences in five different regions indicate clearly (Figs 3b and 6b) that there is a general inverse relationship between the mean heat flow and aftershocks productivity in a region. These results are compatible with the previous observational findings of Mogi (1967), Kisslinger & Jones (1991) and other studies summarized by Utsu et al. (1995). Among the five examined regions, the Landers and Hector-Mine area, the Imperial Valley and the Coso region have seismicity that is generally below the sediments. The corresponding points for these three regions in the productivity versus heat flow plot of Fig. 6(b) fall nearly on a straight line. This indicates that for seismogenic zones with similar lithology (e.g. crystalline rocks), the aftershock productivity is primarily determined by the heat flow (or more correctly the temperature field). The Landers and Hector-Mine area, the San Bernardino region and the Ventura Basin have similar heat flow levels but considerably different thickness of sedimentary covers. The aftershock productivities are lower in the latter two regions, where the sedimentary covers extend partially (in the San Bernardino region) or almost fully (in the Ventura Basin) into the seismogenic zone. The above observations are consistent with the expectations based on the damage rheology model of Ben-Zion & Lyakhovsky (2006).

As mentioned in Section 1, the seismic coupling coefficient \( \chi \) is expected to be inversely proportional to the material parameter \( R \) (eq. 2) and to be directly proportional to the aftershocks productivity in a region (eqs 3a and 3b). Since the maximum event size and seismic hazard in a region increase with increasing seismic coupling, it is important to develop methodologies for estimating the \( \chi \) values in different regions. Based on the damage rheology model, the largest value of \( \chi \) in the study area is expected to be in the Landers and Hector-Mine region, and the lowest values in the Imperial Valley and the Coso region. This is consistent with the facts that the largest earthquake in the analysed 1984–2002 catalogue (the M 7.3 1992 Landers earthquake) is in region B, and the smallest main shocks are in regions A and E (although the relative short duration of the catalogue compared with recurrence times of large earthquakes precludes drawing conclusions from this alone).

A direct estimate of the \( \chi \) value that characterizes a given fault system requires a comparison between the seismic release and geodetically observed deformation in the region. Lohman & McGuire (2007) compared geodetic data with earthquake properties within the Salton Trough area of southern California (green rectangle in region A of Fig. 1) and estimated the \( \chi \) value for their study area to be 0.2. Their results on earthquake sequences in the area are consistent with the expected correlations of Ben-Zion & Lyakhovsky (2006). Most seismically active regions (especially outside the US and Japan) do not have geodetic networks that operate over sufficiently long time- (e.g. >10 yr) and length-scales (e.g. 10 km) to allow direct estimates of the seismic coupling coefficient. An ability to estimate \( \chi \) and the material parameter \( R \), through analysis of aftershocks productivity of the type done in this paper, can therefore be useful for seismic hazard assessment and various other studies. This can be done as follows. From eqs (2) and (3b), the ratios of the \( R \) and \( \chi \) values in different regions can be written as

\[
R_2/R_1 \approx K_1/K_2, \\
\chi_2/\chi_1 \approx (1 + R_1)/(1 + R_2).
\]
Combining these two relations leads to

$$\chi_z \approx \frac{1}{(K_1/K_2)(1/\chi_1 - 1) + 1}. \quad (9)$$

In the absence of any direct information on the seismic coupling (or $R$), one may assume a $\chi$ value for one region and then derive corresponding relative values using the observed ratios of the aftershock productivities for other regions. As mentioned, based on the damage model results of Ben-Zion & Lyakhovsky (2006), we expect the Landers and Hector-Mine area to have relatively low $R$ value and relatively high $\chi$ value. Assuming that area B of Fig. 1 is characterized by $\chi = 0.85$, we can derive from the observed normalized $K$ values of Fig. 6 and eqs (9) and (8) the $\chi$ and $R$ values for all five study areas. The results are summarized in the left-hand side entries of $\chi$ and $R$ in Table 1. In our case, we can also use the estimated value $\chi = 0.2$ of Lohman & McGuire (2007), along with estimated $K$ value for the same area, and then derive from those results the $\chi$ and $R$ values for all other regions. Applying the

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Figure 5. (a) Estimated productivity values of stacked aftershocks versus the $L$ factor that scales the size of the considered aftershock areas for the different regions of Fig. 1. (b) Estimated productivity values for the different regions normalized by the used $L$ factor. The results are stable for $15 < L < 50$. The value $L = 20$ (dashed vertical line) is used for the results in Figs 4 and 6 and Table 1.
Stacking approach of Section 2.2 to the green box in area A of Fig. 1, we obtain a normalized $K = 0.23$ from data associated with that region. The $\chi$ and $R$ values for the different regions based on $\chi = 0.2$ and $K = 0.23$ for the green box in Fig. 1 are summarized in the right-hand side entries of $\chi$ and $R$ in Table 1. The obtained $\chi = 0.82$ for area B of Fig. 1 is close to our assumed 0.85 value.

Observations of earthquake sequences can be done relatively easily. The damage model results of Ben-Zion & Lyakhovsky (2006) and foregoing procedure provide an effective tool for deriving, from properties of aftershock sequences, fundamental parameters that characterize the rheology and seismic potential of a region. Our analysis employs a simple procedure associated with pre-selection of regions, based on supplementary knowledge on clear differences in assumed controlling regional conditions, and mapping the differences of aftershocks rates in given magnitude ranges to the productivity parameter. More sophisticated analyses (e.g. using smoothly varying adjustable selection of regions, assimilation of more detailed information on heat flow, lithology and other relevant information, joint inversion of multiple parameters, etc.) may lead to better estimates of the seismic coupling coefficient. This is left for future work.

ACKNOWLEDGMENTS

We thank David Harte and Ray Brownrigg for providing the SSLib (Statistical Seismology Library, available at http://homepages.paradise.net.nz/david.harte/SSLib/) package that was used for data analysis. The manuscript benefited from comments by Zhigang Peng, two anonymous referees and editor Massimo Coco. The study was supported by the Southern California Earthquake Center (based on NSF Cooperative Agreement EAR-0106924 and USGS Cooperative Agreement 02HQAG0008). We dedicate the paper to Ma Li, an ardent researcher of earthquake phenomena, who passed away untimely on 2008 August 6.

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