Quasi-static and Quasi-dynamic Modeling of Earthquake Failure at Intermediate Scales

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Abstract—We present a model for earthquake failure at intermediate scales (space: 100 m–100 km, time: 100 m/shear - 1000’s of years). The model consists of a segmented strike–slip fault embedded in a 3-D elastic solid as in the framework of BEN-ZION and RICE (1993). The model dynamics is governed by realistic boundary conditions consisting of constant velocity motion of the regions around the fault, static/kinetic friction laws with possible gradual healing, and stress transfer based on the solution of CHINNERY (1963) for static dislocations in an elastic half-space. As a new ingredient, we approximate the dynamic rupture on a continuous time scale using a finite stress propagation velocity (quasi–dynamic model) instead of instantaneous stress transfer (quasi–static model). We compare the quasi–dynamic model with the quasi–static version and its mean field approximation, and discuss the conditions for the occurrence of frequency-size statistics of the Gutenberg–Richter type, the characteristic earthquake type, and the possibility of a spontaneous mode switching from one distribution to the other. We find that the ability of the system to undergo a spontaneous mode switching depends on the range of stress transfer interaction, the cell size, and the level of strength heterogeneities. We also introduce time-dependent log(τ) healing and show that the results can be interpreted in the phase diagram framework. To have a flexible computational environment, we have implemented the model in a modular C++ class library.

Key words: Earthquakes, fault models, dynamic properties, seismicity.

1. Introduction

In recent years various models of earthquake sequences have been developed. Although the verification of such models is difficult due to limited data, considerable progress has been made with respect to the generation and understanding of various seismicity patterns. An important goal is the development of conceptual models, which are simple enough to allow some analytical understanding of the relevant processes, but also produce seismic dynamics that is to some degree realistic. Such models have in general a set of tuning parameters that should not be too large. For

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some parameters empirical values are available, while others can be used to tune the model dynamics towards an expected behavior. The model of Ben-Zion and Rice (1993) of a fault in a 3-D elastic half-space appears to meet these criteria. Using this model, several observed frequency-size and temporal statistics could be explained in terms of structural properties of a given fault.

The most striking feature of seismicity is the frequency-size distribution, which follows in regional domains the Gutenberg-Richter power-law relation. On an individual fault, the situation can be different: although power-law scaling is always observed for a certain range of magnitudes, deviations are found for large magnitudes. On one hand, the Gutenberg-Richter scaling can break down; that is, large events may be suppressed. On the other hand, the large magnitude range can be dominated by a frequently occurring “characteristic” earthquake. Using a mean field approximation of the model of Ben-Zion and Rice (1993), it has been shown that different frequency-size distributions can result from different values of a dynamic weakening coefficient controlling the brittle properties of the fault, and a conservation parameter that determines the amount of stress transfer remaining on the fault (Dahmen et al. 1998; Fisher et al. 1997). For certain parameter values, the system can spontaneously switch from one state to the other. In Hainzl and Zöller (2001) a cellular automaton model of Burridge-Knopoff type (Burridge and Knopoff, 1967; Hainzl et al., 1999) has been analyzed with the result that the degree of disorder and stress concentration can tune the frequency-size statistics. The question whether or not a small earthquake can grow into a system wide event is connected to the degree of smoothness (or roughness) of the stress field. In Ben-Zion (1996) and Ben-Zion et al. (2003) it is argued that the smoothing of long wavelength components of stress can be interpreted as a development of long-range correlations or, in terms of critical phenomena, as an increase of the spatial correlation length, which prepares the fault for a large event (Sornette and Sammis, 1995; Zöller et al. 2001; Zöller and Hainzl, 2002). Consequently, all processes that influence the smoothness of the stress field, e.g., the type of stress transfer during a rupture and quenched heterogeneities, are relevant for the dynamics.

Most existing models for earthquake failure are governed by instantaneous stress transfer (quasi-static) during the rupture process, and in part by unrealistic stress transfer functions like nearest-neighbor interaction or homogeneous stress transfer independent of the position on the fault and the rupture dimension (see Ben-Zion, 2001, for a recent summary of fully dynamic earthquake models). As in the present work, the fault is usually discretized into uniform cells. To account for a more realistic rupture process, the quasi-static approach is extended here to a quasi-dynamic one by introducing a finite communication speed (Gabrielov et al., 1994). Using Chinnery’s solution for a strike-slip fault in a 3-D elastic half-space (Chinnery, 1963), realistic boundary conditions, dynamic weakening, and optionally a gradual time-dependent healing, provides a realistic, but still relatively simple earthquake model on the intra- and the inter-event time scale.
The results indicate that the quasi-dynamic model favors for certain parameter values the occurrence of large events. In particular, the truncation of the Gutenberg-Richter law in the quasi-static approach vanishes. We further show that the ability of a system to undergo a mode switching between the Gutenberg-Richter law and a characteristic earthquake distribution depends on the range of the stress transfer interaction, the cell size, and the strength heterogeneities. If \( \log(t) \) healing is included, the time-dependence can be absorbed in an effective dynamic friction and stress loss in the model without healing.

2. Model Framework

We assume conceptually a hierarchical model that consists of three hierarchies: the system (top level) as a whole contains a set of faults (middle level); each fault is composed of an array of cells (bottom level). The system is embedded in a three-dimensional elastic half-space. At the system level, the interaction between the faults is accounted for. The fault controls the interaction of the cells during an event, while the accumulation and the release of stress takes place on the cell level.

At the present state of model development, only a single rectangular fault is considered. Unless stated otherwise, a fault of 70-km length and 17.5-km depth is covered by a computational grid, divided into 128 \( \times \) 32 uniform cells, where deformational processes are calculated. As discussed in Ben-Zion and Rice (1993), this geometry corresponds approximately to the San Andreas fault near Parkfield, CA. Tectonic loading is imposed by a motion with constant velocity \( v_{pl} = 35 \) mm/year of the regions around the computational grid. The space-dependent loading rate provides realistic boundary conditions. Using the static stress transfer function \( K(i, j; k, l) \) from Chinnery (1963), the tectonic loading for each cell \((i, j)\) is a linear function of time \( t \) and plate velocity \( v_{pl} \):

\[
\Delta \tau(i, j; t) = (-v_{pl} \cdot t) \cdot \sum_{k, l \in \text{fault}} K(i, j; k, l),
\]

where the minus sign stems from the fact that forward (right-lateral) slip of regions around a locked fault segment is equivalent to back (left-lateral) slip of the locked fault segment. The grid of cells is governed by a static/kinetic friction law, i.e., a cell slips initially if the static friction \( \tau_s \) is exceeded. The threshold decreases instantaneously to the dynamic friction \( \tau_d < \tau_s \) and remains there until the earthquake is terminated (model without healing during events). The stress itself drops to the arrest stress \( \tau_a < \tau_d \). This process of dynamic weakening can be parameterized by the dynamic overshoot coefficient \( D = (\tau_s - \tau_d)/(\tau_s - \tau_d) \). Following the description in Dahmen et al. (1998), we set \( \tau_s = 1 \) and \( \tau_a = 0 \) and use the dynamic weakening coefficient \( e = (\tau_s - \tau_d)/\tau_s = 1 - \tau_d \) to connect static/kinetic friction and arrest stress. Consequently, the dynamic weakening parameter
The case ε = 0 represents the unrealistic case of static friction equals kinetic friction, or instantaneous healing. To account for heterogeneities in the brittle properties, some uniformly distributed noise \( \frac{C_0}{C_0} \) is added to \( s_d \).

As yet, the frictional properties of the fault are strongly simplified; the strength envelope, which describes the change of the friction coefficient by means of static and dynamic frictions, is a piecewise constant function (see solid line in Fig. 1). It is, however, known from laboratory experiments that ruptures are governed by more complicated friction laws (Dieterich, 1972; 1978). Most of the experimental results are described by the rate- and state-dependent constitutive law (Ruina, 1983; Scholz, 1998). Within this law, the response to a sudden change of the sliding velocity is an instantaneous effect on the friction coefficient followed by a more gradual evolution.

As a first step towards a more realistic friction law, we introduce a gradual time-dependent healing of the form (see dashed line in Fig. 1)

\[
\tau_d(t) = \tau_{d,0} + C \log(1 + (t/t_0)),
\]

where \( \tau_{d,0} \) is the initial dynamic friction coefficient, \( t_0 \) is a reference time and \( C \) is a free parameter. The healing begins after the instantaneous drop of the friction from \( \tau_s \) to \( \tau_{d,0} \). For our numerical simulations, we parameterize the healing with two parameters \( p_1 \) and \( t_{100} \), instead of \( C \) and \( t_0 \). The value of \( p_1 \) gives the fraction of \( \tau_s - \tau_{d,0} \) that has healed after a time of \( 1 \text{ km}/v_s \), and the value \( t_{100} \) gives the time interval required for complete healing, \( \tau_d(t_{100}) = \tau_s \).
The stress transfer during an earthquake is calculated by means of the three-dimensional solution of Chinnery (1963) for static dislocations on rectangular patches in an elastic Poisson solid with rigidity $\mu = 30\text{GPa}$. In particular, we approximate the $3+1$ dimensional space-time stress transfer by

$$\Delta \tau(i, j; t) = (1 - \gamma) \cdot \sum_{k, l \in \text{fault}} K(i, j; k, l) \Delta u(k, l; t - r/v_s),$$

(3)

where $\Delta u$ is the slip, $r$ is the spatial distance between the cells $(i, j)$ and $(k, l)$, and $v_s$ is the shear-wave velocity. The factor $1 - \gamma \in (0; 1]$ corresponds to a given ratio of rigidities governing during instabilities the self-stiffness of a slipping cell (diagonal elements of the stiffness matrix) and the stress transfer to the surrounding domain (off-diagonal elements). A ratio smaller than 1 represents stress loss during rapid slip on the fault to internal free surfaces in the solid associated with porosity and cracks. We refer to $\gamma$ and $1 - \gamma$ the stress loss parameter and stress conservation parameter, respectively. The slip $\Delta u(i, j)$ of a cell at a position $(i, j)$ is related to the stress drop $\Delta \tau(i, j)$ at the same position through the self-stiffness: $\Delta u(i, j) = \Delta \tau(i, j)/K(i, j; i, j)$.

The size of an earthquake is measured with two quantities: 1. the area, which is proportional to the number of cells that participated in the earthquake failure, and the potency, which is the integral of the slip over the rupture area. This approach extends the model of Ben-Zion and Rice (1993) to a quasi-dynamic procedure with a finite communication speed $v_s$ for stress transfer and a related causal rupture process. In one version of the model, the quasi-dynamic rupture process is calculated on a continuous time scale. This may, however, lead to a numerical explosion during the simulation, for certain parameter values. Therefore, we also study simplifications of the continuous time process, e.g., by discretizing the time scale for the stress transfer during an earthquake.

At the present state, the model is characterized by two separate time scales: the inter-event time scale during which the fault is loaded between two events, and the intra-event time scale defined by the travel time of a shear wave along the fault, where coseismic stress redistribution takes place. During the event, the tectonic loading is neglected. The stress conservation parameter $1 - \gamma$, which controls the amount of stress remaining on the computational grid, is varied between $\gamma \approx 1$ (no stress redistribution) and the conservative case $\gamma = 0$ (stress drop $\tau - \tau_0$ is completely redistributed). In general, $\gamma$ controls the size of the generated earthquakes: for high values of $\gamma$, small amounts of stress are redistributed and the evolution of cascading failure events stops earlier than in corresponding cases of small $\gamma$, for which large earthquakes can develop. In the latter case, numerical problems may occur, because large runaway events that cover the entire fault and have multiple slip episodes of each cell, result in a memory exhaustion on the continuous intra-event time scale. Therefore, we also study a simplification of the quasi-dynamic rupture: the intra-event time scale is discretized into $N$ time intervals. Each slip of a cell is assigned to one of the time intervals. Note that the case $N = 1$ represents the quasi-static model.
used in Ben-Zion and Rice (1993). Although it is mathematically not clear that the limit $N \to \infty$ converges to the continuous time scale, we assume that a value of $N$ exists, which approximates the continuous case with a reasonable accuracy. In our simulations we used $N = 1000$. Given the dimensions of our grid, a stress signal may travel ten times along the fault before the time error becomes comparable to the travel time between neighboring cells. The model simulations are started with a random distribution of initial stress. To account for transient effects in the dynamics, the first 50,000 earthquakes are neglected in each simulation.

### 3. Model Simulations

#### 3.1. Influence of Intra-event Dynamic

Figure 2 gives the frequency-size distribution for different versions of the intra-event time scale. Figure 2(a) shows results calculated with $\gamma = 0.3$ and a realistic value of the dynamic weakening coefficient $\varepsilon = 0.8$ corresponding to a dynamic overshoot coefficient $D = 1.25$ (Madariaga, 1975). Figure 2(b) shows the same calculation for the unrealistic case of $\varepsilon = 0$ and instantaneous healing ($\tau_d = \tau_s$), where the frequency-size statistics is a truncated power law. The results indicate that the continuous time scale (solid line) can be approximated quite well by the discretized time scale with 1000 intervals (dotted line). In the case without large earthquakes, there is no significant difference between the quasi-dynamic model and the quasi-static model (dash-dotted line). In contrast, Figure 2(a) shows a clear fall-off for large events in the quasi-static case. This difference is a stable feature; that is, it is also present for a broad range of parameter values $\varepsilon$ and $\gamma$ that allow large earthquakes to occur. Consequently, the quasi-static approximation seems to
suppress large earthquakes leading to a truncation in the frequency-size statistics. For large earthquakes, the propagation of stress with finite communications speed $v_s$ is obviously a relevant feature.

3.2. The Phase Diagram Framework

In Dahmen et al. (1998) a simplified two parameter mean field version of the quasi-static model of Ben-Zion and Rice (1993) has been analyzed. A “conservation parameter” $c$ (similar to our $1 - \gamma$) has been introduced, and an infinite-range mean-field stress transfer function $G \sim c/N$ ($N =$ total number of cells) was used instead of Chinnery’s solution for the 3-D elastic stress transfer. For this model, a phase diagram spanned by $c$ and the dynamic weakening coefficient $\epsilon$ contains two distinct phases: one phase where the frequency-size distribution follows the Gutenberg-Richter law, and a second phase governed by a spontaneous mode switching between Gutenberg-Richter statistics and the characteristic earthquake distribution. In particular, it is hypothesized that these phases are generic for more realistic stress transfer functions. In the first part of this study, we analyze this claim for various types of interactions. We observe in agreement with Weatherley et al. (2002) that with coarse cell size there is a threshold of the interaction range, below which no mode-switching occurs. However, with small enough cells and strong heterogeneities, mode-switching also occurs for the realistic case of elastic stress transfer. In the second part, we consider the influence of time-dependent $\log(t)$ healing. The results show that the model can be mapped onto the same phase diagram if the net effect of healing is absorbed in effective dynamic threshold and stress loss.

3.2.1. The phase diagram for the quasi-dynamic and elastic model.

Figure 3 shows frequency-size statistics for a fixed dynamic weakening coefficient $\epsilon = 0.6$ as a function of the stress loss factor $\gamma$. High values of $\gamma$ prevent the occurrence of large earthquakes, in that stress that is needed to bring cells to failure is
lost and earthquakes are stopped earlier. Furthermore, the figure shows a transition from a truncated Gutenberg-Richter distribution ($\gamma = 0.2$) to a characteristic earthquake distribution ($\gamma = 0$). For $\gamma \approx 0.125$, the system bifurcates to the two regimes.

Our calculations with the quasi-dynamic elastic model lead (Fig. 4) to a large Gutenberg-Richter regime with small earthquake sizes, and a regime governed by a characteristic earthquake distribution with some short time fluctuations. Note that the factor $c$ in Dahmen et al. (1998) is now replaced by $1 - \gamma$. The characteristic earthquake regime is smaller than the mode switching phase in Dahmen et al. (1998), because in the elastic model a small fraction of the stress drop is lost even for $\gamma = 0$ due to the finite fault size. We have estimated that on average 80% of the stress drop remains on the fault for the employed 70 km and 17.5 km dimensions with $\gamma = 0$. Consequently, the case $\gamma = 0$ for the elastic model corresponds to $c = 0.8$ of the mean field model. This is approximately confirmed by a comparison of Figure 4 with the phase diagram in Dahmen et al. (1998).

3.2.2. Mode-switching between Gutenberg-Richter and characteristic earthquake statistics.

To address the question of a possible mode switching, long simulations are required. Therefore, we use for the following analysis a fault of 10.9-km length and 2.7-km depth divided into $20 \times 5$ uniform cells. Figure 5 shows a typical earthquake sequence (rupture area vs. time) from a catalog representing the characteristic earthquake regime. The quasi-periodically occurring characteristic earthquakes are...
interrupted for short time by clusters of small and intermediate earthquakes. It seems as if the system attempts to switch into the other regime and flips back after a short time, e.g., for $t = 361$ in Figure 5. In Figure 6 we show that the ability of the system

**Figure 5**
Earthquake area as a function of time for the quasi-static model ($N = 20 \times 5$, $\epsilon = 0.6$, $\gamma = 0.03$, no healing during events). The sequence follows the characteristic earthquake distribution, although the system attempts to switch to a Gutenberg-Richter phase, e.g., at $t \approx 361$.

**Figure 6**
Earthquake area as a function of time for the quasi-static model ($N = 20 \times 5$, $\epsilon = 0.6$, no healing during events) and different types of interaction $\sim r^{-x}$. The stress loss factor $\gamma$ has been adjusted in order to bring the system into the transition regime between Gutenberg-Richter and characteristic earthquake distributions. The case $x = 0$ represents the mean field approximation [DAHMEN et al., 1998], while $x = 3$ corresponds to the elastic solution of CHINNERY [1963].
to undergo a mode-switching depends significantly on the kind of stress transfer. We consider interactions between the limit cases of mean field interaction \( \Delta \tau / C r^0 \), where \( r \) is the distance from the slipped cell) and the elastic solution of Chinnery (1963), \( \Delta \tau / C r^3 \). These simulations are performed on a 20 \( \times \) 5 grid with dynamic weakening \( \varepsilon = 0.6 \). Since mode-switching most likely occurs in the transition regime between Gutenberg-Richter and characteristic event distribution, we adjust the stress loss factor \( \gamma \) to be in this part of the parameter space. Each plot in Figure 6 gives results for 10,000 simulated events with stress transfer \( \Delta \tau / C r^x \) and a certain value of the exponent \( x \). The plots show clearly that in simulations with coarse cells, the mode-switching in the mean field approximation degenerates with increasing exponent \( x \) to the short-term fluctuations described above. We also performed similar calculations for a uniform stress transfer on a compact support (circle with radius \( R \)), \( \Delta \tau = \tau_0 \cdot \Theta(r - R) \). Figure 7 shows earthquake sequences as in Figure 6 for this kind of interaction. Again, the mode-switching behavior, which is clearly visible for large radii, degenerates to short-term fluctuations for small radii. Although we did not observe mode-switching of the type described in Dahmen et al. (1998) in earthquake catalogs up to 20,000,000 events, it cannot be ruled out that this behavior may occur with a very large persistence time in one mode.
In sum, we have found that the ability of the system to undergo a mode-switching from the Gutenberg-Richter distribution to the characteristic earthquake distribution with coarse cell size depends on the range of the stress transfer interaction. Similar results were found in (Weatherley et al., 2002). While mode-switching occurs quite frequently for the infinite range mean field interaction, this behavior could not be observed for the more realistic long-range 3-D elastic interaction, although the system attempts to switch from time to time. In this case, the transition from the Gutenberg-Richter regime to the characteristic earthquake regime with coarse grid is continuous; that is, a decrease of $\gamma$ leads to the growth of the average magnitude. The limit case ($\gamma \rightarrow 0$) is the characteristic earthquake distribution with quasi-periodically occurring earthquakes that rupture the entire fault. It is also visible that in contrast to the mean field approximation, intermediate size earthquakes are also present in the characteristic earthquake regime. These observations also hold for the quasi-dynamic model.

Until now we have kept the cell size fixed and have varied only the range of stress transfer interaction. It is, however, possible that smaller numerical cells will produce mode-switching, since such will lead to larger stress concentrations near failure areas and larger stress fluctuations. To investigate this hypothesis we have to consider a grid with numerous cells. Note that a change of the fault dimension in terms of total length and depth produces the same earthquake sequences with a rescaled time axis. Such an analysis requires very time-consuming numerical simulations due to two reasons: 1. A new phase diagram corresponding to Figure 4 must be calculated; 2. the transition between both phases must be scanned with a very high resolution in order to extract the range of parameters within which mode-switching is expected in reasonable short simulations. In this context it is important to note that the persistence time in a certain mode depends not only sensitively on $\gamma$ and $\epsilon$, but also on the number of cells $N$. In the mean field model, the persistence time increases according to $\exp(N)$ (Dahmen et al., 1998).

In Figure 8 we show two simulations using the elastic stress transfer on a $128 \times 50$ grid and values for $\tau_d$ which are uniformly distributed between 0.15 and 0.35 ($\langle \epsilon \rangle = 1 - \langle \tau_d \rangle = 0.75$), resulting in a more heterogeneous distribution of the brittle properties. The two plots refer to different values of the stress loss $\gamma$. The results clearly show a tendency towards mode-switching behavior, e.g., for $t \in [6.4; 9.0]$ in Figure 8(a). We can thus conclude that mode-switching also depends on the degree of heterogeneity in a system, determined by the cell size and the distribution of the brittle properties. While in the constant mean-field interaction, a large fraction of the parameter-space is governed by mode-switching. This phenomenon seems to occur with the more realistic $1/r^3$ elastic interaction in considerably smaller ranges of parameters. However, the region in parameter space producing mode-switching in the case of $1/r^3$ interaction is expected to increase with further decreasing of cell size and increasing of heterogeneities.
3.2.3 Time-dependent healing.

The introduction of time-dependent healing according to Eq. (2) replaces the piecewise constant strength envelope by a function consisting of a strength drop followed by a log($t$) increase of the strength. This kind of healing requires three parameters: the value of $p_1$ (healing rate after 1 km/$\nu_s$ stress propagation), the value $t_{100}$ (time when healing is complete) and the values $\tau_{d,0}$ to which the strength drops when a cells begins to slide. For the present analysis we keep $t_{100}$ fixed ($t_{100} = 100$ km/$\nu_s$) and tune the shape of the healing curve by means of $p_1$ and $\tau_{d,0}$. The stress loss $\gamma$ is an additional parameter independent of the healing.

It is a reasonable assumption that in the case of healing, effective values of $\tau_{d,0}$ and $\gamma$ exist that correspond to $\tau_d$ in the case without healing. To investigate this hypothesis we determine for different values of $p_1$ and $\gamma$ the dynamic threshold $\tau_{d,0}^*$, where the transition between the Gutenberg-Richter law and the characteristic earthquake distribution occurs. The value $\tau_{d,0}^*$ as a function of $p_1$ and $\gamma$ is then compared with the corresponding dynamic threshold $\tau_{d,0}^*$ in the system without healing.

Figure 9 shows for the $20 \times 5$ grid the dependence of $\tau_{d,0}^*$ on $p_1$ for different values of $\gamma$ (Fig. 9(a)) and the dependence of $\tau_{d,0}^*$ on $\gamma$ for different values of $p_1$ (Fig. 9(b)). The curves for the model with healing ($p_1 > 0$) are normalized to the model without healing ($p_1 = 0$). In particular, the vertical axis gives $\tau_{d,0}^*(p_1)/\tau_{d,0}^*(p_1 = 0)$. Note that $\tau_{d,0}^*(p_1 = 0) = \tau_d$ describes the model without healing. In Figure 9(b) the curves are normalized to the horizontal line $\tau_{d,0}^*(p_1)/\tau_{d,0}^*(p_1 = 0) \equiv 1$. Figure 9(a) shows clearly a systematic decrease of $\tau_{d,0}^*$ for growing values of $p_1$. Moreover, $\tau_{d,0}^*$ also decreases as a function of $p_1$, although this decrease is less significant. The dependence of $\tau_{d,0}^*$ on $p_1$ and $\gamma$ can be described by the formula

$$\tau_{d,0}^*(p_1, \gamma) = \tau_d \cdot (1 - s(p_1, \gamma)), \quad \text{(4)}$$
where \( s(p_1, \gamma) \) is given by the curves in Figure 9. This rule connects the model with healing \((p_1 > 0)\) and the model without healing \((p_1 = 0)\). Consequently, \( p_1 \) is not an independent parameter like \( \tau_d \) and \( \gamma \). Rather, the model with healing can be obtained
from the model without healing by a shift in the phase diagram according to Eq. (4). The observation that the dependence of \( s(p_1, \gamma) \) on \( p_1 \) is more pronounced than the dependence on \( \gamma \) arises from the origin of \( p_1 \): this parameter defines the rate of healing; in other words, the rate of increase of \( \tau_d \). It is, therefore, reasonable that an effective dynamic threshold (in the model without healing) exists that can to some extent absorb the time-dependence in the model with healing.

In general, it is an interesting task to also map other models systematically onto the present one with respect to the phase diagram. This would, however, require two independent state variables that allow positioning a given catalog in the phase diagram without pre-knowledge of the parameters. This work is left for the future.

3.3. Area-potency Relations

Finally, we investigate the area-potency relation. Figure 10 shows calculations for \( \varepsilon = 0.8 \) and \( \gamma = 0.0 \) and \( \gamma = 0.3 \), for both the quasi-dynamic and the quasi-static models. The change of slope for the transition from small to large earthquakes is in agreement with the quasi-static simulations in BEN-ZION and RICE (1993) and the data analysis in BEN-ZION and ZHU (2002). The slope for the large earthquakes is close to the exponent in the classical crack relation \( a \sim p^{2/3} \) (KANAMORI and ANDERSON, 1975).

4. Conclusions

We have developed a model for earthquake failure at intermediate scales that is based on the framework of BEN-ZION and RICE (1993). The model is implemented in a flexible C++ class library and therefore allows the incorporation of additional processes and structural fault properties in a straightforward manner. In this study we extended the model of BEN-ZION and RICE (1993) by two important features. First, the stress propagation is modeled on a continuous intra-event time scale using a finite communication speed. This quasi-dynamic model results in more realistic behavior of the stress propagation. Second, we have introduced a gradual time-dependent healing after a cell has slipped. This mechanism modifies the frictional properties from the simple static/kinetic friction towards the rate- and state-dependent constitutive law.

The results indicate that the finite stress propagation velocity is a relevant feature even for the frequency-size statistics. The quasi-dynamic model produces larger events compared to the quasi-static model in which the occurrence of large events is suppressed. This is in agreement with the fully dynamic 2-D calculations of BEN-ZION and RICE (1997). Future activities with the quasi-dynamic version of the model include analysis of slip histories and possible calculations of synthetic seismograms. In another research direction we examined the ability of the model to undergo a spontaneous mode-switching between the Gutenberg-Richter law and the charac-
teristic earthquake distribution. While the mean field model, which is characterized by a constant stress transfer function, exhibits clear mode-switching behavior, this is not the case in the model with the coarse grid and 3-D elastic stress transfer function. Instead, it is found that in the characteristic earthquake regime, the system attempts to switch into the Gutenberg-Richter regime, but flips back after a very short time. However, if the cell size is decreased and the degree of strength heterogeneity is increased, mode-switching behavior is observed, even for the $1/r^3$ elastic stress transfer function. This is in agreement with similar studies in more complex models (Ben-Zion et al., 1999; Lyakhovsky et al., 2001). Remaining important questions are which mechanisms are responsible for this behavior and whether these mechanisms are relevant for real faults.

The incorporation of time-dependent healing leads to a modification of the phase diagram shown in Figure 4. As we have shown, the frequency-size event distribution for a model with a log($t$) healing can be reproduced by a model without healing and with different values of $\gamma$ and $\tau_d$. It is desirable to find two independent state variables that will allow quantification of a catalog and assignment of unique values of $\gamma$ and $\tau_d$. With such state variables one could map other model classes onto the present model.

Due to the modular design of our model code, it is easy to include more structural features and mechanisms into the model, e.g., a more refined frictional behavior and heterogeneities in the distribution of the arrest stress. A detailed analysis of statistical properties and seismicity patterns with various model versions and ranges of parameters will result in a deeper understanding of natural seismicity.

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